Decision-making in interconnected multiagent networks: roles of frustration and social commitment

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joint work with

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Network Dynamics in the Social, Economic, and Financial Sciences,

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Outline

1. **Problem:**
   - Government formation dynamics in multiparty democracies
2. **Model:**
   - Network with antagonistic relationships: signed graphs and structural balance
   - Dynamics of opinion forming on signed multiagent networks
   - Computing level of structural unbalance
   - Dynamics of opinion forming in structurally balanced / unbalanced networks
3. **Application:**
   - Government formation process using signed parliamentary networks

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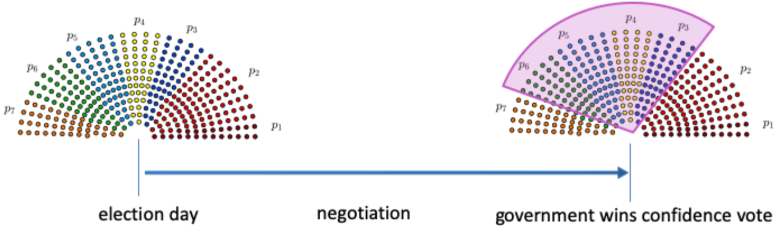
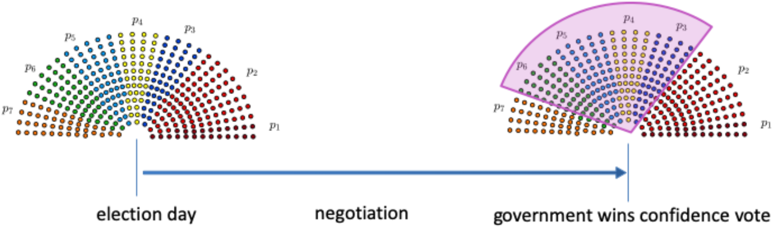
Government formation process

- Government formation in multiparty democracies:



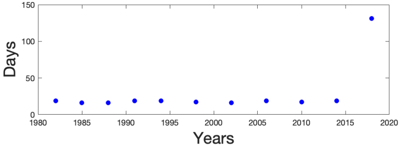
Government formation process

- Government formation in multiparty democracies:



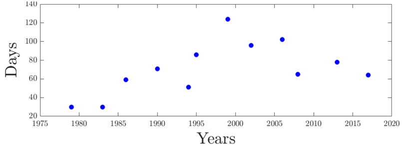
- Sometimes it happens that government negotiation talks take a very long time

Sweden, 2018: 134 days

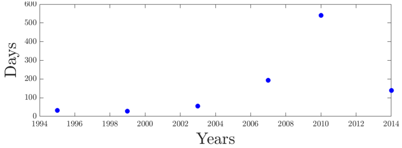


Government formation process

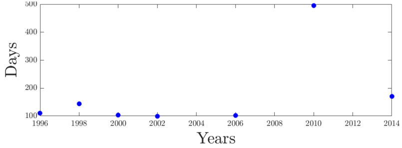
Austria 1999: 124 days



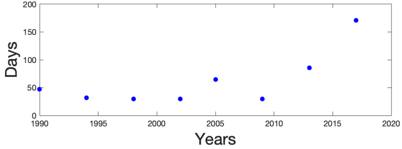
Belgium 2010: 541 days



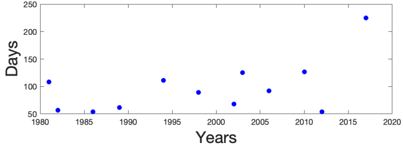
Bosnia-Herzegovina 2010: 495 days



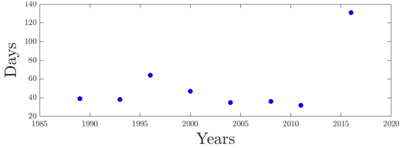
Germany 2017: 171 days



Netherland 2017: 225 days

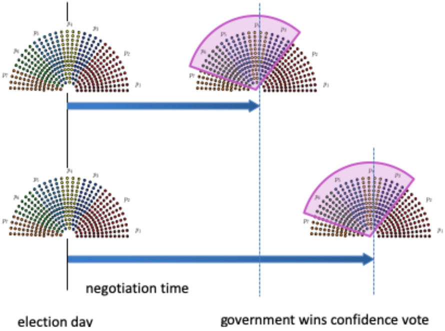


Spain 2015+2016: 365+131 days



Government formation process

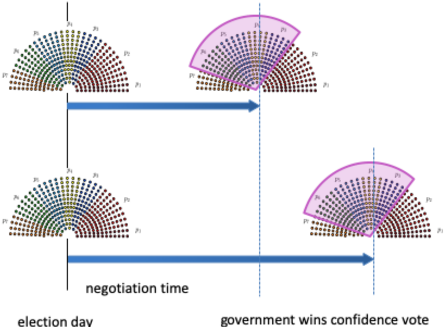
Question: what determines the duration of the negotiation phase?



- in political sciences: game-theoretical models of bargaining processes

Government formation process

Question: what determines the duration of the negotiation phase?



- in political sciences: game-theoretical models of bargaining processes

Tasks: develop a dynamical model that can capture and explain the duration of the negotiation phase

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Collective decision models: examples

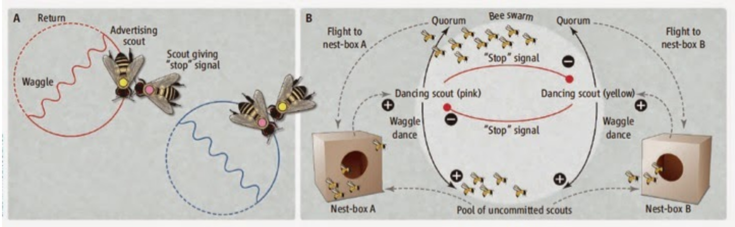
- animal groups as "multiagent systems"

cross or not cross?

migrate?

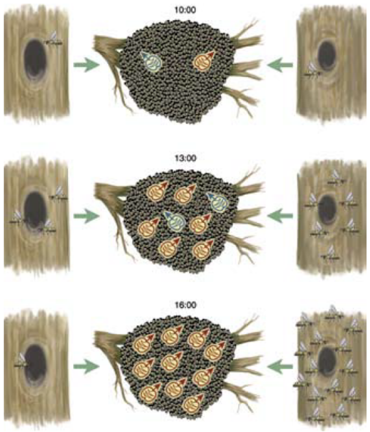
left or right?

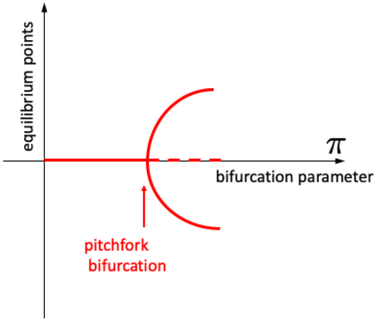
Example: bees deciding to relocate to a new hive



Bees decision making as a bifurcation

Example: bees deciding to relocate to a new hive





Seeley et al. Am. Scientist, 2006

N. Leonard. IFAC World Congress, 2014

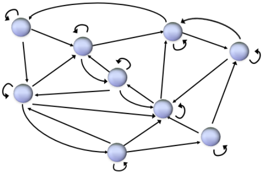
Distributed decision-making model

$$\dot{x} = -\Delta x + \pi A \psi(x)$$

Gray,...,Leonard. Multi-agent decision-making dynamics inspired by honeybees. IEEE Trans Contr. Netw. Sys. 2018

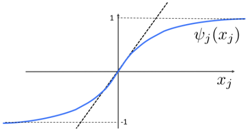
- states: x = vector of decisions
- negative self-loops: "inertia" of the agents

$$\Delta = \mathrm{diag}\big(\delta_1, \ldots, \delta_n\big) \qquad \delta_i > 0$$



- interactions: – graph $\mathcal{G}(A)$
  – influences: sigmoidal functions $\Longrightarrow$ saturations

$$\psi(x) = \begin{bmatrix} \psi_1(x_1) \\ \vdots \\ \psi_n(x_n) \end{bmatrix}, \qquad \frac{\partial \psi_i(x_i)}{\partial x_i} > 0 \qquad \frac{\partial \psi_i(0)}{\partial x_i} = 1$$



Distributed decision-making model (cont'd)

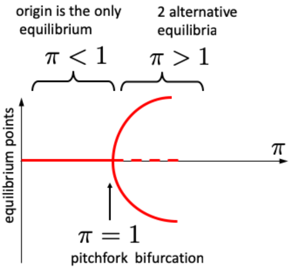
$$\dot{x} = -\Delta x + \pi A \psi(x)$$

- Laplacian assumption:

$$L = \Delta - A \quad \text{is a Laplacian}$$

$$\implies \quad \delta_i = \sum_{j=1}^{n} a_{ij}$$

- Scalar bifurcation parameter: π = social commitment ≥ 0
  Interpretation: π is the amount of interaction among the agents

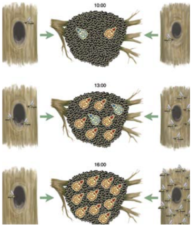


Distributed decision-making model (cont'd)

Applications

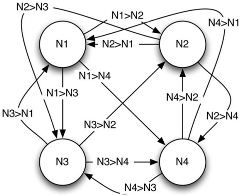
Animal group decision

I.D. Couzin, N. Leonard



Neuronal networks

J. Hopfield



Social Networks

Social networks as (signed) graphs

- **Nodes**: individuals
- **Edges**: interactions
- **Assumption**: agents form their opinion based on the influences of their neighbors
- **Choose:** plausible form of the dynamics

$$\dot{x} = -\Delta x + \pi A \psi(x)$$

Social networks as (signed) graphs

- **Nodes**: individuals
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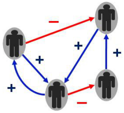
- **Assumption**: agents form their opinion based on the influences of their neighbors
- **Choose:** plausible form of the dynamics

$$\dot{x} = -\Delta x + \pi A \psi(x)$$

- **Extra assumption**: individuals can be "friends" or "enemies"
  - friends (cooperation, alliance, trust): positive edge
  - enemies (competition, rivalry, mistrust): negative edge

$\Longrightarrow A =$ "sociomatrix" is a signed matrix

$$A = (a_{ij}) \quad a_{ij} \lesseqgtr 0$$

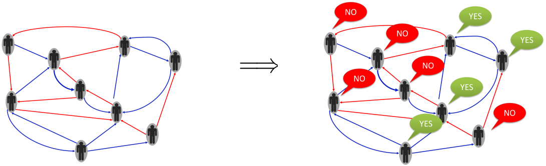


Social networks as (signed) graphs

Tasks: predicting the collective decision of the agents in the model

$$\dot{x} = -\Delta x + \pi A \psi(x)$$

based on knowledge of A when varying π



- Intuitively: agents form their opinion based on the influences of their neighbors
    1. align with opinions of "friends"
    2. oppose opinions of "enemies"

$$\text{sign}(\text{Jacobian}) = \text{sign}(A)$$

Example: consensus

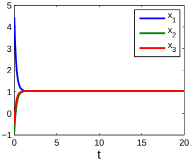
Consensus on nonnegative graphs

$A \geq 0 \implies$ nonnegative Laplacian

$$L = \Delta - A, \qquad \delta_i = \sum_{j=1}^{n} a_{ij}$$

- $-L$ always stable
- $\lambda_1(L) = 0$ always an eigenvalue
- consensus

$$\dot{x} = -Lx$$



Example: consensus

Consensus on nonnegative graphs

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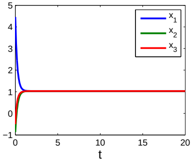


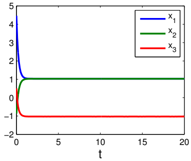
Consensus on signed graphs

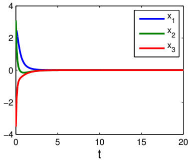
$A \lesseqgtr 0 \implies$ signed Laplacian

$$L_s = \Delta - A, \qquad \delta_i = \sum_{j=1}^{n} |a_{ij}|$$

- $-L_s$ stable or asymptotically stable
- $\lambda_1(L_s) = 0$ may or may not be an eigenvalue
- consensus

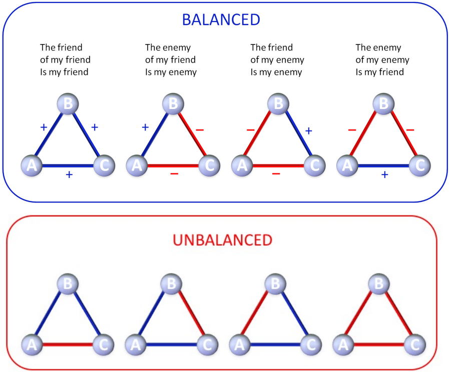
$$\dot{x} = -L_s x$$





Structural balance: the enemy of my enemy...

- in social network theory: certain social relationships (represented as signed graphs) are "more stressful" than others



F. Heider. Attitudes and cognitive organization. J Psychol. 1946

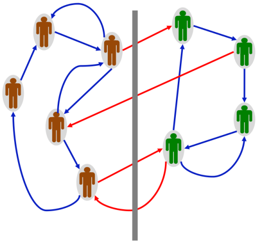
- generalization to any signed graph $\Longrightarrow$ structural balance

Structural balance

Definition A signed graph $\mathcal{G}(A) = \{\mathcal{V}, \mathcal{E}, A\}$ is said structurally balanced if $\exists$ partition of the nodes $\mathcal{V}_1$, $\mathcal{V}_2$, $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$, $\mathcal{V}_1 \cap \mathcal{V}_2 = 0$ such that

- $a_{ij} \geqslant 0 \ \forall \ v_i, \ v_j \in \mathcal{V}_q$,
- $a_{ij} \leqslant 0 \ \forall \ v_i \in \mathcal{V}_q, \ v_j \in \mathcal{V}_r, \ q \neq r$.

It is said *structurally unbalanced* otherwise.



- two individuals on the same side of the cut set are "friends"
- two individuals on different sides of the cut set are "enemies"

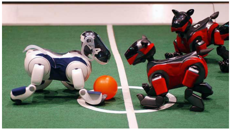
D. Cartwright and F. Harary, Structural balance: a generalization of Heider's Theory, Psychological Review, 1956.

D. Easley and J. Kleinberg, Networks, Crowds, and Markets. Reasoning About a Highly Connected World, Cambridge, 2010

Examples

Two-party parliamentary systems

Team sports



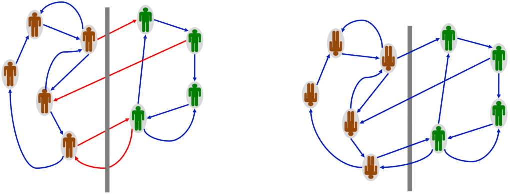
International alliances

Structural balance

Lemma A signed graph $\mathcal{G}(A)$ is structurally balanced *iff* any of the following equivalent conditions holds:

1. all cycles of $\mathcal{G}(A)$ are positive;
2. $\exists$ a diagonal signature matrix $D = \text{diag}(\pm 1)$ such that DAD is nonnegative;
3. the signed Laplacian L_s has $\lambda_1(L_s) = 0$

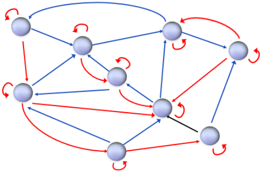
$$DAD = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & 1 \end{bmatrix} \begin{bmatrix} & a_{12} & & a_{14} & a_{15} \\ a_{21} & & & a_{24} & \\ & a_{32} & & & a_{35} \\ & a_{42} & a_{43} & & \\ a_{51} & a_{52} & a_{53} & & \end{bmatrix} \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & 1 \end{bmatrix}$$



Distributed decision-making (signed) model

$$\dot{x} = -\Delta x + \pi A \psi(x)$$

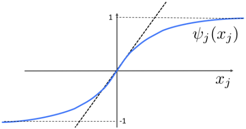
- states: x = vector of decisions
- self-loops: "inertia" of the agents



$$\Delta = \mathrm{diag}(\delta_1, \ldots, \delta_n) \qquad \delta_i = \sum_{j=1}^{n} |a_{ij}|$$

- interactions: – graph $\mathcal{G}(A)$
  – A symmetrizable $\Longrightarrow \lambda_i(A)$ real
  – influences: sigmoidal functions $\Longrightarrow$ saturations

$$\psi(x) = \begin{bmatrix} \psi_1(x_1) \\ \vdots \\ \psi_n(x_n) \end{bmatrix}, \qquad \frac{\partial \psi_i(x_i)}{\partial x_i} > 0 \qquad \frac{\partial \psi_i(0)}{\partial x_i} = 1$$



Opinion forming in signed social networks: model

- "normalized" form:

$$\dot{x} = \Delta\big(-x + \pi \underbrace{H}_{\Delta^{-1}A} \psi(x)\big)$$

- Laplacian assumption:

$$\delta_i = \sum_j |a_{ij}| \qquad \Longrightarrow \qquad 1 = \sum_j |h_{ij}|$$

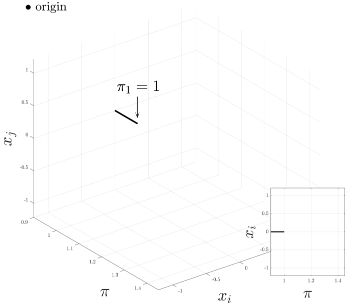
$\Longrightarrow L_s = \Delta - A$ is a signed Laplacian
$\Longrightarrow \mathcal{L}_s = I - H$ is "normalized" signed Laplacian

- Scalar bifurcation parameter: π = social commitment ≥ 0
  Interpretation: π is the amount of interaction among the agents

Opinion forming on structurally balance social networks

$$\dot{x} = \Delta\left(-x + \pi H \psi(x)\right)$$

Cases:	**Meaning**
$\pi < 1$	not enough social commitment: no decision



Bifurcation diagram (x_i, π, x_j)

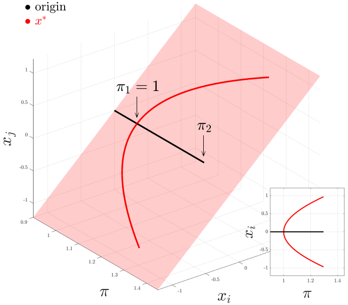
first bifurcation: $\pi_1 = \dfrac{1}{1 - \lambda_1(\mathcal{L}_s)} = 1$

Fontan, Altafini, "Multiequilibria analysis for a class of collective decision-making networked syst.", *IEEE TCNS*, 2018.

Opinion forming on structurally balance social networks

$$\dot{x} = \Delta\left(-x + \pi H \psi(x)\right)$$

Cases:	Meaning
$\pi < 1$	not enough social commitment: no decision
$1 < \pi < \pi_2$	right commitment: two alternative polarized decisions x^*



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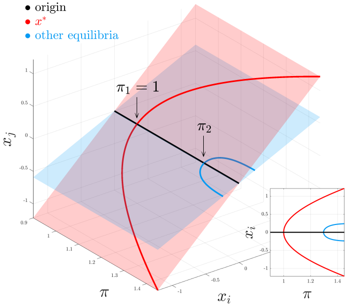
Opinion forming on structurally balance social networks

$$\dot{x} = \Delta\left(-x + \pi H \psi(x)\right)$$

Cases:	**Meaning**
$\pi < 1$	not enough social commitment: no decision
$1 < \pi < \pi_2$	right commitment: two alternative polarized decisions x^*
$\pi > \pi_2$	overcommitment: multiple decisions



Bifurcation diagram (x_i, π, x_j)

second bifurcation: $\pi_2 = \dfrac{1}{1 - \lambda_2(\mathcal{L}_s)}$ $\quad(\lambda_2(\mathcal{L}_s) =$ algebraic connectivity$)$

Fontan, Altafini, "Multiequilibria analysis for a class of collective decision-making networked syst.", *IEEE TCNS*, 2018.

Opinion forming on structurally balance social networks

Theorem: Given the system

$$\dot{x} = \Delta\left(-x + \pi H \psi(x)\right)$$

for which $\exists\ D$ s.t. DHD is nonnegative and irreducible, then:

- for $\pi < \pi_1 = \frac{1}{1-\lambda_1(\mathcal{L}_s)} = 1$, $x^* = 0$ is a globally asymptotically stable equilibrium
- when $\pi = 1$, the system undergoes a pitchfork bifurcation, with x^* becoming unstable and two new locally asymptotically stable equilibria $x^*_{1,2} \in D\mathbb{R}^n_{\pm}$ appear;
- when $\pi = \pi_2 = \frac{1}{1-\lambda_2(\mathcal{L}_s)}$, the system undergoes a second pitchfork bifurcation, and new equilibria appear.

Proof:

- Singularity analysis of bifurcations via Lyapuonv-Schmidt reduction;
- Perron-Frobenius theorem

Opinion forming on structurally balance social networks

Proof: First bifurcation at $\pi = 1$:

- Lyapunov-Schmidt reduction:

$$\Phi(x) = -x + \pi H \psi(x) = 0$$

- at $\pi = 1$ the Jacobian $J = \frac{\partial \Phi(0,1)}{\partial x} = -I + H$ is singular
- $w, v =$ left, right eigenvector of J relative to 0
- $E = I - vw^T =$ projection operator onto $\mathrm{range}(J) = \big(\mathrm{span}(w)\big)^{\perp}$
- Split x into $x = (r, y)$

$$r = Ex \in \big(\mathrm{span}(w)\big)^{\perp} \qquad y = (I - E)x \in \mathrm{span}(w)$$

- split $\Phi(x)$ accordingly

$$E\Phi(x) = 0 \qquad (I - E)\Phi(x) = 0$$

- implicit function theorem:

$$E\Phi(x) = 0 \quad \Longrightarrow \quad r = R(y, \pi)$$

- $\Longrightarrow$ (1-dim) center manifold

$$g(y, \pi) = w^T (I - E)\Phi(y + R(y, \pi), \pi) = 0$$

Opinion forming on structurally balance social networks

- enough to check the partial derivatives

$$g = g_y = g_{yy} = g_\pi = 0, \qquad g_{yyy} g_{\pi y} < 0$$

$\Longrightarrow$ recognition problem for a pitchfork bifurcation is solved.

Second bifurcation at $\pi_2 > 1$: same procedure for the Fiedler eigenvector

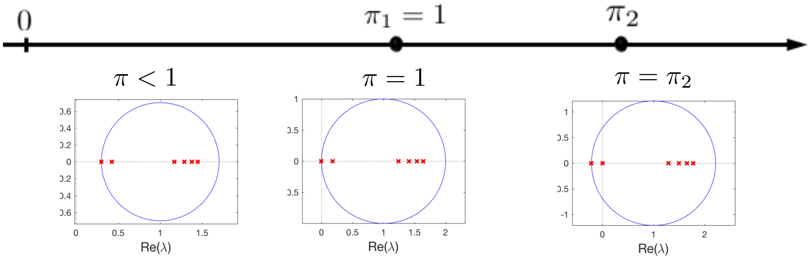
Opinion forming on structurally balance social networks

- enough to check the partial derivatives

$$g = g_y = g_{yy} = g_\pi = 0, \qquad g_{yyy} g_{\pi y} < 0$$

$\implies$ recognition problem for a pitchfork bifurcation is solved.

Second bifurcation at $\pi_2 > 1$: same procedure for the Fiedler eigenvector



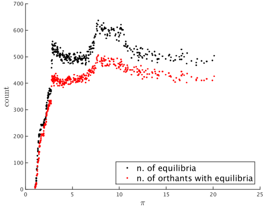
$I - \pi H$ is diagonally dominated	$I - H$ is Laplacian $\lambda_n(H) = 1$ $\lambda_1(I - H) = 0$	$I - \pi_2 H$ is unstable $\pi_2 \lambda_{n-1}(H) = 1$ $\lambda_2(I - \pi_2 H) = 0$

Opinion forming on structurally balance social networks

- for $\pi > \pi_2$: many new equilibria (stable/unstable)

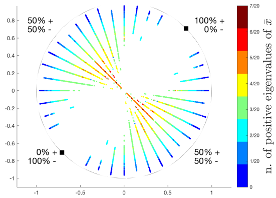
Example $n = 20$

- n. of orthants: $> 10^6$
- n. of equilibria: grows exponentially with n
- numerical analysis: 500 values of π, 10^4 trials each



- location of new equilibria $\bar{x}$ for all identical ψ_i

$$\|\bar{x}\| \leq \|x^*\|$$

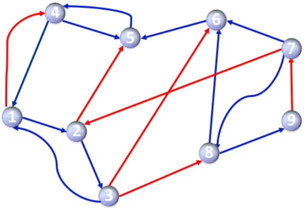


Structurally unbalanced graphs

- A signed graph $\mathcal{G}(A)$ in general is not structurally balanced

Proposition A signed graph $\mathcal{G}(A)$ is structurally unbalanced *iff* any of the following equivalent conditions holds:

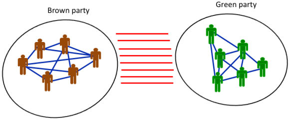
1. not all cycles of $\mathcal{G}(A)$ are positive;
2. No diagonal signature matrix $D = \mathrm{diag}(\pm 1)$ exists such that DAD is nonnegative;
3. the signed Laplacian $\mathcal{L}_s$ has $\lambda_1(\mathcal{L}_s) > 0$



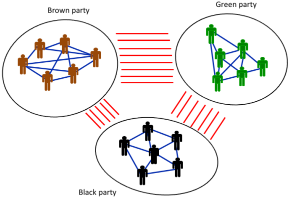
$\mathcal{G}(H)$ structurally balanced vs. unbalanced

Example: parliamentary system

Two-party system
$\mathcal{G}(H)$ structurally balanced



Three-party system
$\mathcal{G}(H)$ structurally unbalanced



$\mathcal{G}(H)$ structurally balanced vs. unbalanced

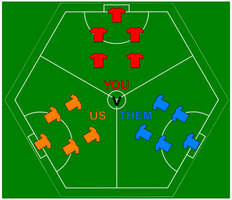
Example: football

Normal football

$\mathcal{G}(H)$ structurally balanced

Three-sided football

$\mathcal{G}(H)$ structurally unbalanced



- much more tactical and difficult to play than normal football
- plenty of team "alliances" and "betrayals" during the game
- "organized confusion"

$\mathcal{G}(H)$ structurally balanced vs. unbalanced

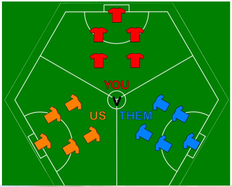
Example: football

Normal football

$\mathcal{G}(H)$ structurally balanced

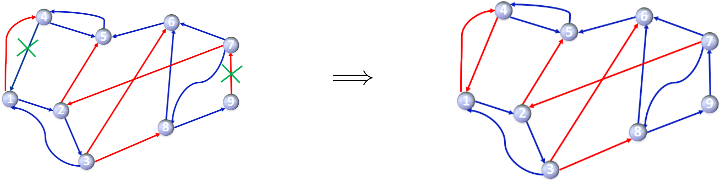
Three-sided football

$\mathcal{G}(H)$ structurally unbalanced



Computing the level of structural unbalance

- How "distant" is a graph from structural balance?
- intuitively: the least number of edges that must be removed (or switched of sign) in order to get a structurally balanced graph



- computation is NP-hard
- heuristics:
  - direct approach: counting cycles $\longrightarrow$ unfeasible
  - in statistical physics: computing the ground state of an Ising spin glass
  - in computer science: MAX-CUT or MAX-XORSAT problems

Computing the level of structural unbalance

- To measure distance to structural balance

Definitions

- Frustration = minimum of an energy-like functional

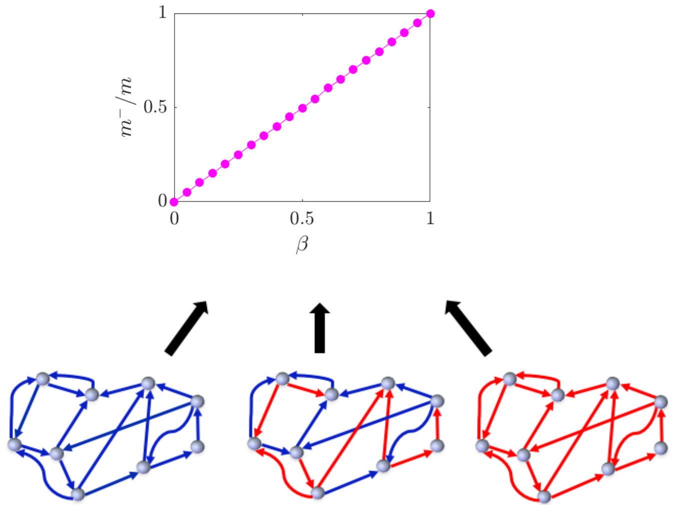
$$\epsilon(H) = \min_{\substack{D=\mathrm{diag}(d_1,\ldots,d_n)\\ d_i=\pm 1}} \frac{1}{2}\sum_{i\neq j}\left(|\mathcal{L}_s| - D\mathcal{L}_s D\right)_{ij}$$

- Algebraic conflict = smallest eigenvalue of $\mathcal{L}_s$

$$\xi(H) = \lambda_1(\mathcal{L}_s)$$

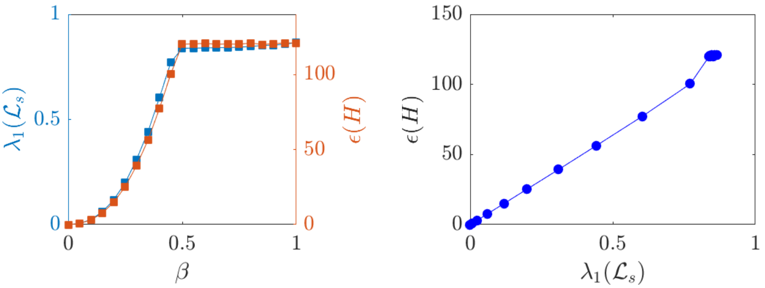
Computing the level of structural unbalance

Example: Erdős-Rényi networks with varying amount of negative edges



Computing the level of structural unbalance

- Algebraic conflict / Frustration index



- $\epsilon(H)$ and $\lambda_1(\mathcal{L}_s)$ are proportional

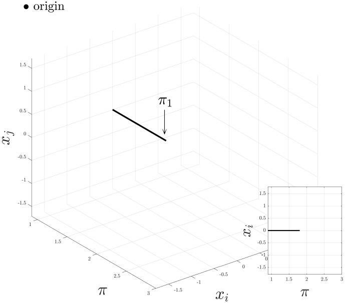
$$\epsilon(H) \approx \lambda_1(\mathcal{L}_s)$$

- both grow with β, then saturate at around $\beta \approx 0.5$

Opinion forming on structurally unbalance social networks

$$\dot{x} = \Delta\left(-x + \pi H \psi(x)\right)$$

Cases:	**Meaning**
$\pi < \pi_1$	not enough social commitment: no decision



Bifurcation diagram (x_i, π, x_j)

first bifurcation: $\pi_1 = \dfrac{1}{1 - \lambda_1(\mathcal{L}_s)}$

Fontan, Altafini, "Achieving a decision in antagonistic multiagent networks", *CDC*, 2018.

Opinion forming on structurally unbalance social networks

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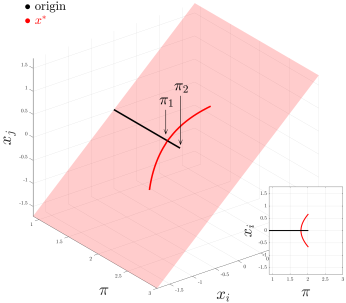
$\pi_1 < \pi < \pi_2$	right commitment: two alternative polarized decisions x^*



Bifurcation diagram (x_i, π, x_j)

first bifurcation: $\pi_1 = \dfrac{1}{1 - \lambda_1(\mathcal{L}_s)}$ second bifurcation: $\pi_2 = \dfrac{1}{1 - \lambda_2(\mathcal{L}_s)}$

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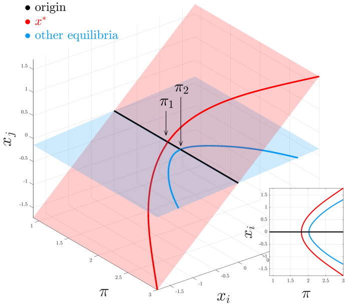
Opinion forming on structurally unbalance social networks

$$\dot{x} = \Delta\left(-x + \pi H \psi(x)\right)$$

Cases:	**Meaning**
$\pi < \pi_1$	not enough social commitment: no decision
$\pi_1 < \pi < \pi_2$	right commitment: two alternative polarized decisions x^*
$\pi > \pi_2$	overcommitment: multiple decisions



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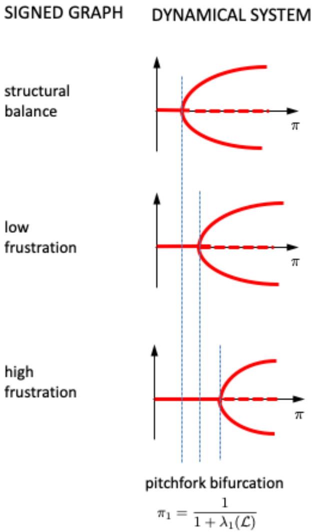
Summary



- $\lambda_1(\mathcal{L}_s)$ grows with the frustration
- $\pi_1 = \frac{1}{1-\lambda_1(\mathcal{L}_s)}$ grows with $\lambda_1(\mathcal{L}_s)$
- the larger π_1, the larger is the social effort needed to achieve a decision
- the higher the frustration, the more difficult it is to achieve a nontrivial decision

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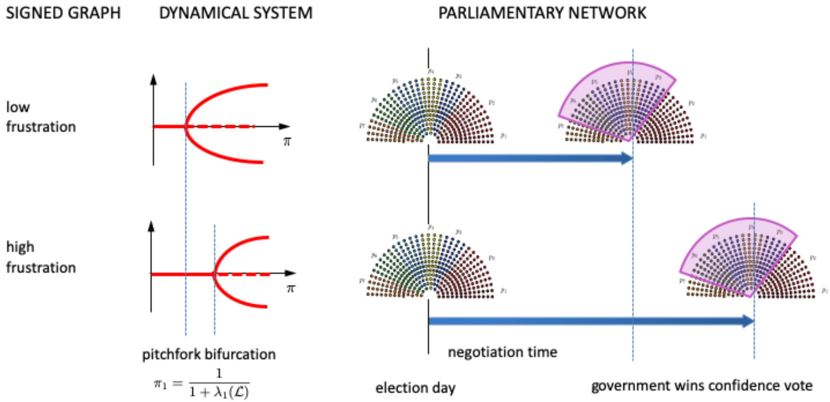
Application: Government formation process

Question: Is the process of government formation "sensitive" to the amount of frustration?

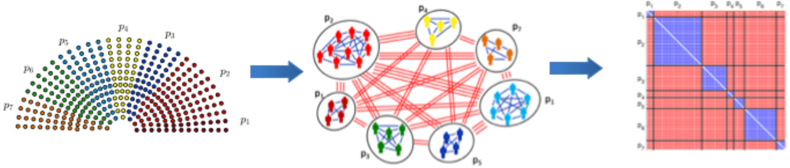


Government formation in parliamentary networks

1. quantification of “social effort”: days to government = n. of days required to get a confidence vote from parliament
2. build a parliamentary network for a multiparty parliament:

Government formation in parliamentary networks

1. quantification of "social effort": days to government = n. of days required to get a confidence vote from parliament
2. build a parliamentary network for a multiparty parliament:
   Scenario I:
   - all MPs of one party are friends (+1 edge)
   - all MPs from different parties are rival (-1 edge)



⟹ fully connected block-structured unweighted signed graph

⟹ frustration can be computed exactly

Government formation in parliamentary networks

- Data analyzed: 29 European nations

Albania
Andorra
Austria
Belgium
Bosnia Herzegovi
Bulgaria
Croatia
Czech Republic
Denmark
Estonia
Finland
Germany
Greece
Hungary
Iceland
Ireland
Italy
Latvia
Luxembourg
Macedonia
Moldova
Netherlands
Norway
Serbia
Slovakia
Slovenia
Spain
Sweden
UK

- datasets: Manifesto Project, Parliaments and Governments database, Wikipedia, Chapter Hill surveys, etc.
- time span: 1980-2018

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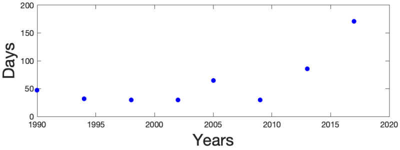
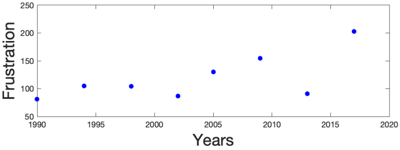
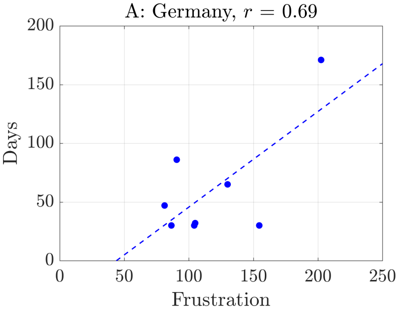
Government formation in parliamentary networks

- Example: Germany

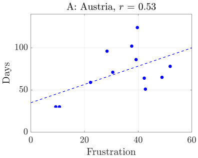




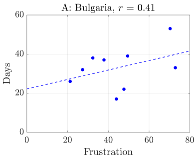


Government formation in parliamentary networks

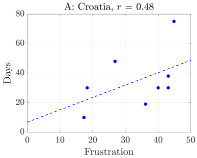
Austria



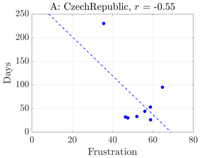
Bulgaria



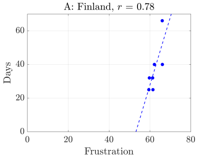
Croatia



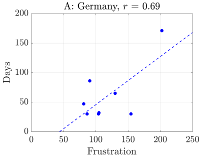
Czech Rep.



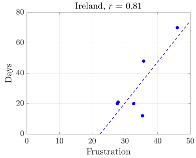
Finland



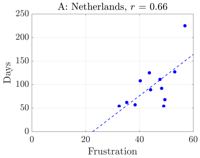
Germany



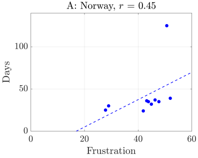
Ireland



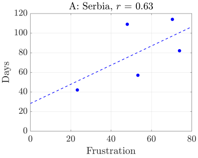
Netherlands



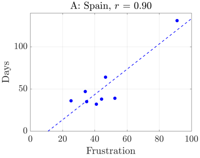
Norway



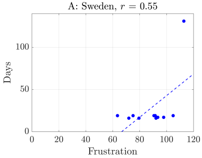
Serbia



Spain

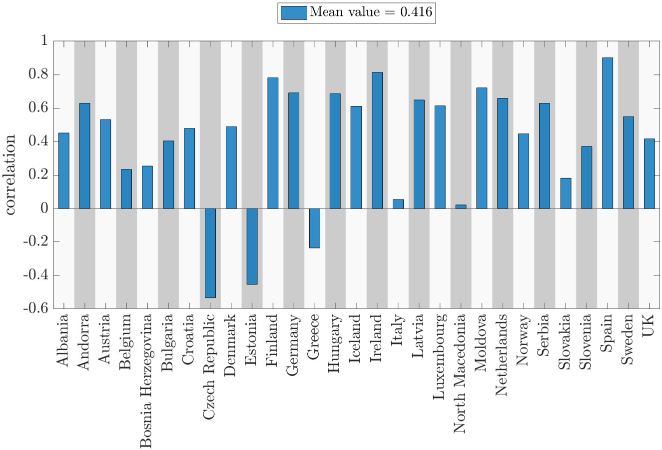


Sweden



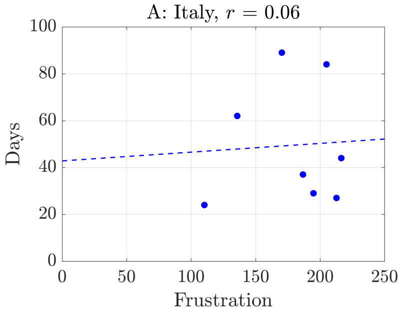
Government formation in parliamentary networks

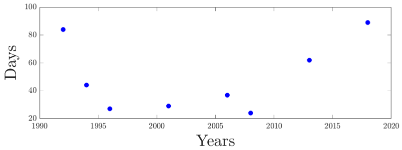
- Results: correlation between frustration and days-to-government (mean for each nation)

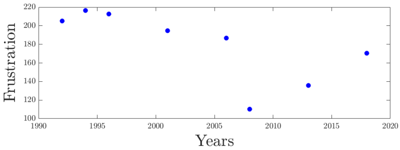


Government formation in parliamentary networks

- How about Italy?



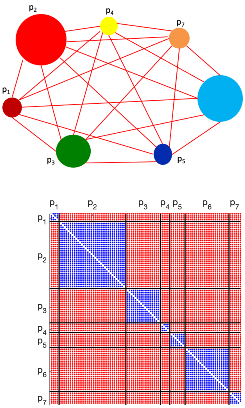




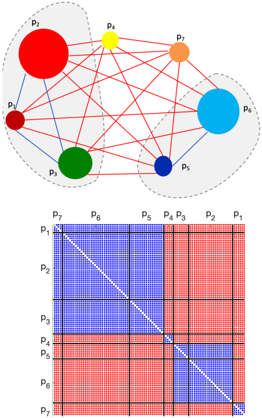
Government formation in parliamentary networks

- Refinements: choose edge weights in a more appropriate way

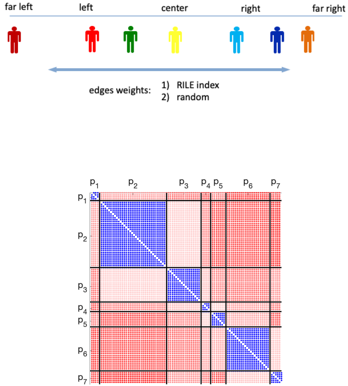
“All-against-all”



Coalitions



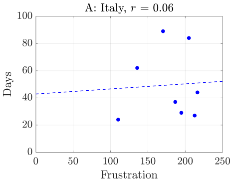
“Left-right” index



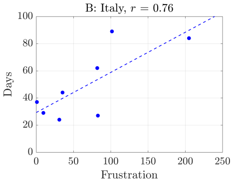
Government formation in parliamentary networks

- Example: Italy

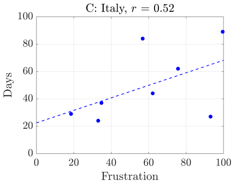
All-against-all



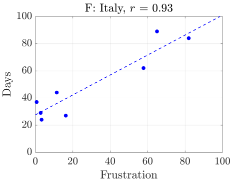
Coalitions



"Left-right" index (RILE)



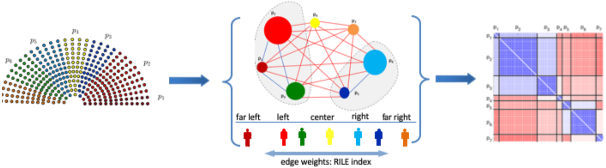
Coalitions + "Left-right" index + "optimized" weights

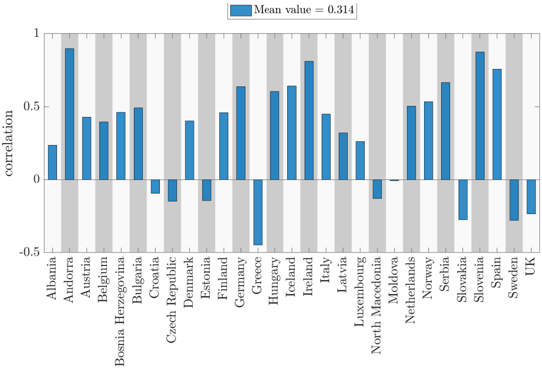


Government formation in parliamentary networks

Scenario II:

1. party coalitions
2. RILE

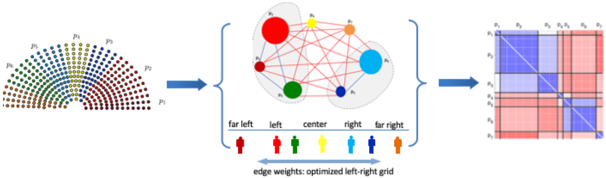


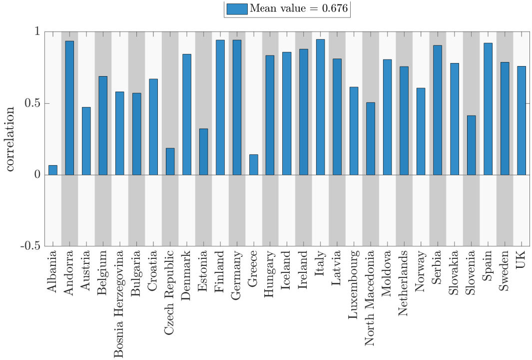


Government formation in parliamentary networks

Scenario III:

1. party coalitions
2. optimized Left-Right grid



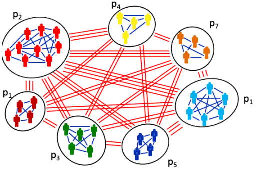


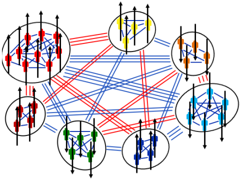
Frustration and energy landscape

- Energy of the "Ising spin glass"

$$e(D) = \frac{1}{2} \sum_{i \neq j} (|\mathcal{L}_s| - D\mathcal{L}_s D)_{ij}$$

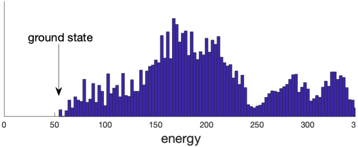
$D = \mathrm{diagblock}(\pm 1)$ “spin up”, “spin down”





- changing D: $e(D)$ changes
- frustration corresponds to the energy of the “ground state” D_{best}:

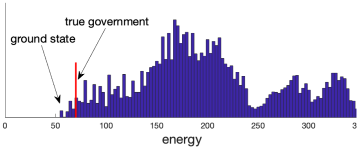
$$\epsilon(H) = e(D_{\mathrm{best}})$$

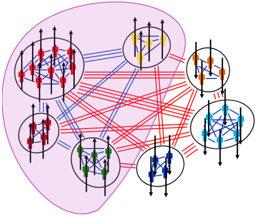


Frustration and energy landscape

- "true government" corresponds to D_{gov}, of energy

$$e(D_{\text{gov}}) = \frac{1}{2} \sum_{i \neq j} \left(|\mathcal{L}_s| - D_{\text{gov}} \mathcal{L}_s D_{\text{gov}} \right)_{ij}$$





Frustration and energy landscape

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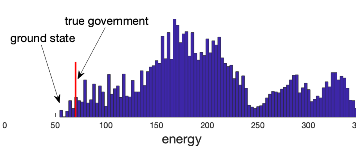
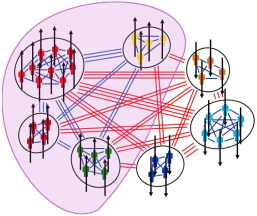
$$e(D_{\text{gov}}) = \frac{1}{2} \sum_{i \neq j} \left(|\mathcal{L}_s| - D_{\text{gov}} \mathcal{L}_s D_{\text{gov}} \right)_{ij}$$



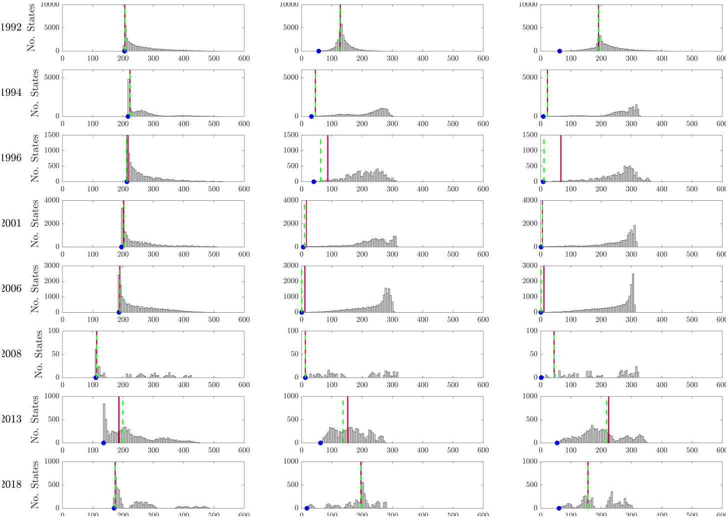


Question: how close is $e(D_{\text{gov}})$ to $e(D_{\text{best}})$?

Frustration and energy landscape

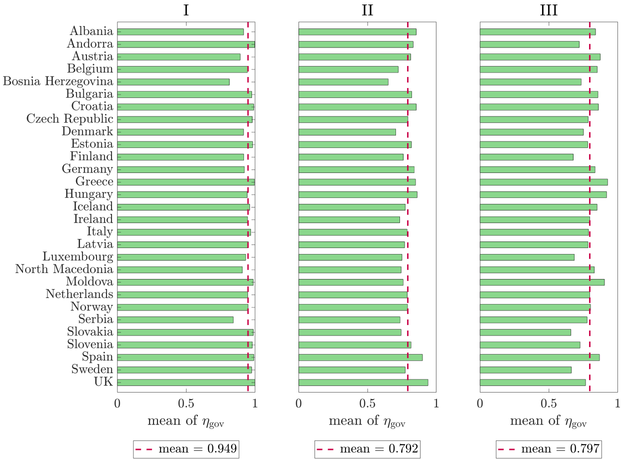
Example: Italy

Scenario I   Scenario II   Scenario III



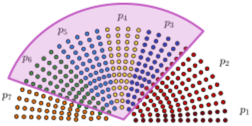
Frustration and energy landscape

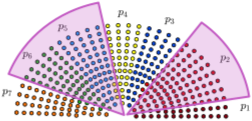
- Energy gap: $\eta_{\text{gov}} = 1 - \frac{e(D_{\text{gov}}) - e(D_{\text{best}})}{\max_D e(D) - e(D_{\text{best}})}$



Government composition

Question: can we predict successful government coalitions?

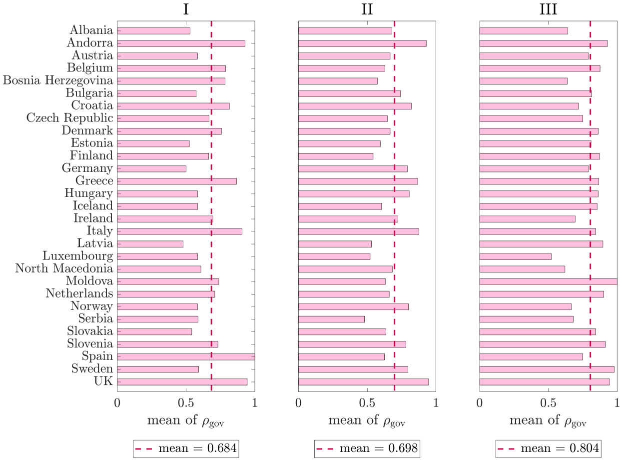




- $\mathcal{P}_{\text{best,maj}}$ = group of parties forming a majority in the ground state
- $\mathcal{P}_{\text{gov}}$ = group of parties forming a majority in the ground state

$$\rho_{\text{gov}} = \frac{\text{card}(\mathcal{P}_{\text{best,maj}} \cap \mathcal{P}_{\text{gov}})}{\text{card}(\mathcal{P}_{\text{gov}})}$$

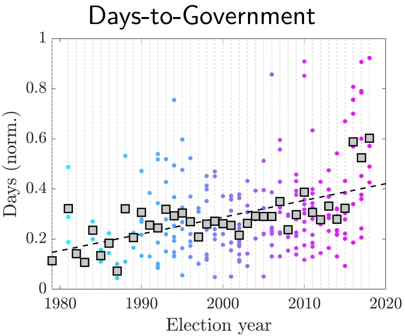
Government composition

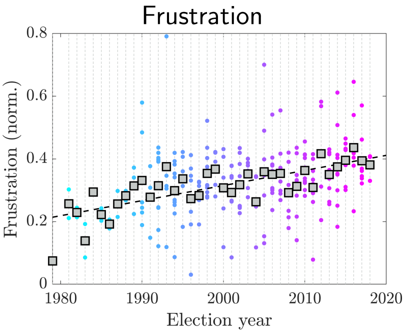


- complication: minority governments...

Pan-European yearly trends

- Data from different countries can be compared after normalization





- In the last 40 years, the duration of the post-election government negotiation phase has more than doubled
- Why? Perhaps because the frustration of our parliamentary networks has nearly doubled...

Conclusion

- Aim: provide a dynamical model able to explain the dynamics of government formation in multiparty democracies
- Model: collective decision making on signed graphs
  - structurally balanced graph
    - more predictable dynamics (monotone system)
    - low "social commitment" for bifurcation
  - structurally unbalanced graph:
    - amount of frustration influences the decision process
    - the higher frustration, the higher is the social commitment for bifurcation
- Duration of government formation process correlates strongly with the frustration of the parliament network

Thank you!

Banksy, Devolved Parliament, 2009

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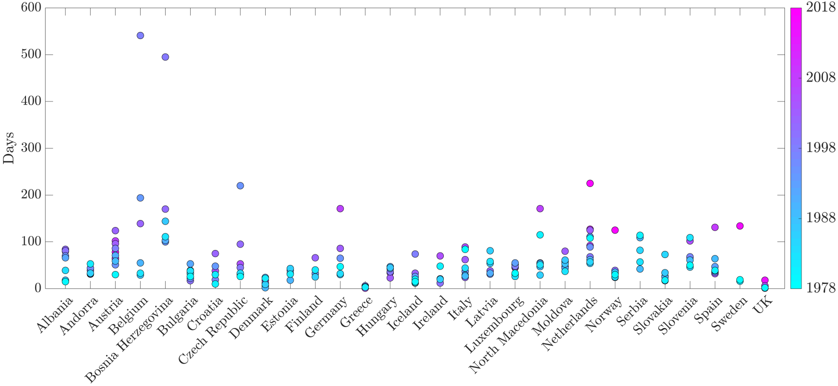
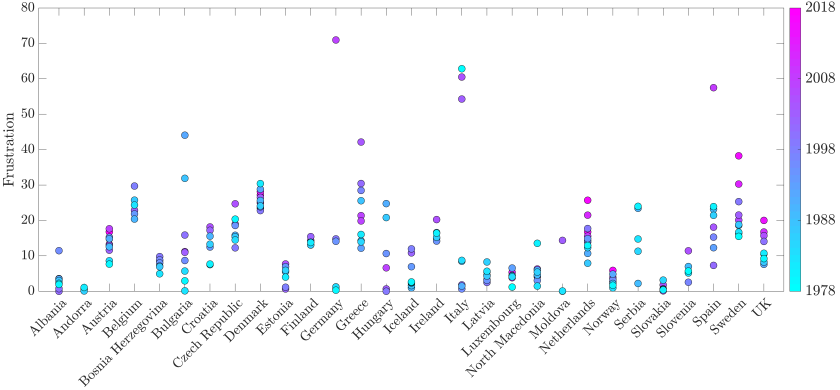
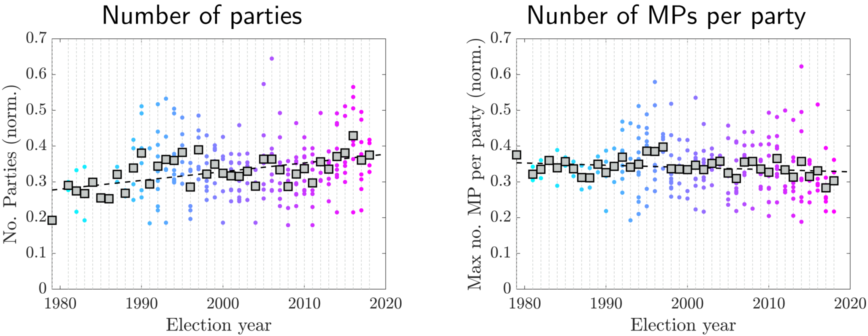
Duration of government negotiations



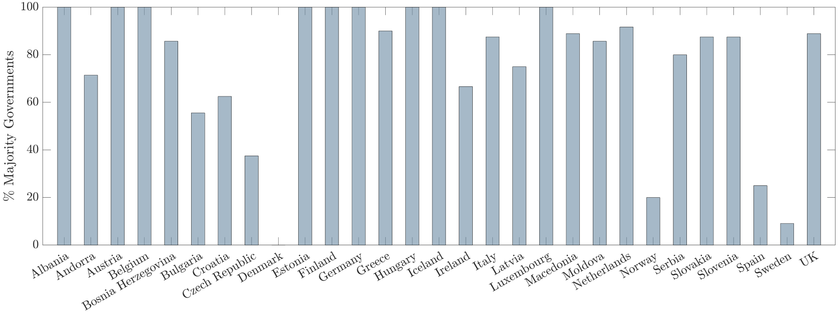
Frustration (Scenario I)



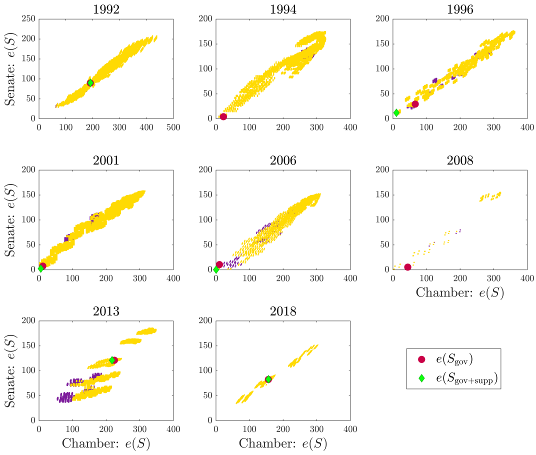
Pan-European yearly trends



Fraction of majority governments



Italy: energy of Lower chamber vs Senate



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