

Virtual Reinforcement of Power Grids: A Feedback Optimization Approach

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Joint work with



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Future power systems: challenges and opportunities

Fluctuating renewable energy sources

- poor short-range prediction
- correlated uncertainty

Inverter-based generation

- control flexibility
- tight operating specifications

Electric mobility

- large additional demand
- new spatial-temporal patterns



Congestion of the power distribution infrastructure



Power distribution infrastructure has limited power transfer capacity

- overvoltage / undervoltage
 - quality of service (loads)
 - protections
 - disconnection of power inverters

Overvoltage / undervoltage can be prevented

- by curtailing generation / loads (expensive!)
- by injecting / drawing reactive power

Resource sharing problem. Today, cooperative solutions only.

Power distribution infrastructure network



Resource sharing – Volt/VAR regulation

- *u* controllable input
- w uncontrollable input
- y performance output

reactive power injection of generators

power demand of loads, power generation

voltage of generators

 $h(\cdot, \cdot)$ input-output steady-state relation y = h(u, w)

AC power flow eqs.

Efficient resource sharing

minimize _{u} $f(u)$	cost of control effort
subject to $y \in \mathcal{Y}$	specifications on performance output $y = h(u, w)$
$u \in \mathcal{U}$	actuation constraints

see also: flexible loads, data networks, traffic, ...

Computational approach

Efficient resource sharing

minimize _{u} $f(u)$	cost of control effort
subject to $y \in \mathcal{Y}$	specifications on performance output $y = h(u, w)$
$u \in \mathcal{U}$	actuation constraints

Optimal dispatch:

- 1. Measure/estimate the uncontrollable input w
- 2. Solve the non-convex optimization problem (OPF)
- **3.** Dispatch control u

V. A. Evangelopoulos, P. S. Georgilakis, N. D. Hatziargyriou "Optimal operation of smart distribution networks: A review of models, methods and future research" Electric Power Systems Research (2016)

...and many others

Experimental result

Experimental distribution feeder SYSLAB at DTU, Denmark.



- controllable inputs: reactive power injection PV1, PV2, Battery
- uncontrollable inputs: active power injections PV1, PV2, Battery, Load

$$\begin{array}{ll} \text{minimize} & \sum_i (q_i/q_i^{\text{max}})^2 \\ \text{subject to} & q_i \in [q_i^{\min} \; q_i^{\max}] \\ & v_i \in [v^{\min} \; v^{\max}] \\ & v = h(q,w) \end{array}$$

optimization problem

Optimal dispatch



OPF-based dispatch

Feedforward optimization



- complex optimal decision
- operational constraints
- MIMO (multi-input/output)
- highly model-based
- requires full measurement
- (computationally intensive)

y=h(u,w)

Resource sharing problem h uncertain, w unknown

Power distribution grids

- unmonitored loads
- parametric uncertainty (cables)
- model uncertainty (loads)
- steady state tracking errors
- measurement bias / noise

Feedback vs feedforward

Feedforward optimization



- complex optimal decision
- operational constraints
- MIMO (multi-input/output)
- highly model-based
- requires full measurement
- (computationally intensive)





- robust to model uncertainty
- rejects unmeasured disturbances
- fast response
- requires exogeneous set-points

Feedback Volt/VAr control



Static and dynamic feedback laws No communication

- Turitsyn et al. (2011)
- Jahangiri & Aliprantis (2013)
- Cavraro & Carli (2015)
- Farivar, Zhou, & Chen (2015)
- Kundu, Backhaus, & Hiskens (2013)
- Yeh, Gayme, & Low (2012)
- Samadi et al. (2014)
- Kekatos et al. (2015)
- Zhu & Liu (2015)
- Li, Gu, & Dahleh (2014)
- VDE-AR-N 4105 standard (2018)
- ENTSO-E / EU Comm. Reg. 631 (2016)
- IEEE 1547.2018 standard (2018)

Heuristic feedback control design

- VDE-AR-N 4105 standard (2018)
- ENTSO-E / EU Comm. Reg. 631 (2016)
- IEEE 1547.2018 standard (2018)



• Model free – relies only on $\frac{\partial v_h}{\partial a_h} > 0$

• Proportional tracking of a **nominal voltage reference** v = 1

Experiment: droop Volt/VAr control



Decentralized feedback control

Suboptimality gap

Decentralized feedback strategies **cannot** guarantee a feasible voltage profile (even when it exists).

Def: Decentralized feedback

$$q_h(t+1) = g_h(q_h(t), v_h(t))$$

where $g_h : [q_{\min}, q_{\max}] \times \mathbb{R}_{\geq 0} \rightarrow [q_{\min}, q_{\max}]$ satisfies

- **1.** $g_h(q, v)$ continuous in v for all q
- **2.** $g_h(q, v)$ weakly decreasing in v
- **3.** $g_h(q, v) g_h(q', v) < q q'$ for all q > q'

A wide class that contains all the strategies mentioned before (and more).

Equilibria of decentralized strategies

Individual equilibria - agent i

For any v_i^* , there is a unique $q_i^* = F_i(v_i^*)$ which satisfies $q_i^* = g_i(q_i^*, v_i^*)$. Moreover, F_i is a continuous, weakly decreasing function of v_i^* .

The equilibria of the system of interconnected agents are the solutions of

$q^* = F(v^*)$	individual agents
$v^* = Xq^* + b$	linearized grid equations

Unique system equilibrium

Sandberg & Willson (1969) \rightarrow Existence & uniqueness of sol. of F(x) + Ax = b

 \rightarrow There exists a unique equilibrium voltage profile $v^* = XF(v^*) + b$

Constructive proof



For any value of the grid parameters, there exists a value of q_2 , q_3^{max} , q_1^{max} such that the unique equilibrium is unfeasible for all decentralized strategies

$$(q^*, v^*) \notin [q^{\min}, q^{\max}] \times [v^{\min}, v^{\max}]$$

even if a (non equilibrium) feasible solution exists.

S. Bolognani, R. Carli, G. Cavraro, S. Zampieri "On the need for communication for voltage regulation of power distribution grids" IEEE Transactions on Control of Network Systems (2019)

Feedback optimization



Proposal: a feedback optimization approach to inherit the best of both

- Robustness against uncertainty in the model h
- Rejection of unmeasured disturbances w
- No need for exogenous set-points
- Guaranteed tracking of the optimal constrained solution

Control design specifications





- Input saturation: $u \in U$ at all times (hard constraint)
- **Closed-loop trajectory:** $y \in \mathcal{Y}$ at steady state
- Optimality: The closed-loop system converges to the constrained solution

Optimization perspective		Control perspective	
Algorithms as dynamical systems [Lessard et al., 2014], [Wilson et al., 2018]	Ex as	kisting feedback systems interpreted solving opt. problem	
\rightarrow implemented via the physics	\rightarrow	general objective + constraints	

Optimization algorithms as dynamical systems

Feedback optimization design \leftrightarrow continuous-time limit of iterative algorithms

Gradient Flows on Matrix Manifolds

[Brockett, 1991], [Bloch et al., 1992], [Helmke & Moore, 1994], ...

Interior-point methods

[Karmarkar, 1984], [Khachian, 1979], [Faybusovich, 1992], ...

Acceleration & Momentum methods

[Su et al., 2014], [Wibisono et al, 2016], [Krichene et al., 2015], [Wilson et al., 2016], [Lessard et al., 2016], ...

Saddle-Point Flows

[Arrow et al., 1958], [Kose, 1956], [Feijer & Paganini, 2010], [Cherukuri et al., 2017], [Holding & Lestas, 2014], [Cortés & Niederländer, 2018], [Qu & Li, 2018], ...

In continuous-time, most algorithms reduce to either (projected) gradient flows (w/ w/o momentum), (projected) Newton flows, or (projected) saddle-point flows.

Example: Projected saddle-flow



Constraint substitution ("certainty equivalence design")

assume steady state y = h(u, w)

Partial Lagrangian (dualize the output constraints only)

$$\mathcal{L}(u,\lambda) := f(u) + \lambda^{\top} g(h(u,w))$$

Example: Projected saddle flow

$$\mathcal{L}(u,\lambda) := f(u) + \lambda^{\top} g(h(u,w))$$

Primal projected Newton descent + Dual projected gradient ascent

$$\dot{u} = \Pi_{T_u \mathcal{U}} \left[-(\nabla^2 f)^{-1} \nabla_u \mathcal{L}(u, \lambda) \right]$$

$$\dot{\lambda} = \prod_{\geq 0} \left[\alpha \nabla_{\lambda} \mathcal{L}(u, \lambda) \right]$$



Differential inclusion if proj not unique

Example: Projected saddle flow

$$\mathcal{L}(u,\lambda) := f(u) + \lambda^{\top} g(h(u,w))$$

Primal projected Newton descent + Dual projected gradient ascent

$$\begin{split} \dot{u} &= \Pi_{T_u \mathcal{U}} \left[-(\nabla^2 f)^{-1} \nabla_u \mathcal{L}(u, \lambda) \right] \\ &= \Pi_{T_u \mathcal{U}} \left[-(\nabla^2 f)^{-1} \left(\nabla_u f(u) + \underbrace{\nabla_u h(u, w)^\top}_{\text{model}} \nabla g(\underbrace{h(u, w)}_{\text{meas.}})^\top \lambda \right) \right] \\ \dot{\lambda} &= \Pi_{\geq 0} \left[\alpha \nabla_\lambda \mathcal{L}(u, \lambda) \right] \\ &= \Pi_{\geq 0} \left[\alpha g\left(\underbrace{h(u, w)}_{\text{meas.}} \right) \right] \end{split}$$

Discrete time implementation (dual update)

$$\dot{\lambda} = \Pi_{\geq 0} \left[\alpha \ g \left(\underbrace{h(u, w)}_{\text{meas.}} \right) \right] = \Pi_{\geq 0} \left[\alpha \ g(\boldsymbol{y}) \right]$$

- forward Euler step
- projection in the standard metric \rightarrow element-wise saturation

$$\begin{split} \lambda(t+1) &= \arg\min_{\lambda \ge 0} \|\lambda - (\lambda(t) + \alpha g(y))\| \\ &= \max\left\{0, \lambda(t) + \alpha g(y)\right\} \end{split}$$

Dual update \rightarrow fully decentralized integrator of the constraint violations

Discrete time implementation (primal update)

$$\dot{u} = \Pi_{T_u \mathcal{U}} \left[\underbrace{-(\nabla^2 f)^{-1} \left(\nabla_u f(u) + \underbrace{\nabla_u h(u, w)^\top}_{\text{model}} \nabla g(y)^\top \lambda \right)}_{\delta u} \right]$$

- forward Euler integration $u(t) + \delta u$
- projection step in the same metric $(\nabla^2 f)^{-1}$

$$u(t+1) = \arg\min_{u \in \mathcal{U}} \left\| u - (u(t) + \delta u(t)) \right\|_{(\nabla^2 f)^{-1}}$$

Primal update \rightarrow centralized **quadratic program**

Sensitivity approximation

Back to the Volt/VAr problem:

- quadratic cost function f(u)
- $\blacksquare \text{ simple } \mathcal{Y}$
- simple U
- sensitivity $\nabla_u h(u, w)$
 - can be estimated from cable data / experiments
 - conjecture: very robust against approximation

 $\begin{aligned} q^\top M q \\ v_i^{\min} &\leq v_i \leq v_i^{\max} \\ q_i^{\min} &\leq q_i \leq q_i^{\max} \\ \approx \text{susceptance matrix } X \end{aligned}$



Experiment: Feedback Optimization



Experiment: Feedback Optimization



Open question: characterization of this robustness





Virtual grid reinforcement



A promising unified approach

Work since 2013 by: Low, Li, Dörfler, Bolognani, Zampieri, Simpson-Porco, Zhao, Dall'Anese,

Simonetto, De Persis, Gan, Topcu, Bernstein, Jokic, ...

D. K. Molzahn, F. Dörfler, H. Sandberg, S. H. Low, S. Chakrabarti, R. Baldick, J. Lavaei "A Survey of Distributed Optimization and Control Algorithms for Electric Power Systems" IEEE Transactions on Smart Grid (2017)

> F. Dörfler, S. Bolognani, J. W. Simpson-Porco, S. Grammatico "Distributed Control and Optimization for Autonomous Power Grids" European Control Conference (2019)



UNICORN A Unified Control Framework for Real-Time Power System Operation







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CLOSED-LOOP ANALYSIS

Stability



Two reasons why it may fail to converge to the OPF solution

- Irregularity of the domain \mathcal{U}, \mathcal{Y}
- Interplay with the dynamics of the grid $y \approx h(u, w)$

WELL-POSEDNESS AND DOMAIN REGULARITY

Power flow manifold

- Ambient space for the steady-state map $y = h(u, w) \rightarrow x = \begin{bmatrix} \theta \\ p \\ z \end{bmatrix}$
- Set of all grid states that satisfy the **AC** power flow equations F(x) = 0

 \rightarrow power flow manifold $\mathcal{M} := \{x \mid F(x) = 0\}$

Regular submanifold of dimension 2n



Induced trajectory

- Steady state *h* ⇔ Attractive manifold
- Feedback (gradient) control law u
 u = ... induces a trajectory

 $x(t) \in \mathcal{M}$

Bolognani & Dörfler "Fast power system analysis via implicit linearization of the power flow manifold" Allerton Conference (2015)

Projected dynamical systems





- $\mathcal{U},\mathcal{Y} \quad \rightarrow \quad \mathcal{X} \subset \mathcal{M}$
- Projected vector field (a set-valued map) with respect to a metric g

$$\Pi^g_{\mathcal{X}}[f](x) := \underset{v \in T_x \mathcal{X}}{\arg\min} \|v - f(x)\|_{g(x)}$$

Initial value problem (differential inclusion)

$$\dot{x} \in \Pi^g_{\mathcal{X}}[f](x), \qquad x(0) = x_0 \in \mathcal{X}$$

 Well posedness of this trajectory (existence, uniqueness) is (for the most part) function of the regularity of the domain X

Well-posedness results (static)



Krasovskii \rightarrow La Salle \rightarrow convergence of projected gradient systems

Hauswirth, Bolognani, & Dörfler "Projected Dynamical Systems on Irregular, Non-Euclidean Domains for Nonlinear Optimization" arXiv (2018)

Time-varying projected dynamical systems (with I. Subotić)

expanding domain



shrinking domain



New definition of temporal tangent "cone"

$$v \in T_x^t \mathcal{X} \Leftrightarrow \begin{cases} \exists x_k \to x, \delta_k \to 0^+ : \\ \frac{x_k - x}{\delta_k} \to v \quad \text{and} \quad x_k \in \mathcal{X}(t + \delta_k) \end{cases}$$

- $\rightarrow~$ not necessarily a cone \Rightarrow can be empty
 - Define time-varying projected vector field

$$\Pi_{\mathcal{X}}[f](x,t) := \underset{v \in T_x^t \mathcal{X}}{\arg\min} \|v - f(x)\|$$

- \rightarrow initial value problem
 - $\dot{x} \in \Pi_{\mathcal{X}}[f](x), \qquad x(0) = x_0 \in \mathcal{X}$
- $\Rightarrow \text{ Existence of Krasovskii solutions if } \mathcal{X}(t) \text{ is a forward Lipschitz continuous}$

Time varying domain

Limit on domain shrinking **Constraint geometry** t < 0t < 0 $\mathcal{X}(t)$ $\mathcal{X}(t)$ $\mathcal{X}(t)$ $\chi(t)$ 8 8 t = 0t = 0 $\chi(0)$ $\mathcal{X}(t)$ $\mathcal{X}(t)$ а 8 t > 0t > 0 $\mathcal{X}(t)$ $\mathcal{X}(t)$ p(t $\begin{bmatrix} \nabla h(x) \\ \nabla q_{I(x)}(h) \end{bmatrix}$ full rank

A. Hauswirth, I. Subotic, S. Bolognani, G. Hug, F. Dörfler "Time-varying Projected Dynamical Systems with Applications to Feedback Optimization of Power Systems" IEEE Conference on Decision and Control (2018)

Genericity results

LICQ

 $\begin{bmatrix} \nabla h(x) \\ \nabla g_{I(x)}(h) \end{bmatrix}$ full rank % f(x) = 0 is generically satisfied in static OPF

 \rightarrow well-posedness & uniqueness of Lagrange multipliers

- Ioad perturbation or shunt admittance perturbation OK
- Ine parameter perturbation NOT OK (structural ill-posedness)

A. Hauswirth, S. Bolognani, G. Hug, F. Dörfler "Generic Existence of Unique Lagrange Multipliers in AC Optimal Power Flow" IEEE Control Systems Letters (2018)

Remark: Non-existence of a feasible trajectory can be interpreted as a lack of **control authority**: no finite control effort will maintain the grid state inside the desired bounds, or to track the prescribed reference.

INTERCONNECTED DYNAMICS

Gradient-based Feedback Optimization



Feedback on state
$$x$$
, Cost $f(u, x)$

Optimization Dynamics

Variable-metric gradient descent

 $\dot{u} = -Q(u)\nabla \tilde{f}(u)$

- $Q(u) \succ 0$ for all $u \in \mathbb{R}^p$
- $\bullet \ \tilde{f}(u) := f(u, h(u, w))$

•
$$\nabla \tilde{f}(u) = \nabla_u f + \nabla h^T \nabla_y f$$

Plant Dynamics

Exponentially stable system

 $\dot{x} = \phi(x, u)$

with steady-state map x = h(u, w)

Interconnection

$$egin{aligned} \dot{x} &= \phi(x, oldsymbol{u}) \ \dot{u} &= -Q(u) \left(
abla_u f(u, oldsymbol{x}) +
abla h^T
abla_y f(u, oldsymbol{x})
ight) \end{aligned}$$

Interconnected dynamics (gradient FO)

Theorem

Assume

Physical system exponentially stable with Lyapunov function W(x, u) s.t.

 $\dot{W}(x,u) \le -\gamma ||x - h(u)||^2$ $||\nabla_u W(x,u)|| \le \zeta ||x - h(u)||.$

• f(u, x) has compact level sets and *L*-Lipschitz gradient.

Then, all trajectories converge to the set of KKT points whenever

$$\sup_{u\in\mathbb{R}^p}\|Q(u)\|<\frac{\gamma}{\zeta L}\,.$$

Furthermore,

- Asymptotically stable equilibrium ⇒ strict local minimizer
- Strict local minimizer ⇒ stable equilibrium
- \rightarrow If f convex and h(u,w) linear, then convergence to set of global minimizers.

Interconnected dynamics (gradient FO)

Vanilla GD

Choose $Q = \varepsilon I_n$. Stability is guaranteed if

 $\varepsilon \leq \frac{\gamma}{\zeta L}$

 \Rightarrow prescription on global control gain

Projected GD

Control signal u constrained to set \mathcal{U} (in case of actuator saturation).

 $\dot{u} = \Pi_{T_u \mathcal{U}} [-\varepsilon \nabla \tilde{f}(u)]$

 \Rightarrow stable if $\varepsilon \leq \frac{\gamma}{\zeta L}$ (same bound)

Newton GD

Choose $Q(u) = (\nabla^2 f(u, h(u)))^{-1}$ (if $f \mu$ -strongly cvx and twice diff'ble) Stability is guaranteed if

 $\frac{L}{\mu} \leq \frac{\gamma}{\zeta}$

 \Rightarrow invariant under scaling of J

Not

- Subgradient methods
- Accelerated gradient method

General feedback optimization controllers

General *Slow* Dynamics

 $\dot{u} = \varepsilon g(h(u), u, z)$ $\dot{z} = \varepsilon k(h(u), u, z)$

- Saddle-point flows
- → requires exponential stability (open problem!) [Qu & Li, 2018]
 - Projected dynamics
- → primal variables: any-time constraints
- → dual variables: steady-state constraints

Theorem

- (g(x,u,z),k(x,u,z)) is L-Lipschitz in x
- (g(h(u), u, z), k(h(u), u, z)) is ℓ -Lipschitz
- \exists Lyap fct V(u, z) for the slow dynamics

$$\begin{split} \dot{V}(u,z) &\leq -\mu \|e(u,z)\|^2 \\ \|\nabla V(u,z)\| &\leq \kappa \|e(u,z)\| \end{split}$$

• \exists Lyap fct W(x, u) for the plant

$$\dot{W}(x,u) \le -\gamma \|x - h(u)\|^2$$
$$|\nabla_u W(x,u)\| \le \zeta \|x - h(u)\|$$

Then, asymptotic stability is guaranteed if

$$\epsilon < \frac{\gamma}{\zeta L(1+\frac{\kappa\ell}{\mu})}$$
.

Highlights and comparison

Weak assumptions on plant

- internal stability
- exponential convergence

Weak assumptions on cost

- Lipschitz gradient
- no convexity required

- potentially conservative bound (Singular Perturbation Analysis)
- directly useful for control design (no LMI/IQC stability test) [Nelson et al. 2017], [Colombino et al. 2018]
- analysis applicable to many continuous-time optimization algorithms

Hauswirth, Bolognani, Hug, & Dörfler "Timescale Separation in Autonomous Optimization" arXiv (2019)

CONCLUSIONS

E zürich

Conclusions

- Integral feedback control to solve constrained optimization problems
- A sound mathematical framework: projected dynamical systems
 - existence of solutions, time varying constraints, disconnected regions, ...
- Stability / convergence guarantees
 - singular-perturbation analysis yields design specifications
- A new unified approach to real-time power systems operations
 - any-time feasibility + steady state optimality
 - robustness to model mismatch / unmeasured disturbances / uncertain dynamics
 - almost-model-free design (only "sensitivities" are needed)
- Resource sharing problem: beyond power grids?



Real-Time Power System Operation





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