A model of anonymous influence with anti-conformist agents

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- We give exact description of the convergence by providing all possible absorbing classes and their conditions of existence.
- A further analysis is done supposing a large number of agents and considering several typical situations (e.g., small proportion of anti-conformists). This permits to demonstrate the possibility of occurrence of chaotic situations and opinion reversal.

Outline

1. **The model**
2. Analysis of convergence
3. Particular cases

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- Assumption 1: The state of the society is ruled by a homogeneous Markov chain.
- Assumption 2: Evolution of the opinion is *anonymous*, i.e., it depends only on the *number* of agents saying 'yes' at previous time step.

Aggregation rules

- *aggregation rule:* (individual to each agent) $p : \{0, 1, \dots, n\} \to [0, 1]$; $s \mapsto p(s)$: probability of saying 'yes' knowing that s agents say 'yes'.

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- Let p_i be agent i's aggregation rule. Supposing that the updating of opinion is independent accross the agents, the probability of transition from a state S to a state T is
$$\lambda_{S,T} = \prod_{i \in T} p_i(s) \prod_{i \notin T} (1 - p_i(s)).$$

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- Assumption 3: all purely conformist agents have in common the same minimum number of 'yes' for them to say 'yes' with positive chance, and the same minimum number of 'yes' for them to say 'yes' with probability 1 (similarly for anti-conformists):

$$\min\{s \mid p_i(s) > 0\} =: l^c + 1$$

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- l^c, l^a: maximum number of 'yes' without any effect on the probability of saying 'yes'
- r^c, r^a: maximum number of 'no' without any effect on the probability of saying 'yes'
- each mixed agent i has a convex combination coefficient α_i.

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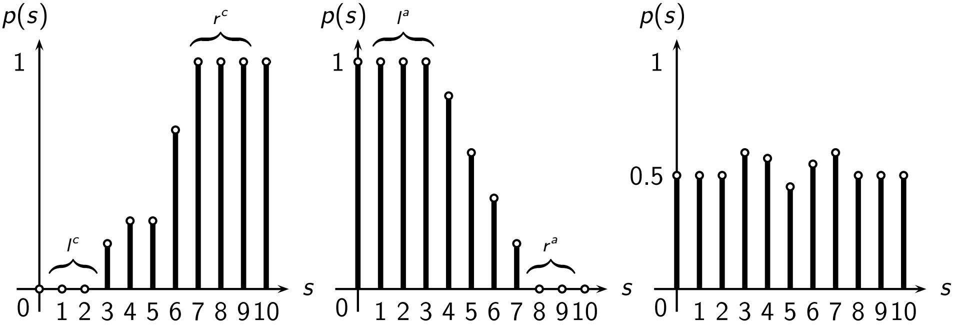

Figure: Typical aggregation rules for conformist agents (left), anti-conformists (center), and mixed agents (right) with $n = 10$ agents. The latter is obtained by mixing the 2 first ones with $\alpha = 0.5$

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  - When $\alpha_i < 1/2$, product A is chosen if $s < s_0$ (anti-conformism)
- Uncertainty in the decision: if the difference of utilities is smaller than Δ, the probability of choosing A is proportional to the difference of utilities

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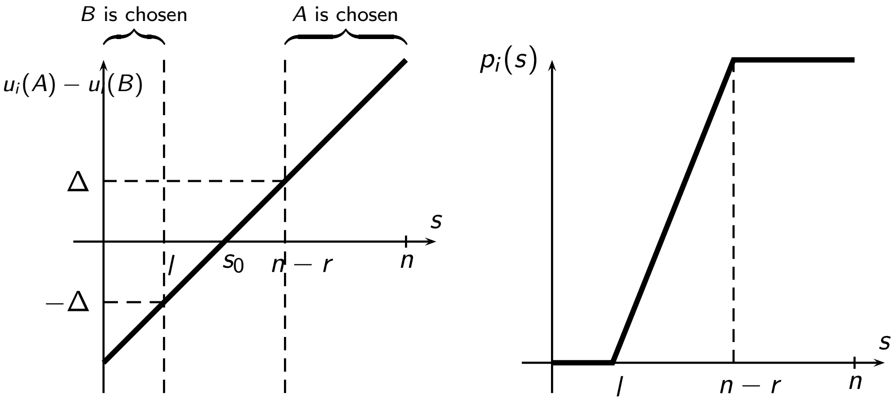

Figure: Choice among two products A and B: difference of utility functions with $\alpha_i > 1/2$ (left) and the resulting aggregation rule p_i (right)

Transition matrix and absorbing classes

- For a Markov chain with set of states E and transition matrix Λ and its associated digraph Γ, a *class* is a subset C of states such that for all states $e, f \in C$, there is a path in $\Gamma_{|C}$ from e to f, and C is maximal w.r.t. inclusion for this property.

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- An absorbing class C is *periodic of period k* if it can be partitioned in blocks $C_1, \ldots, C_k$ such that for $i = 1, \ldots, k$, every outgoing arc of every state $e \in C_i$ goes to some state in C_{i+1}, with the convention $C_{k+1} = C_1$.

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- When each $C_1, \ldots, C_k$ reduces to a single state, one may speak of *cycle of length k*

Basic properties of the transition matrix

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- We distinguish the *pure case* ($N^m = \emptyset$) and the *mixed case* ($N^m \neq \emptyset$).

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- We have the sure transitions

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- Hence we get

	$0 \leq s \leq l^c$	$l^c < s < n - r^c$	$n - r^c \leq s \leq n$
$0 \leq s \leq l^a$	N^a	$T \in [N^a, N]$	N
$l^a < s < n - r^a$	$T \in [\emptyset, N^a]$	$T \in 2^N$	$T \in [N^c, N]$
$n - r^a \leq s \leq n$	$\emptyset$	$T \in [\emptyset, N^c]$	N^c

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- Also

$$
\begin{array}{llll}
(i \in N^m) & p_i(s) = 0 & \Leftrightarrow & n - r^a \leq s \leq l^c \\
 & p_i(s) = 1 & \Leftrightarrow & n - r^c \leq s \leq l^a,
\end{array}
$$

and $0 < p_i(s) < 1$ for all other cases.

Basic properties of the transition matrix: the mixed case

- We have $\lambda_{\emptyset,T} > 0$ for every $T \in [N^a, N^a \cup N^m]$, and $\lambda_{N,T} > 0$ for every $T \in [N^c, N^c \cup N^m]$.
- Also

$$(i \in N^m) \qquad p_i(s) = 0 \quad \Leftrightarrow \quad n - r^a \leq s \leq l^c$$
$$p_i(s) = 1 \quad \Leftrightarrow \quad n - r^c \leq s \leq l^a,$$

  and $0 < p_i(s) < 1$ for all other cases.
- Hence

	$0 \leq s \leq l^c$	$l^c < s < n - r^c$	$n - r^c \leq s \leq n$
$0 \leq s \leq l^a$	$T \in [N^a, N^a \cup N^m]$	$T \in [N^a, N]$	N
$l^a < s < n - r^a$	$T \in [\emptyset, N^a \cup N^m]$	$T \in 2^N$	$T \in [N^c, N]$
$n - r^a \leq s \leq n$	$\emptyset$	$T \in [\emptyset, N^c \cup N^m]$	$T \in [N^c, N^c \cup N^m]$

Outline

1. The model

2. **Analysis of convergence**

3. Particular cases

Notation

- A sure transition $\lambda_{S,T} = 1$ is denoted by $S \xrightarrow{1} T$.

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- The notation is extended to collections of sets:

$$\mathcal{S} \xrightarrow{1} \mathcal{T} \quad \Leftrightarrow \quad \forall T \in \mathcal{T}, \exists S \in \mathcal{S} \text{ s.t. } \lambda_{S,T} > 0$$
$$\text{and } \forall S \in \mathcal{S}, \forall T \notin \mathcal{T}, \lambda_{S,T} = 0.$$

Case with no anti-conformists

With $N^a = \emptyset$, the transition table reduces to

$0 \leq s \leq l^c$	$l^c < s < n - r^c$	$n - r^c \leq s \leq n$
$\emptyset$	2^N	N

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$\emptyset$	2^N	N

Hence only the consensus states $\emptyset, N$ are absorbing states, and there is no other absorbing class.

Case with no conformists

With $N^c = \emptyset$, the transition table reduces to

$0 \leq s \leq l^a$	$l^a < s < n - r^a$	$n - r^a \leq s \leq n$
N	2^N	$\emptyset$

Case with no conformists

With $N^c = \emptyset$, the transition table reduces to

$0 \leq s \leq l^a$	$l^a < s < n - r^a$	$n - r^a \leq s \leq n$
N	2^N	$\emptyset$

Then there is only one absorbing class which is the cycle $\emptyset \xrightarrow{1} N \xrightarrow{1} \emptyset$.

The general result for the pure case ($N^m = \emptyset$) (1/3)

Assume that $N^m = \emptyset$, $N^a \neq \emptyset$ and $N^c \neq \emptyset$. There are twenty possible absorbing classes, grouped in the following categories:

(i) **Polarization:**

(1) N^a if and only if $n^c \geq (n - l^c) \vee (n - l^a)$;
(2) N^c if and only if $n^c \geq (n - r^c) \vee (n - r^a)$;

(ii) **Cycles:**

(3) $N^a \xrightarrow{1} \emptyset \xrightarrow{1} N^a$ if and only if $n - l^c \leq n^c \leq r^a$;
(4) $N^c \xrightarrow{1} N \xrightarrow{1} N^c$ if and only if $n - r^c \leq n^c \leq l^a$;
(5) $N^a \xrightarrow{1} N^c \xrightarrow{1} N^a$ if and only if $n^c \leq l^c \wedge l^a \wedge r^c \wedge r^a$;
(6) $\emptyset \xrightarrow{1} N^a \xrightarrow{1} N^c \xrightarrow{1} \emptyset$ if and only if $n^c \leq r^c \wedge r^a \wedge l^c$ and $n^c \geq n - r^a$;
(7) $N^a \xrightarrow{1} N \xrightarrow{1} N^c \xrightarrow{1} N^a$ if and only if $n^c \leq l^c \wedge l^a \wedge r^c$ and $n^c \geq n - l^a$;

(iii) **Fuzzy cycles:**

(8) $N^a \xrightarrow{1} [\emptyset, N^c] \xrightarrow{1} N^a$ if and only if $n^c \leq l^c \wedge l^a \wedge r^a$ and $r^c < n^c < n - l^c$;

(9) $N^c \xrightarrow{1} [N^a, N] \xrightarrow{1} N^c$ if and only if $n^c \leq r^c \wedge r^a \wedge l^a$ and $l^c < n^c < n - r^c$;

(10) $[\emptyset, N^c] \xrightarrow{1} [N^a, N] \xrightarrow{1} [\emptyset, N^c]$ if and only if $r^c \vee l^c < n^c \leq r^a \wedge l^a \wedge (n - l^c - 1) \wedge (n - r^c - 1)$;

(iv) **Fuzzy polarization:**

(11) $[\emptyset, N^a]$ if and only if $(n - l^c) \vee (r^a + 1) \leq n^c < n - l^a$;

(12) $[N^c, N]$ if and only if $(n - r^c) \vee (l^a + 1) \leq n^c < n - r^a$;

The general result for the pure case ($N^m = \emptyset$) (3/3)

(v) **Chaotic polarization:**

(13) $[\emptyset, N^a] \cup [\emptyset, N^c]$ if and only if $l^c \geq n - r^a$ and
$n^c \in \big( ]r^c, n - l^c[\cap]l^a, n - r^c[\big) \cup \Big(\big( ]l^a, n - r^a[\cup]l^c, n - r^c[\big) \cap]0, r^c] \Big)$;

(14) $[N^a, N] \cup [N^c, N]$ if and only if $l^a \geq n - r^c$ and
$n^c \in \big( ]l^c, n - r^c[\cap]r^a, n - l^c[\big) \cup \Big(\big( ]r^a, n - l^a[\cup]r^c, n - l^c[\big) \cap]0, l^c] \Big)$;

(15) $[\emptyset, N^a] \cup \{N^c\}$ if and only if $l^c + r^c = n - 1$, $r^a \geq r^c$, $l^c > l^a$ and
$l^a < n^c < (n - r^a) \wedge (n - l^c)$;

(16) $[N^c, N] \cup \{N^a\}$ if and only if $l^c + r^c = n - 1$, $l^a \geq l^c$, $r^c > r^a$ and
$r^a < n^c < (n - l^a) \wedge (n - r^c)$;

(17) $[\emptyset, N^c] \cup \{N^a\}$ if and only if $l^a + r^a = n - 1$, $l^c \geq l^a$, $n^c < n - r^c$ and
$n^c \in]r^c, n - l^c[\cup]l^c, r^c]$;

(18) $[N^a, N] \cup \{N^c\}$ if and only if $l^a + r^a = n - 1$, $r^c \geq r^a$, $n^c < n - l^c$ and
$n^c \in]l^c, n - r^c[\cup]r^c, l^c]$.

(19) $[\emptyset, N^a] \cup [N^c, N]$ if and only if $l^c + r^c = n - 1$ and
$l^a \vee r^a < n^c \leq l^c \wedge r^c$;

(vi) **Chaos:**

(20) 2^N otherwise.

General comments

- Cases (1) to (20) are not exclusive (see, e.g., cases (1) and (2) are both possible when $l^a = l^c$ and $r^a = r^c$)

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General comments

- Cases (1) to (20) are not exclusive (see, e.g., cases (1) and (2) are both possible when $l^a = l^c$ and $r^a = r^c$)
- A necessary condition for the existence of cycles and periodic classes (5) to (10) is that $n^a > n^c$
- There is no symmetry between N^a and N^c (e.g., $[\emptyset, N^a]$ is possible but not $[\emptyset, N^c]$). Rather, the symmetry is $S \longleftrightarrow N \setminus S$: all classes can be paired by taking complement of sets: (1) and (2), (3) and (4), (6) and (7), etc. Classes (5), (10), (19) and (20) are complement of themselves.

Case with only mixed agents

With $N^m = N$, the transition table becomes:

	$0 \leq s < n - r^c$	$n - r^c \leq s \leq l^a$	$l^a < s \leq n$
$0 \leq s < n - r^a$	2^N	N	2^N
$n - r^a \leq s \leq l^c$	$\emptyset$	does not occur	$\emptyset$
$l^c < s \leq n$	2^N	N	2^N

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$n - r^a \leq s \leq l^c$	$\emptyset$	does not occur	$\emptyset$
$l^c < s \leq n$	2^N	N	2^N

We can see that $\emptyset, N$ are not absorbing states, hence the only absorbing class is 2^N.

The general mixed case (1/3)

Assume that $N^m \neq \emptyset$, $N^a \neq \emptyset$ and $N^c \neq \emptyset$. Let $\overline{N}^a = N^a \cup N^m$ and $\overline{N}^c = N^c \cup N^m$. There are twenty possible absorbing classes which are:

(i) **Fuzzy polarization:**

(1) $[N^a, \overline{N}^a]$ if and only if $n^c \geq (n - l^c) \vee (n - l^a)$;
(2) $[N^c, \overline{N}^c]$ if and only if $n^c \geq (n - r^c) \vee (n - r^a)$;
(3) $[\emptyset, \overline{N}^a]$ if and only if $(n - l^c) \vee (r^a + 1) \leq n^c < n - l^a$;
(4) $[N^c, N]$ if and only if $(n - r^c) \vee (l^a + 1) \leq n^c < n - r^a$;

The general mixed case (2/3)

(ii) **Fuzzy cycles:**

(5) $[N^a, \overline{N}^a] \xrightarrow{1} \emptyset \xrightarrow{1} [N^a, \overline{N}^a]$ if and only $n - l^c \leq n^c \leq r^a - n^m$;

(6) $[N^c, \overline{N}^c] \xrightarrow{1} N \xrightarrow{1} [N^c, \overline{N}^c]$ if and only if $n - r^c \leq n^c \leq l^a - n^m$;

(7) $[N^a, \overline{N}^a] \xrightarrow{1} [N^c, \overline{N}^c] \xrightarrow{1} [N^a, \overline{N}^a]$ if and only if $n^c + n^m \leq l^c \wedge l^a \wedge r^c \wedge r^a$;

(8) $\emptyset \xrightarrow{1} [N^a, \overline{N}^a] \xrightarrow{1} [N^c, \overline{N}^c] \xrightarrow{1} \emptyset$ if and only if $n^c + n^m \leq r^c \wedge r^a \wedge l^c$ and $n^c \geq n - r^a$;

(9) $[N^a, \overline{N}^a] \xrightarrow{1} N \xrightarrow{1} [N^c, \overline{N}^c] \xrightarrow{1} [N^a, \overline{N}^a]$ if and only if $n^c + n^m \leq l^c \wedge l^a \wedge r^c$ and $n^c \geq n - l^a$;

(10) $[N^a, \overline{N}^a] \xrightarrow{1} [\emptyset, \overline{N}^c] \xrightarrow{1} [N^a, \overline{N}^a]$ if and only if $n^c + n^m \leq l^c \wedge l^a \wedge r^a$ and $r^c - n^m < n^c < n - l^c$;

(11) $[N^c, \overline{N}^c] \xrightarrow{1} [N^a, N] \xrightarrow{1} [N^c, \overline{N}^c]$ if and only if $n^c + n^m \leq r^c \wedge r^a \wedge l^a$ and $l^c - n^m < n^c < n - r^c$;

(12) $[\emptyset, \overline{N}^c] \xrightarrow{1} [N^a, N] \xrightarrow{1} [\emptyset, \overline{N}^c]$ if and only if $l^c \vee r^c < n^c + n^m \leq r^a \wedge l^a \wedge (n - (l^c + 1)) \wedge (n - (r^c + 1))$;

The general mixed case (3/3)

- **Chaotic polarization:**

(13) $[\emptyset, \overline{N}^a] \cup [\emptyset, \overline{N}^c]$ if and only if $l^c \geq n - r^a$ and

$n^c \in \big(]r^c - n^m, n - l^c[\cap]l^a, n - r^c[\big)\cup\big((]l^a - n^m, n - r^a[\cup]l^c - n^m, n - r^c$

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(15) $[\emptyset, \overline{N}^a] \cup [N^c, \overline{N}^c]$ if and only if $l^c + r^c = n - 1$, $r^c \leq r^a$, $l^c > l^a$, $n^c < n - l^c$ and $l^a < n^c + n^m < n - r^a$;

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(19) $[\emptyset, \overline{N}^a] \cup [N^c, N]$ if and only if $l^c + r^c = n - 1$ and $l^a \vee r^a < n^c \leq l^c \wedge r^c$;

- **Chaos:**

(20) 2^N otherwise.

Outline

Extreme values for l^a, l^c, r^a, r^c

- $l^c = l^a = r^c = r^a = 0$.
  Under this assumption, the table of transitions reduces to its central box 2^N. Then only class 2^N remains. All special phenomena (polarization, cycles, etc.) disappear: they are only due to the presence of thresholds in the aggregation rules.

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- $l^a + r^a = l^c + r^c = n - 1$ (aggregation rules become threshold functions).
  Under this assumption, the middle row and middle column of the table of transitions disappear: only classes (1), (2) and cycles (3) and (7) remain: fuzzy and chaotic behaviors disappear.

Outline

1. The model
2. Analysis of convergence
3. **Particular cases**
    3.1. Extreme values for l^a, l^c, r^a, r^c
    3.2. **n tends to infinity**

Notation

- We divide all parameters by n and make n tends to infinity.
- Hence, $n^a, l^a, r^a, n^c, l^c, r^c \in [0, 1]$ and

$$n^c = 1 - n^a$$
$$l^a + r^a < 1$$
$$l^c + r^c < 1,$$

- Given an aggregation rule with l, r specified, we introduce

$$\gamma = \frac{1}{1 - r - l}$$

  Note that $\gamma \in \left[\frac{1}{1-l}, \infty\right[$.
- Interpretation: l is the *firing threshold*, γ the *reactiveness*, $1 - r$ is the *saturation threshold*.
- We assume throughout $N^m = \emptyset$.

Situation 1: $l^a = l^c = l$ and $r^a = r^c = r$

Only the following absorbing classes are possible:

1. N^a if and only if $n^a \leq l$; (*polarization*)
2. N^c if and only if $n^a \leq 1 - l - \frac{1}{\gamma}$; (*polarization*)
3. $N^a \xrightarrow{1} N^c \xrightarrow{1} N^a$ if and only if $n^a \geq 1 - l$ and $n^a \geq \frac{1}{\gamma} + l$; (*cycle*)
4. 2^N otherwise. (*chaos*)

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4. 2^N otherwise. (*chaos*)

We make a "phase diagram" with the three parameters n^a, l, γ showing the possible absorbing classes, keeping in mind that $\gamma \geq \frac{1}{1-l}$.

Situation 1: $l^a = l^c = l$ and $r^a = r^c = r$

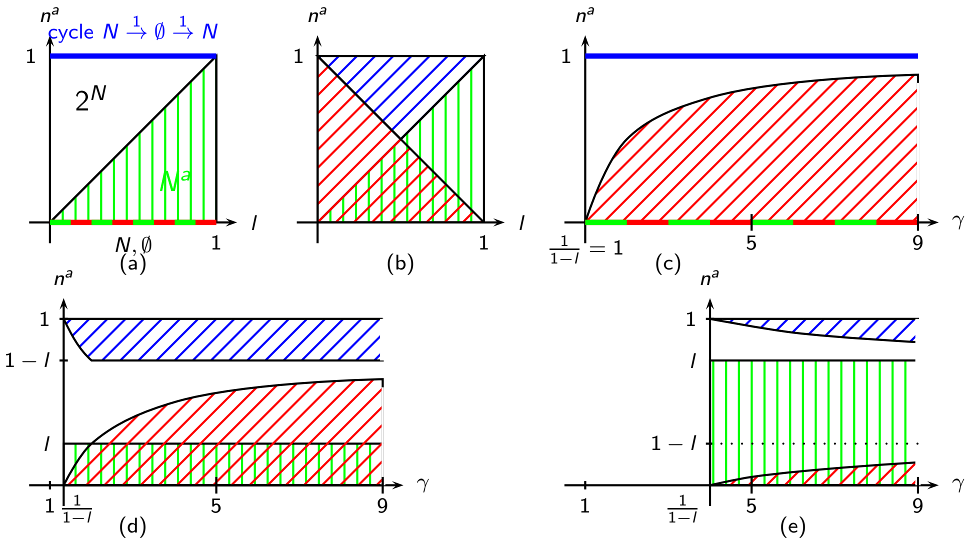

Figure: Phase diagram for Situation 1: (a) γ has minimum value $\frac{1}{1-l}$ $(r = 0)$; (b) $\gamma \to \infty$; (c) $l = 0$; (d) $l \in [0, 1/2]$; (e) $l \in [1/2, 1[$. Color code: white$=2^N$, blue=cycle, red$=N^c$, green$=N^a$

Comments on Situation 1

- When n^a increases from 0 to 1, we go from consensus, next to polarization, next to chaos, and finally to a cycle.

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- Similarly, when the firing threshold is high (e), there is a cascade effect leading all conformists to say 'no', if the proportion of anti-conformists is not too small but less than the firing threshold.
- The two cases (c) and (e) show how, in a society of conformists, the opinion can be manipulated by introducing a relatively small proportion of anti-conformists. The final opinion depends essentially on the firing threshold.

Situation 2: $l^a = r^a$ and $l^c = r^c$

Note that l^a, l^c vary in $[0, 1/2[$, and $\gamma^a = \frac{1}{1-2l^a}$, $\gamma^c = \frac{1}{1-2l^c}$.

Situation 2: $l^a = r^a$ and $l^c = r^c$

Note that l^a, l^c vary in $[0, 1/2[$, and $\gamma^a = \frac{1}{1-2l^a}$, $\gamma^c = \frac{1}{1-2l^c}$.
The possible absorbing classes are:

- N^a, N^c iff $n^a \leq l^a$ and $n^a \leq l^c$ (*polarization*)
- cycle $N^a \xrightarrow{1} N^c \xrightarrow{1} N^a$ iff $n^a \geq 1 - l^a$ and $n^a \geq 1 - l^c$ (*cycle*)
- periodic class $[\emptyset, N^c] \xrightarrow{1} [N^a, N] \xrightarrow{1} [\emptyset, N^c]$ iff $n^a \geq 1 - l^a$ and $l^c < n^a < 1 - l^c$ (*fuzzy cycle*)
- $[\emptyset, N^a], [N^c, N]$ iff $n^a \leq l^c$ and $l^a < n^a < 1 - l^a$ (*fuzzy polarization*)
- 2^N (*chaos*) otherwise.

Situation 2: $l^a = r^a$ and $l^c = r^c$

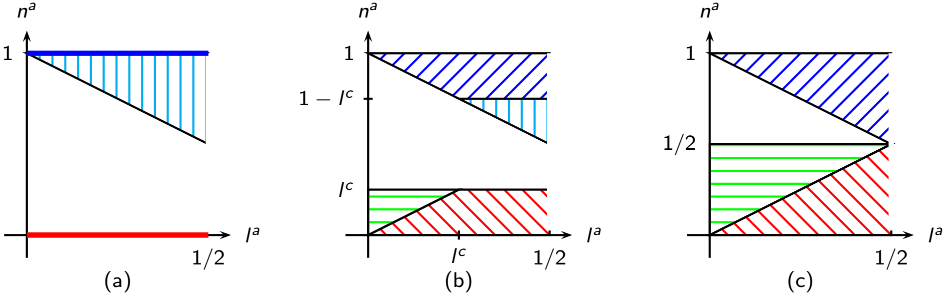

Figure: Phase diagram for Situation 2: (a) $l^c = 0$; (b) $l^c \in]0, 1/2[$; (c) $l^c \to 1/2$. Color code: white=chaos, blue=cycle, cyan=fuzzy cycle, red=polarization, green=fuzzy polarization

Comments on Situation 2

- Compared to Situation 1, the chaos case takes a relatively large area, which grows as l^c or l^a tend to 0. In particular, when conformist agents have a low reactivity, a very small proportion of anti-conformists in the society suffices to make it chaotic.

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- Compared to Situation 1, the chaos case takes a relatively large area, which grows as l^c or l^a tend to 0. In particular, when conformist agents have a low reactivity, a very small proportion of anti-conformists in the society suffices to make it chaotic.
- Contrarily to Situation 1, there is no cascade effect, because the absorbing states N^a and N^c always appear together. This polarization effect happens if the anti-conformists are not "seen" by the conformists, and all the more since the anti-conformists are reactive. Less reactive anti-conformists have a tendency to provoke fuzzy polarization.

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- A limit phenomenon happens when l^a, l^c, n^a tend all together to $1/2$: a kind of “triple point” appears (see (c)), in the sense that the three types of behavior (polarization, fuzzy polarization and cycle) happen together, which is also visible for Situation 1 (Figure 3(b)).

Situation 3: The case where n^a tends to 0

Let us put $n^a = \epsilon > 0$, arbitrarily small. Therefore, $n^c = 1 - \epsilon$. The following classes remain possible:

- (1) N^a iff $l^c \wedge l^a \geq \epsilon$; ('no' consensus)
- (2) N^c iff $r^c \wedge r^a \geq \epsilon$; ('yes' consensus)
- (3) $N^a \xrightarrow{1} \emptyset \xrightarrow{1} N^a$ iff $l^c \geq \epsilon$ and $r^a \geq 1 - \epsilon$; ($\approx$ (1))
- (4) $N^c \xrightarrow{1} N \xrightarrow{1} N^c$ iff $r^c \geq \epsilon$ and $l^a \geq 1 - \epsilon$; ($\approx$ (2))
- (11) $[\emptyset, N^a]$ iff $l^a < \epsilon$, $l^c \geq \epsilon$ and $r^a < 1 - \epsilon$; ($\approx$ (1))
- (12) $[N^c, N]$ iff $r^a < \epsilon$, $r^c \geq \epsilon$ and $l^a < 1 - \epsilon$; ($\approx$ (2))
- (13) $[\emptyset, N^a] \cup [\emptyset, N^c]$ iff $l^c < \epsilon$, $r^c < \epsilon$ and $r^a > 1 - \epsilon$; (chaotic 'no')
- (14) $[N^a, N] \cup [N^c, N]$ iff $l^c < \epsilon$, $r^c < \epsilon$ and $l^a > 1 - \epsilon$; (chaotic 'yes')
- (20) 2^N otherwise.

Situation 3: The case where n^a tends to 0

Suppose $l^c, r^c > \epsilon$: the conformists cannot "see" the anti-conformists.

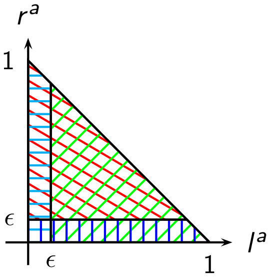

Figure: Phase diagram for Situation 3, with $l^c, r^c > \epsilon$. Color code: green=N^a, red=N^c, cyan=almost consensus 'no' ((3) or (11)), blue=almost consensus 'yes' (4) or (12)

As N^a, N^c are always possible, no cascades of 'yes' or 'no' may occur. The situation is almost identical to the conformist model.

Situation 3: The case where n^a tends to 0

Suppose that the conformists have very small l^c, r^c ($< \epsilon$).

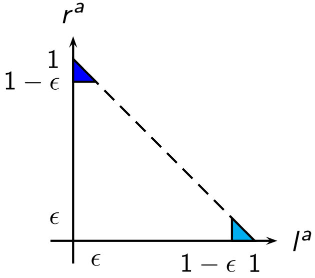

Figure: Phase diagram for Situation 3, with $l^c, r^c < \epsilon$. Color code: blue=chaotic 'no' (13), cyan=chaotic 'yes' (14), white=chaos (20)

No consensus is possible, even in a weak sense, and only chaotic situations arise.

Outline

Example of cascade

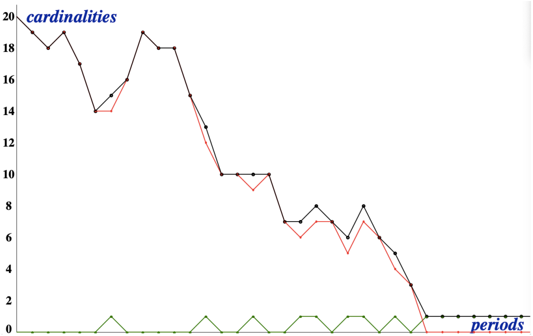

Figure: Evolution of the number of 'yes' (red: conformists, green: anticonformists, black: total) with 19 conformists and 1 anti-conformist, $l = 1$, $r = 0$.

This corresponds to situation 1 ($l^a = l^c = l$ and $r^a = r^c = r$). Starting from N ('yes' consensus), a society with only conformist agents would remain for ever in the same state. Introducing one anti-conformist makes the society converges to almost a 'no' consensus.

Example of shock (context: adoption of new technology)

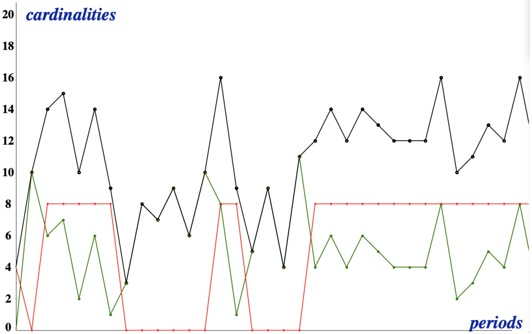

Figure: Evolution of the number of adopters of product A (red: conformists, green: anticonformists, black: total) illustrating the existence of shocks for conformists

Shocks are present in classes (15) to (19), when conformists or anti-conformists have threshold function. Here $n^c = 8$, $n^a = 12$, $l^a = r^a = 0$ and $l^c + 1 = r^c = 10$, leading to class $[\emptyset, N^a] \cup [N^c, N]$ (19).

Some take-home messages

- The introduction of anti-conformists in a society, even in a very small proportion, prevents from reaching a consensus and causes either polarization or various instabilities: cycles, chaotic behavior, etc.

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- Special phenomena like polarization, cycles and their more or less fuzzy versions are only due to the presence of thresholds (firing threshold and saturation threshold) in the aggregation rule. If on the contrary $l^a = r^a = l^c = r^c = 0$, there is only one class containing all states (chaos).

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- Mixed agents do not change the number of possible absorbing classes, but their presence blurs them, because the opinion of mixed agents always oscillate between conformism and anti-conformism.

Thank you for your attention!

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