Towards a Theory of Scalable Control

Anders Rantzer

Automatic Control LTH
Lund University
Sweden

New Generation of Heating Networks

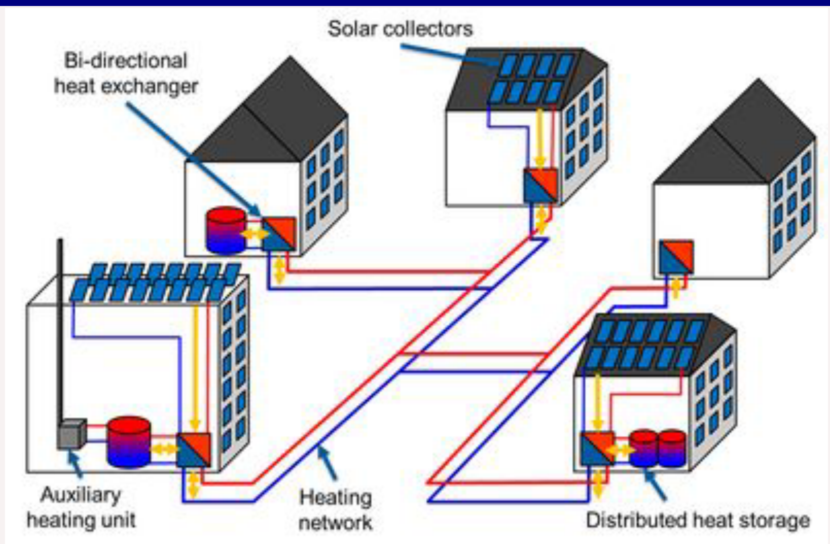

LUNDS NYA STADSDEL BRUNNSHÖG 23 september 2019 12:12

Lund först i världen med ljummen fjärrvärme

Kranen öppnas på tisdag. Brunnshög blir den första stadsdelen i världen som får ett lågvärmenät. Vattnet är tjugo grader kallare än i det vanliga fjärrvärmenätet. Men vad ska det vara bra för?

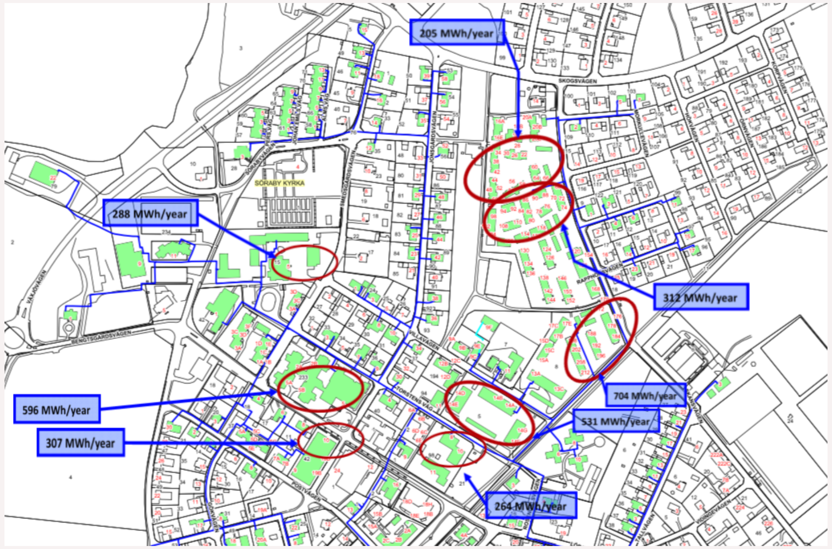
Kalle Kniivilä + Följ

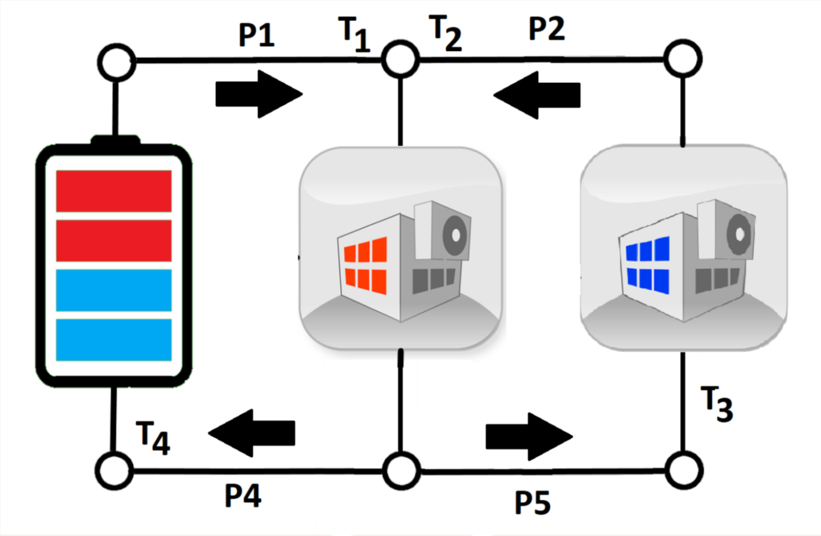
New Generation of Heating Networks

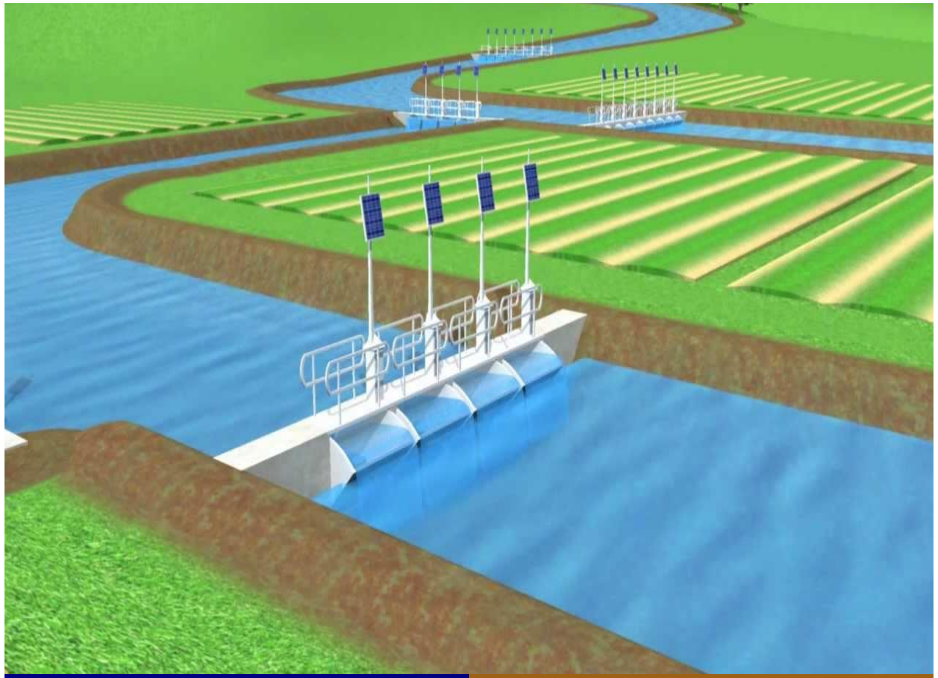


Fossil Free Sweden 2045 !

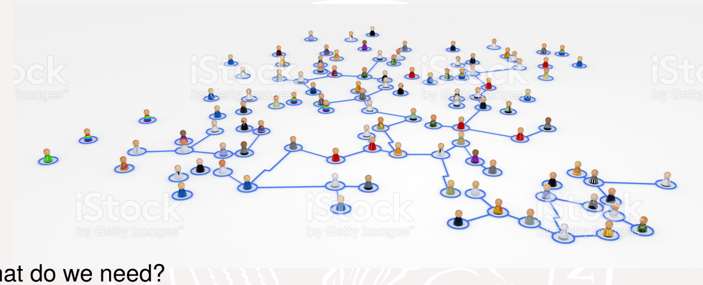
Heating Networks







Towards a Scalable Theory of Control



What do we need?

- Scalable Synthesis
- Scalable Verification
- Scalable Modeling
- Scalable Objectives

Background

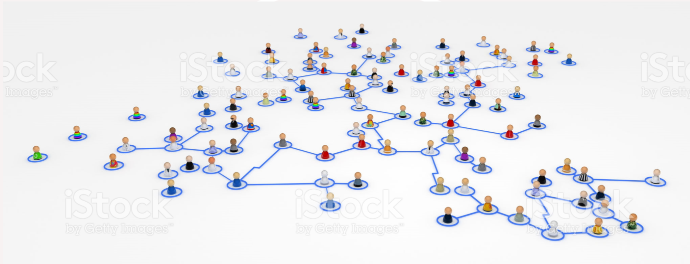
Books on large-scale control and coordination:

Mesarovic, Macko, Takahara (1970)
Singh, Titli (1978)
Findeisen (1980)

Optimization of structured controllers:

Spatially invariant systems: Bamieh, Paganini, Dahleh (2002)
Distributed controllers: D'Andrea and Dullerud (2003)
Quadratic invariance: Rotkowitz and Lall (2002)
Low rank coordination: Madjidian and Mirkin (2014)
Scalability using positivity: Rantzer (2015)
Systems Level Synthesis: Wang, Matni, Doyle (2018)

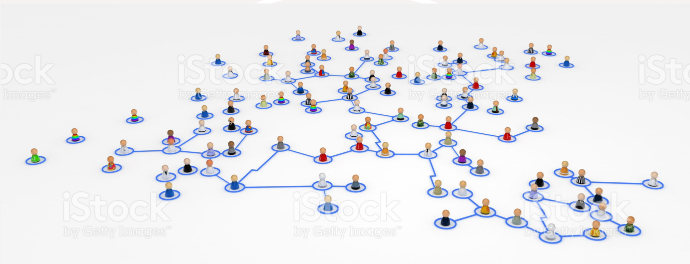
Three Widespread Myths



Three widespread myths

- Scalable controllers are hard to optimize
- H_∞ optimal controllers are not scalable
- H_2 optimal controllers are not scalable

Three Widespread Myths



Three widespread myths

- Scalable controllers are hard to optimize
- H_∞ optimal controllers are not scalable
- H_2 optimal controllers are not scalable

They are all wrong!

Outline

- **Network realizable control** (Systems Level Synthesis)
  - Don't look for sparse transfer functions
  - Connection to Internal Model Control
- Scalable H_∞ optimal synthesis
  - H_∞ optimal static controllers
  - H_∞ optimal dynamic controllers
- Scalable H_2 optimal synthesis
  - a transportation example
- Concluding remarks

Example: River Dams

Consider a model of three water dams along a river:

$$\begin{aligned} x_1(t+1) &= 0.9x_1(t) - u_1(t) \\ x_2(t+1) &= 0.1x_1(t) + 0.8x_2(t) + u_1(t) - u_2(t) \\ x_3(t+1) &= 0.2x_2(t) + 0.7x_3(t) + u_2(t) - u_3(t) \end{aligned}$$

Information propagates downstream.

The transfer function from (u_1, u_2, u_3) to (x_1, x_2, x_3) is triangular:

$$\mathbf{P}(z) = \begin{bmatrix} * & 0 & 0 \\ * & * & 0 \\ * & * & * \end{bmatrix}$$

The localized structure of the state realization is lost in the transfer matrix.

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Closed Loop Convexity

In general, specifications are computationally tractable only if they are convex constraints on the closed loop map

$$\mathbf{C}(I + \mathbf{PC})^{-1}.$$

Sparsity constraints on the matrix $\mathbf{C}$ are not closed loop convex and very difficult to enforce.

However, there is a better choice...

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However, there is a better choice...

Network Realizability

Given a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, a transfer matrix $\mathbf{G}$ is said to be *network realizable on* $\mathcal{G}$ if it has a stabilizable and detectable realization $\mathbf{G}(z) = C(zI - A)^{-1}B + D$ with

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{ccc|ccc} A_{11} & \dots & A_{1N} & B_1 & & 0 \\ \vdots & & \vdots & & \ddots & \\ A_{N1} & \dots & A_{NN} & 0 & & B_N \\ \hline C_{11} & \dots & C_{1n} & D_1 & & 0 \\ \vdots & & \vdots & & \ddots & \\ C_{N1} & \dots & C_{NN} & 0 & & D_N \end{array}\right]$$

where $A_{ij} = 0$ and $C_{ij} = 0$ for $(i, j) \notin \mathcal{E}$.

Network Realizability is a Closed Loop Convex Property!

Theorem 1
Network Realizability is preserved by addition and proper inversion!
If $\mathbf{G}_1$ and $\mathbf{G}_2$ are stable and network realizable, then so is $\mathbf{G}_1\mathbf{G}_2$.

The proof is straightforward
For example, if $\mathbf{G}(z) = C(zI - A)^{-1}B + D$ and D is invertible, then $\mathbf{G}^{-1}$ has the realization

$$\left[\begin{array}{c|c} A - BD^{-1}C & BD^{-1} \\ \hline -D^{-1}C & D^{-1} \end{array}\right].$$

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Network Realizable Control of Stable Plants

Theorem 2
Suppose that the transfer matrix $\mathbf{P}$ is strictly proper, stable and network realizable on $\mathcal{G}$. Then, the controller $\mathbf{C}$ is stabilizing and network realizable if and only if $\mathbf{Q} = \mathbf{C}(I + \mathbf{PC})^{-1}$ is stable and network realizable.

Enforcing network realizability of $\mathbf{Q}$ can be done by convex optimization.

But how do we construct a realization of $\mathbf{C}$ after optimizing $\mathbf{Q}$?

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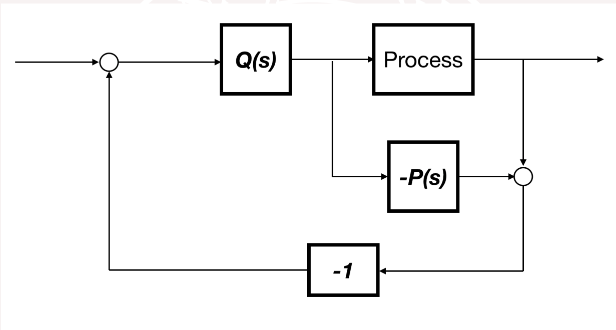
Internal Model Control (IMC)

[Garcia/Morari, 1982]



If both $\mathbf{P}$ and $\mathbf{Q}$ have network realizations on a given graph, then after proper ordering of the states and block partitioning of the matrices also the IMC controller will be a network realization on that graph.

Example: River Dams

$$\begin{bmatrix} x_1^+ \\ x_2^+ \\ x_3^+ \end{bmatrix} = \begin{bmatrix} 0.9 & 0 & 0 \\ 0.1 & 0.8 & 0 \\ 0 & 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -u_1 \\ u_1 - u_2 \\ u_2 - u_3 \end{bmatrix}$$

Let $\mathbf{Q}(z) = E(zI - A)^{-1}F$ be the desired map from reference to input:

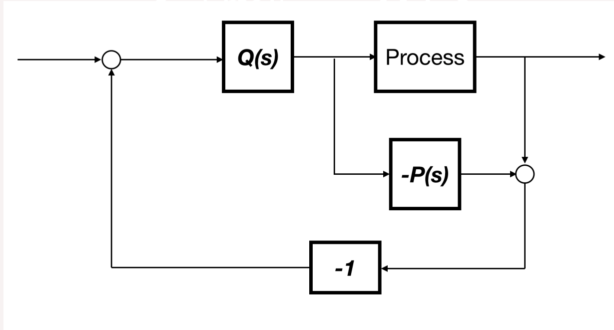
$$\left[\begin{array}{c|c} E & F \\ \hline G & H \end{array}\right] = \left[\begin{array}{ccc|ccc} E_{11} & 0 & 0 & F_1 & 0 & 0 \\ E_{21} & E_{22} & 0 & 0 & F_2 & 0 \\ 0 & E_{32} & E_{33} & 0 & 0 & F_3 \\ \hline G_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_3 & 0 & 0 & 0 \end{array}\right]$$

Example: River Dams

Then the controller $\mathbf{C} = \mathbf{Q}(I - \mathbf{P}\mathbf{Q})^{-1}$ has the realization

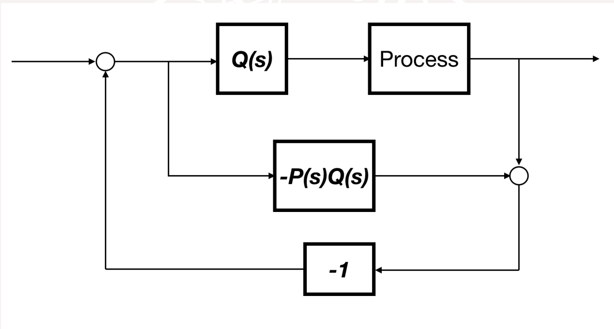
$$\begin{bmatrix} \hat{x}_1^+ \\ \xi_1^+ \\ \hat{x}_2^+ \\ \xi_2^+ \\ \hat{x}_3^+ \\ \xi_3^+ \end{bmatrix} = \left[\begin{array}{cc|cc|cc} 0.9 & -G_1 & 0 & 0 & 0 & 0 \\ F_1 & E_{11} & 0 & 0 & 0 & 0 \\ \hline 0.1 & G_1 & 0.8 & -G_2 & 0 & 0 \\ 0 & E_{21} & F_2 & E_{22} & 0 & 0 \\ \hline 0 & 0 & 0.2 & G_2 & 0.7 & -G_3 \\ 0 & 0 & 0 & E_{32} & F_3 & E_{33} \end{array}\right] \begin{bmatrix} \hat{x}_1 \\ \xi_1 \\ \hat{x}_2 \\ \xi_2 \\ \hat{x}_3 \\ \xi_3 \end{bmatrix} - \begin{bmatrix} 0 \\ e_1 \\ 0 \\ e_2 \\ 0 \\ e_3 \end{bmatrix}.$$

Internal Model Control (IMC)



What if $P(s)$ is unstable?

Internal Model Control (IMC)



Design stable $Q(s)$ to make also $Q(s)P(s)$ stable!

Network Realizability is a Closed Loop Convex Property!

Theorem 3
Consider $\mathbf{P}$ and $\mathbf{C}$ such that $\mathbf{P}$ is strictly proper and define the closed loop

$$\mathbf{H} = \begin{bmatrix} I & -\mathbf{P} \\ \mathbf{C} & I \end{bmatrix}^{-1} = \begin{bmatrix} (I+\mathbf{PC})^{-1} & \mathbf{P}(I+\mathbf{CP})^{-1} \\ -\mathbf{C}(I+\mathbf{PC})^{-1} & (I+\mathbf{CP})^{-1} \end{bmatrix}.$$

Then the following two statements are equivalent:

(i) Both $\mathbf{P}$ and $\mathbf{C}$ are network realizable on $\mathcal{G}$.
(ii) $\mathbf{H}$ is network realizable on $\mathcal{G}$.

The following two statements are also equivalent:

(iii) Both $\mathbf{PC}$ and $\mathbf{C}$ are network realizable on $\mathcal{G}$.
(iv) Both $(I+\mathbf{PC})^{-1}$ and $\mathbf{C}(I+\mathbf{PC})^{-1}$ are network realizable on $\mathcal{G}$.

Outline

- Network realizable control
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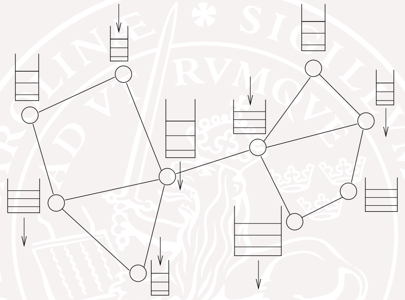
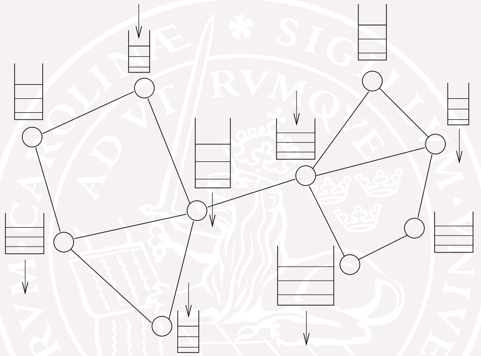
- **Scalable H_∞ optimal synthesis**
  - H_∞ optimal static controllers
  - H_∞ optimal dynamic controllers
- Scalable H_2 optimal synthesis
  - a transportation example
- Concluding remarks

Dynamic Buffer Networks



- Producers, consumers and storages
- Examples: water, power, traffic, data
- Discrete/continuous, stochastic/deterministic
- Multiple commodities, human interaction

Scalable Synthesis for Dynamic Buffer Networks



A common approach is to just focus on stationary optimality or equilibrium conditions and ignore dynamics. ("No arbitrage")

When is this justified for a control system?

H_∞ Optimal Static Control on Networks

Problem:
Given a graph $(\mathcal{V}, \mathcal{E})$ and

$$\dot{x}_i = a_i x_i + \sum_{(i,j)\in\mathcal{E}} (u_{ij} - u_{ji}) + w_i \qquad i \in \mathcal{V}$$

find control law $u = Kx$ that minimizes the H_∞ norm of the map from w to (x, u).

Solution:
An optimal control law when $a_i < 0$ is given by

$$u_{ij} = x_i/a_i - x_j/a_j \qquad (i,j) \in \mathcal{E}.$$

[Lidström/Rantzer, ACC2016]

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[Lidström/Rantzer, ACC2016]

Example

$$\dot{x} = \underbrace{-\operatorname{diag}(1,3,2)}_{A}\, x + \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}}_{B} u + w$$

Find controller $u = Kx$ that minimizes the H_∞ gain from w to (x, u). The Riccati solution gives

$$K_1 = \begin{bmatrix} 0.93 & -0.11 & 0.00 \\ -0.05 & -0.17 & -0.01 \\ 0.04 & 0.16 & -0.26 \end{bmatrix}$$

Our theorem gives another optimal solution:

$$K_2 = \begin{bmatrix} 1 & -\frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & \frac{1}{3} & -\frac{1}{2} \end{bmatrix}$$

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Structure Preserving Static Feedback

Problem

Consider the system $\dot{x} = Ax + Bu + w$ with A symmetric and Hurwitz. Find a state feedback controller $u = Kx$ that minimizes the H_∞ norm of the map from w to (x, u) in the closed loop system $\dot{x} = (A + BK)x + w$.

Theorem

A solution is given by $u = K_* x$ where $K_* = B^T A^{-1}$. The minimal value of the norm is $\sqrt{\|(A^2 + BB^T)^{-1}\|}$.

Proof idea

$K_* = B^T A^{-1}$ minimizes the static gain. Other frequencies are better off.

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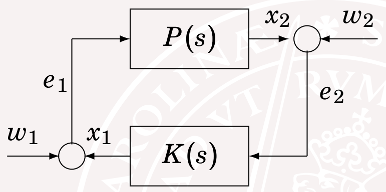
Frequency Weighted Specifications



$$e_1 = Ke_2 + w_1$$
$$e_2 = Pe_1 + w_2$$

Disturbance rejection:
The transfer functions $(I+PK)^{-1}$ and $(I+PK)^{-1}P$ should be small for low frequencies. ("Integral action")

Measurement errors:
The transfer functions $K(I+PK)^{-1}$ and $PK(I+PK)^{-1}$ should be small for high frequencies.

Structure Preserving H_∞ Control

Theorem
Let $P(s) = (sI - A)^{-1}B$ with A symmetric negative definite. The problem

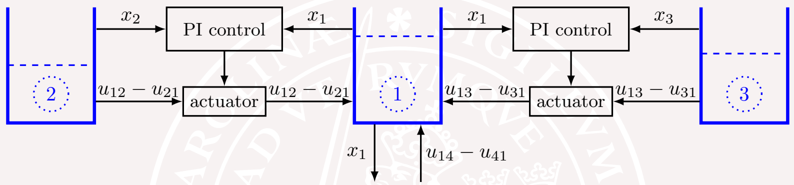
$$\begin{aligned} &\text{Minimize} && \|(I+KP)^{-1}K\|_\infty \\ &\text{subject to} && \|\tfrac{1}{s}P(I+KP)^{-1}\|_\infty \leq \tau \\ & && K \text{ stabilizing} \end{aligned}$$

is solved by

$$\widehat{K}(s) = \frac{\|(A^{-1}B)^\dagger\|}{\tau}\left(B^TA^{-2} - \frac{1}{s}B^TA^{-1}\right).$$

provided that $\tau \geq \sqrt{\|B^TA^{-4}B\|}$.

Optimal Network Control with Edge Integrators



Given a graph $(\mathcal{V}, \mathcal{E})$, let $P(s)$ be the transfer matrix from u to x given by $\dot{x}_i = a_i x_i + \sum_{(i,j)\in\mathcal{E}}(u_{ij} - u_{ji})$, $i \in \mathcal{V}$ with $a_i < 0$. Then $\widehat{K}(s)$ is a separate PI controller for each graph edge:

$$\begin{cases} \dot{z}_{ij} = k(x_i/a_i - x_j/a_j) \\ u_{ij} = z_{ij} - x_i/a_i^2 + x_j/a_j^2 \end{cases}$$

(Works if the graph is a tree!)

Optimal Network Control with Node Integrators

Given a graph $(\mathcal{V}, \mathcal{E})$, let the plant be given by

$$\dot{x}_i = a_i x_i + b_i u_i + \sum_{(i,j)\in\mathcal{E}} (u_{ij} - u_{ji}) \qquad i \in \mathcal{V}$$

with $a_i < 0$. Then $\widehat{K}(s)$ is the map from x to u given by

$$\begin{cases} \dot{z}_i = x_i \\ u_{ij} = z_i/a_i - x_i/a_i^2 - z_j/a_j + x_j/a_j^2 \\ u_i = b_i(z_i/a_i - x_i/a_i^2) \end{cases}$$

(There is a problem if all b_i are zero. Why?)

Limitations due to Graph Structure

Let $A = -I$ while B is an oriented incidence matrix, e.g.

$$B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{bmatrix}.$$

If the graph is a tree, B has full column rank and

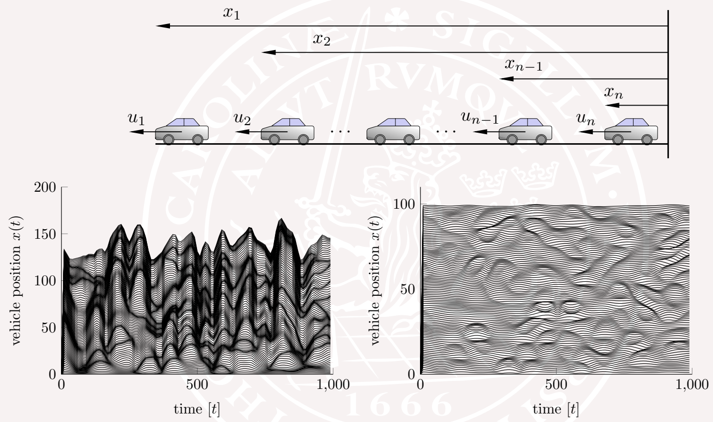
$$\|F_{\widehat{K}}\|_\infty = \sqrt{\|(B^T B)^{-1}\|} = \frac{1}{\sqrt{\lambda_2}},$$

where λ_2 is the algebraic connectivity of the graph. In two extreme cases, a star graph and a one-dimensional path, λ_2 is

$$1 \quad \text{and} \quad \left(2\sin\frac{\pi}{2n}\right)^2$$

respectively.

Relative versus absolute measurements



Centralized information does not help, as long as only local relative measurements are available!

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$\mathbf{H}_2$ Optimal Control

1. Optimal state feedback gains are dense.
2. They are unique.

Bad for large scale problems!

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Why do we want sparse feedback matrices?

$$u = \begin{bmatrix} & * & & & & * & & & & * & & * & & * & \\ * & * & & & & & & & & & * & * & * & * & \\ & & & & & & & & & & * & * & & * & \\ & & & * & & & & * & & & & & & * & * \\ * & * & & * & & * & & * & & * & * & * & & & * \\ & & & & & * & * & & & * & & & & * & * \\ * & * & * & & & & & * & & * & & * & & * & * \\ * & & * & * & & & * & & * & & & & & & \\ & & & & * & & & & & & * & * & & & \\ * & & & & * & & & * & & * & & & & & * \end{bmatrix} x$$

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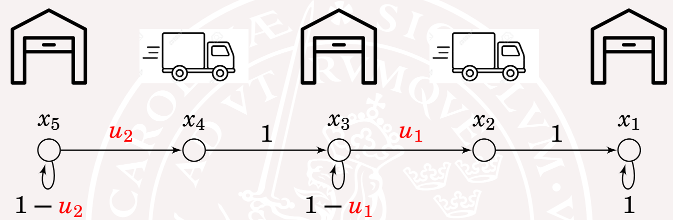
Sparsity $\Longrightarrow$ Distributed computation of u.

Why do we want sparse feedback matrices?

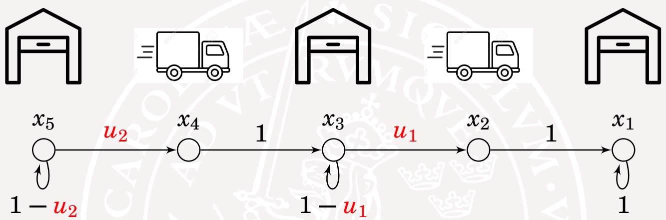
Sparsity $\not\Longleftarrow$ Distributed computation of u.

A Curious Example



European Control Conference, Limassol, Cyprus 2018
M. Heyden, R. Pates, A. Rantzer
A Structured Linear Quadratic Controller for Transportation Problems

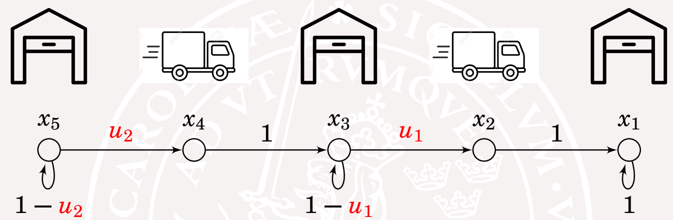
A Curious Example



$$\min_{u[0],u[1],\dots} \sum_{k=0}^{\infty} a^{2k}\left(x_1[k]^2 + x_3[k]^2 + x_5[k]^2\right)$$

$$\text{s.t. } x[k+1] = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x[k] + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} u[k],\ x[0] \in \mathbb{R}^5.$$

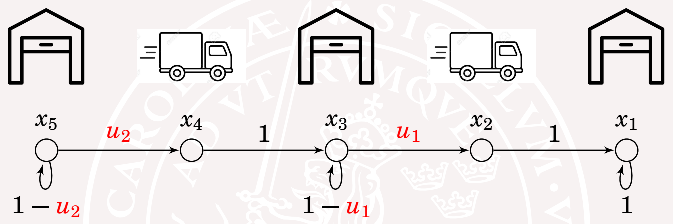
A Curious Example



Solve:

$$a^2A^TXA - X - aA^TXB\underbrace{a\left(B^TXB + R\right)^{-1}B^TXA}_{K_{\text{opt}}} + Q = 0.$$

A Curious Example



There is a pattern...

$$K_{\text{opt}} = \begin{bmatrix} * & * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & 0 \\ * & * & * & * & * & * & * & * & * \end{bmatrix}$$

Why do we want sparse feedback matrices?

$$K_{\text{opt}} = \begin{bmatrix} a^2+1 & 0 \\ 0 & a^4+a^2+1 \end{bmatrix}^{-1} \begin{bmatrix} -a^3 & -a^3 & a & a & 0 \\ -a^5 & -a^5 & -a^5 & -a^5 & a\left(a^2+1\right) \end{bmatrix}$$

$$= \begin{bmatrix} p_1(a) & 0 \\ -a^2 & \dfrac{p_2(a)}{p_1(a)} \end{bmatrix}^{-1} \begin{bmatrix} -a^3 & -a^3 & a & a & 0 \\ 0 & 0 & -a^3 & -a^3 & a \end{bmatrix}$$

Why do we want sparse feedback matrices?

This pattern continues

$$u = \begin{bmatrix} * & 0 & & & \\ * & * & 0 & & \\ 0 & * & * & 0 & \\ & 0 & * & * & 0 \end{bmatrix}^{-1} \begin{bmatrix} * & * & * & 0 & & & & & & \\ 0 & * & * & * & * & 0 & & & & \\ 0 & 0 & 0 & * & * & * & * & 0 & & \\ & & 0 & 0 & 0 & * & * & * & * & 0 \end{bmatrix} x$$

Distributed feedback law:

$$u_i = -\frac{p_i(a)}{p_{i-1}(a)} \left(a^{-3}x_{2i-1} + a^{-3}x_{2i} - ax_{2i+1} - ax_{2i+2} + a^2 u_{i-1}\right).$$

No sparse feedback matrix!!!

Why do we want sparse feedback matrices?

This pattern continues

$$u = \begin{bmatrix} * & 0 & & & \\ * & * & 0 & & \\ 0 & * & * & 0 & \\ & 0 & * & * & 0 \end{bmatrix}^{-1} \begin{bmatrix} * & * & * & 0 & & & & & & \\ 0 & * & * & * & * & 0 & & & & \\ 0 & 0 & 0 & * & * & * & * & 0 & & \\ & & 0 & 0 & 0 & * & * & * & * & 0 \end{bmatrix} x$$

Distributed feedback law:

$$u_i = -\frac{p_i(a)}{p_{i-1}(a)} \left(a^{-3}x_{2i-1} + a^{-3}x_{2i} - ax_{2i+1} - ax_{2i+2} + a^2 u_{i-1}\right).$$

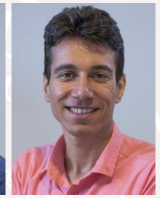
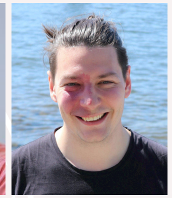
No sparse feedback matrix!!!

Conclusions

- Network realizable control
  - Don't look for sparse transfer functions
  - Connection to Internal Model Control
- Scalable H_∞ optimal synthesis
  - H_∞ optimal static controllers
  - H_∞ optimal dynamic controllers
- Scalable H_2 optimal synthesis
  - a transportation example
- Concluding remarks

Thanks



Carolina Bergeling Richard Pates Martin Heyden Hamed Sadeghi Rijad Alisic

Anders Rantzer Towards a Theory of Scalable Control