Physical Flow over Networks: analysis, control & computation

Workshop on Resilient Control of Infrastructure Networks
Politecnico di Torino
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USC Viterbi
School of Engineering

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Control of Civil Infrastructure Networks

	Date	Location	MW	Customers	Primary cause
1	14-Aug-2003	Eastern U.S., Canada	57,669	15,330,850	Cascading failure
2	13-Mar-1989	Quebec, New York	19,400	*5,828,000*	Solar flare, cascade
3	18-Apr-1988	Eastern U.S., Canada	18,500	2,800,000	Ice storm
4	10-Aug-1996	Western U.S.	12,500	7,500,000	Cascading failure

Recent large North American blackouts

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Recent large North American blackouts

Challenges:

- nonlinearities
- robustness to uncertainty & disruptions
- computational complexity

Outline

- Capacity Computation for the Static Case
- Dynamical Case
  - robustness to uncertainty vs. loss in capacity
  - optimal control of cascading failure
- Lessons From the Field

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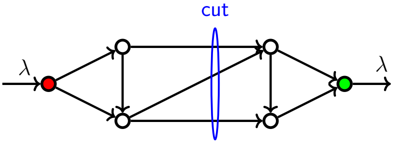
Capacity of Static Flow Network

- flow conservation
- link-wise capacity constraint



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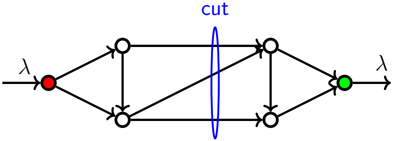

- network capacity $= \underbrace{\min_{\text{cut}} \sum_{i \in \text{cut}} c_i}_{\text{min cut capacity}}$

- network robustness $=$ min cut capacity $- \lambda$

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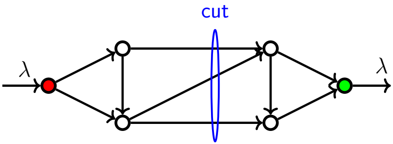

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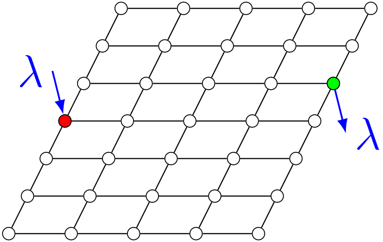
- network robustness $=$ min cut capacity $- \lambda$

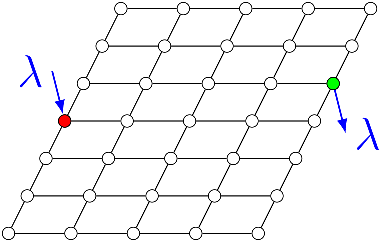
- additional physical constraints, and control mechanisms
- dynamics

Capacity of DC Networks



- $w \geq 0$: link weights

Capacity of DC Networks



- $w \geq 0$: link weights
- flow conservation + Ohm $\implies f(w, \lambda)$
  - linear in λ

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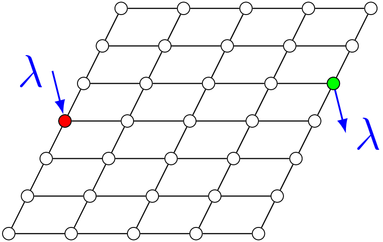

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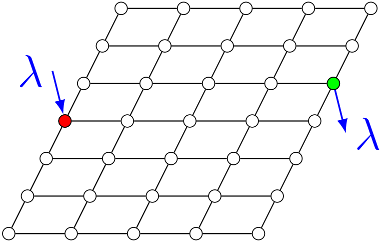

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$$\begin{aligned} \max \quad & \lambda \\ \text{s.t.} \quad & f(w,\lambda) \leq c \\ & w \in [w^{\text{low}}, w^{\text{up}}] \end{aligned}$$

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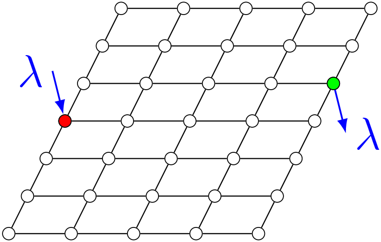

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$$\begin{aligned} \max \quad & \lambda \\ \text{s.t.} \quad & f(w,\lambda) \leq c \\ & w \in [w^{\text{low}}, w^{\text{up}}] \end{aligned}$$

- tree or $w^{\text{low}} = 0$

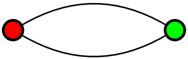
  $\Longrightarrow$ network capacity = min cut capacity [CKMST11]

- non-convex and non-differentiable in general

[CKMST11]: Christiano et al., *Electrical flows, laplacian systems, and faster approximation of maximum flow in undirected graphs*, STOC 2011.

Network Reduction

Parallel



- linear fractional program

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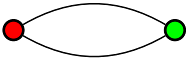

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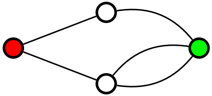

$= ?$

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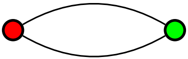

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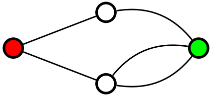

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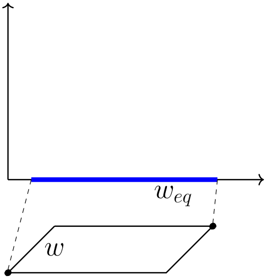

$w_{\text{eq}} \sim$ Thevenin effective resistance

Network Reduction

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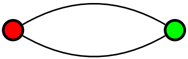

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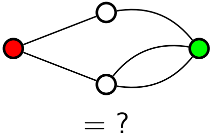

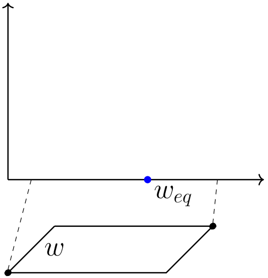

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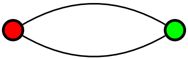

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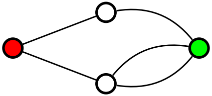

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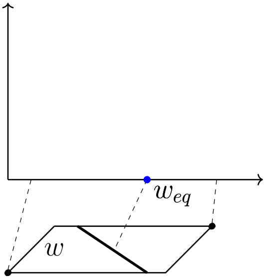

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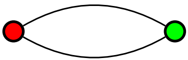

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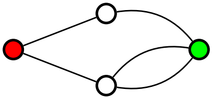

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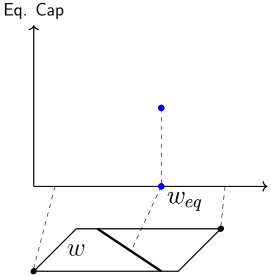

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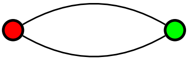

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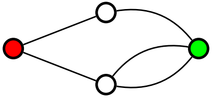

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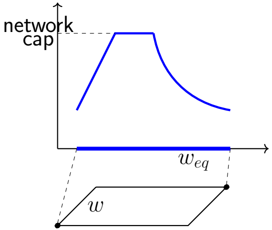

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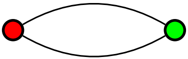

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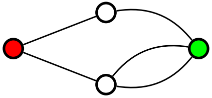

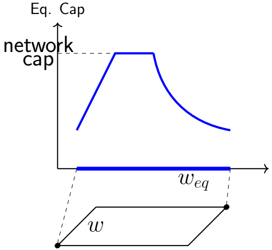

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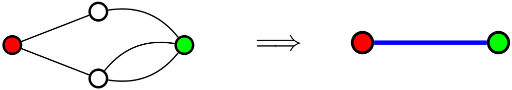
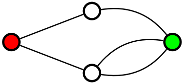
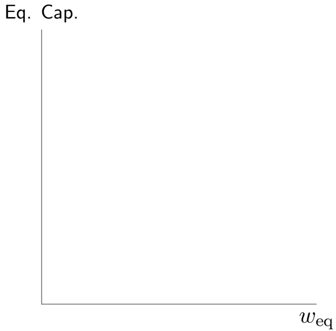


An Incremental Approach to Aggregation





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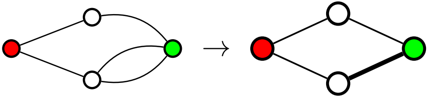

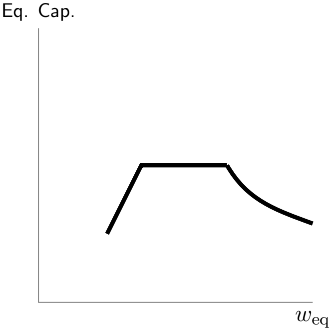

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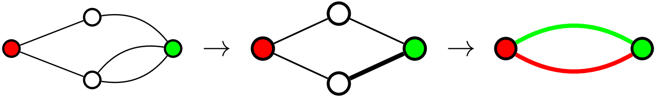

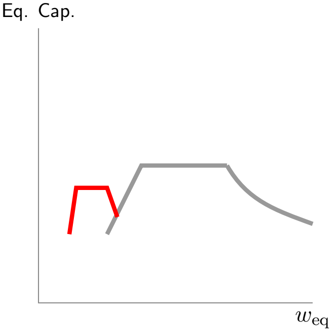

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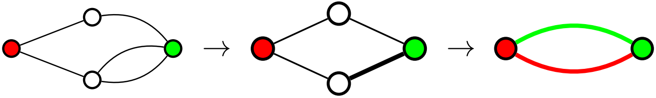

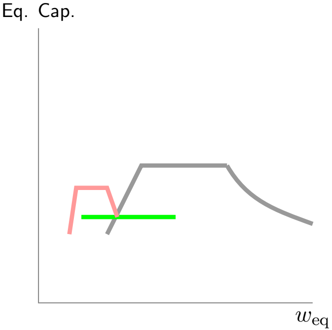

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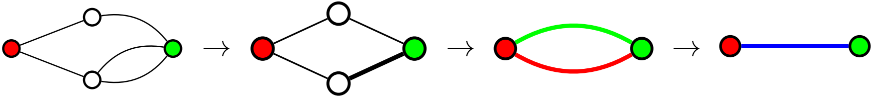

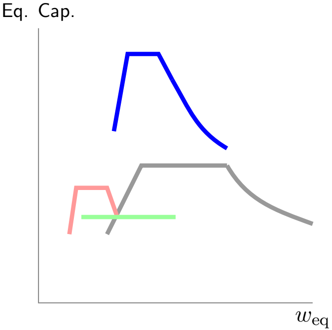

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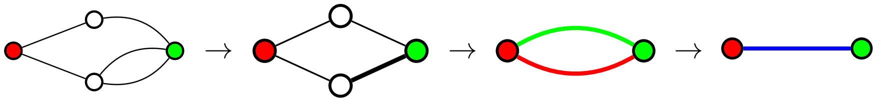
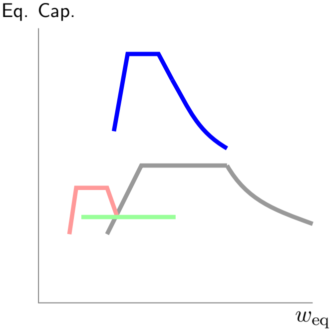
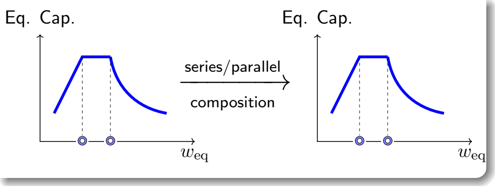
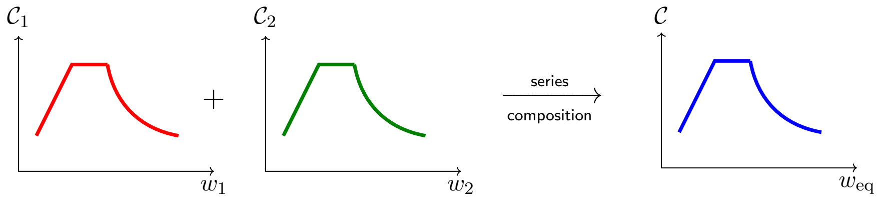






- analytical computation of ◎

Proof Sketch



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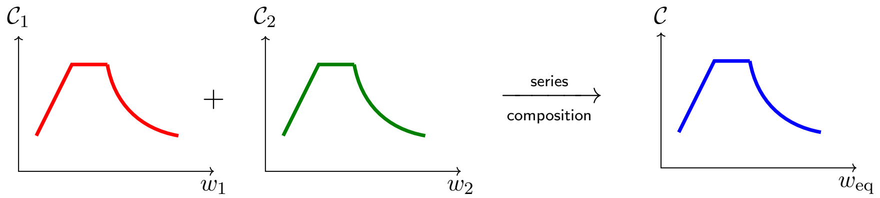

$$\begin{bmatrix} \mathcal{C}_1^{-1} \\ \mathcal{C}_2^{-1} \end{bmatrix} \text{ monotone} + \underbrace{w_{\mathrm{eq}} \text{ monotone}}_{\text{Thevenin}} \implies \mathcal{C}^{-1} = w_{\mathrm{eq}} \circ \begin{bmatrix} \mathcal{C}_1^{-1} \\ \mathcal{C}_2^{-1} \end{bmatrix} \text{ monotone}$$

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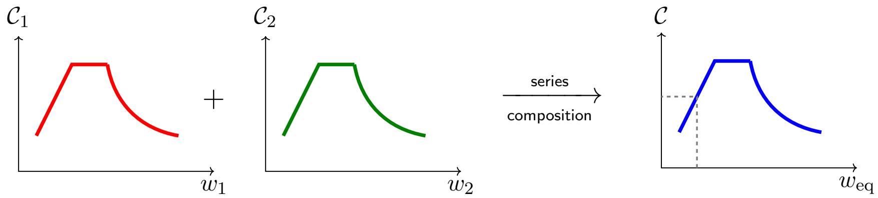

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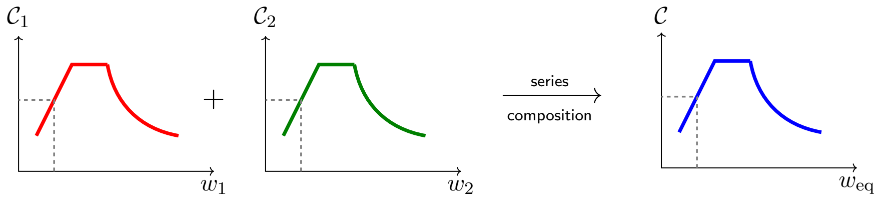

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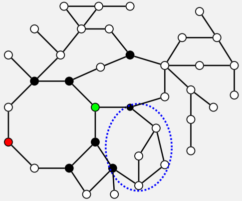
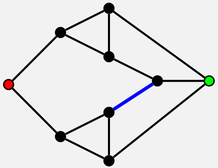
General Networks



IEEE 39 network

[non-convex, non-differentiable]

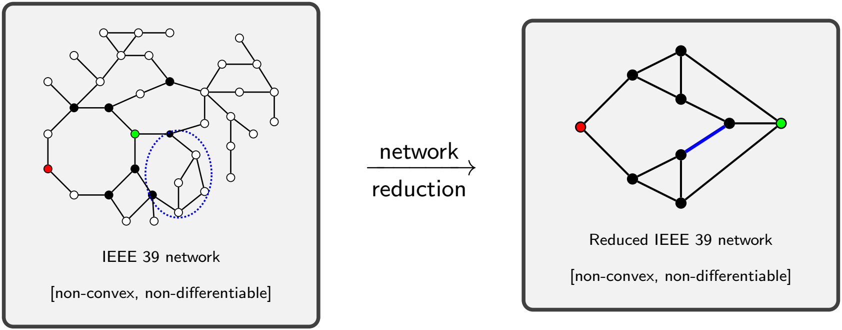
network → reduction



Reduced IEEE 39 network

[non-convex, non-differentiable]

General Networks



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network reduction

Reduced IEEE 39 network

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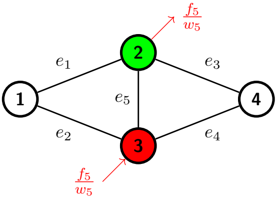
	computation time
original space	3.28×10^{27} years (anticipated)
reduced space	59.3 hours

Elements of Gradient Algorithm

Flow-Weight Jacobian

$$\frac{\partial f_i}{\partial w_j} = \frac{f_j}{w_j}\left(\delta_{i=j} - f_i(w, A_j)\right)$$



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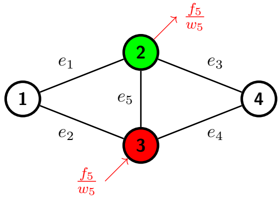

Eq. Cap.(w): local min $\Longrightarrow$ global min

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- **Dynamical Case**
  - **robustness to uncertainty vs. loss in capacity**
  - optimal control of cascading failure
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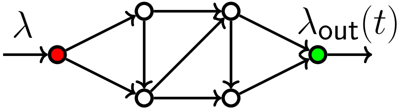
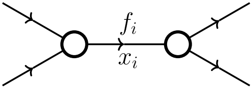
Dynamical Network Flow





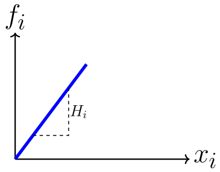
x_i : mass on link i

$R(x)$: routing matrix

Mass Conservation

$$\dot{x} = \underbrace{\lambda + R^T(x)f(x)}_{\text{inflow}} - \underbrace{f(x)}_{\text{outflow}}$$

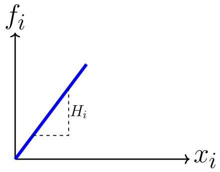
Robustness to Uncertainty



$R(x) \equiv R+$ linear $f \implies \dot{x} = \left(R^T - I\right) Hx + \lambda$

- $x^* := H^{-1}(I - R^T)^{-1}\lambda$ is GAS

Robustness to Uncertainty



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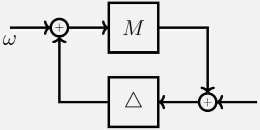
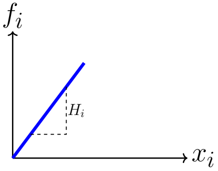
$$H \to \bar{H} + \triangle \tilde{H}$$

$\xrightarrow{\text{[ZDG96]}}$



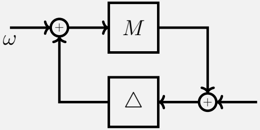
Robustness to Uncertainty



$R(x) \equiv R+$ linear $f \implies \dot{x} = \left(R^T - I\right) Hx + \lambda$

- $x^* := H^{-1}(I - R^T)^{-1}\lambda$ is GAS

$H \to \bar{H} + \triangle \tilde{H}$ $\xrightarrow{\text{[ZDG96]}}$



stochastic $\triangle$: [BF18]

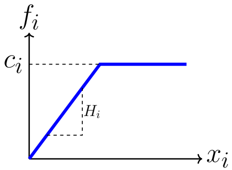
[ZDG96]: K. Zhou, J. Doyle and K. Glover, *Robust and Optimal Control*, 1996.

[BF18]: B. Bamieh and M. Filo, *An input-output approach to structured stochastic uncertainty*, 2018.

Capacitated Network Flow Dynamics

$$\dot{x} = \underbrace{\left(R^T - I\right) H}_{A} x + \underbrace{\left(I - R^T\right)}_{B} u + \lambda$$

$y = Hx$



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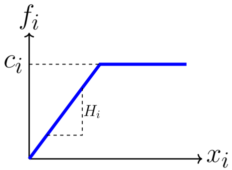

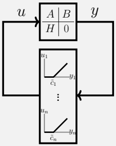

- piecewise linear analysis

  [GMD03]

Capacitated Network Flow Dynamics

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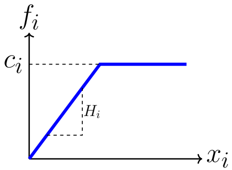
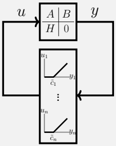




- piecewise linear analysis

[GMD03]

vs

[CKADF13], . . .

- global contraction analysis
- dynamic routing $R(x)$
- dynamic scheduling

[GMD03] M. Gonçalves, A. Megretski, M. A. Dahleh, *Global analysis of piecewise linear systems using impact maps and surface Lyapunov functions*, TAC 03.

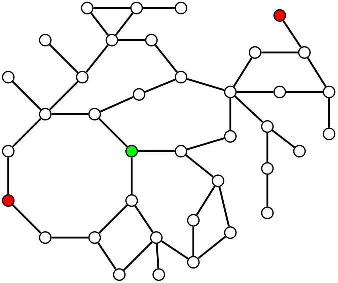
[CKADF13] G. Como, KS, D. Acemoglu, M. A. Dahleh, E. Frazzoli, *Robust distributed routing in dynamical networks*, TAC 13.

Outline

- Capacity Computation for the Static Case
- **Dynamical Case**
  - robustness to uncertainty vs. loss in capacity
  - **optimal control of cascading failure**
- Lessons From the Field

Cascading Failure in DC Networks

- tripping of overloaded lines



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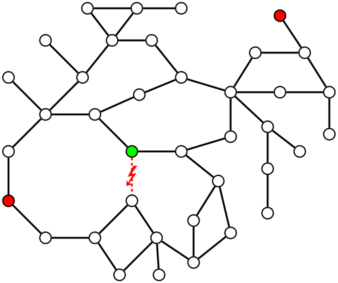

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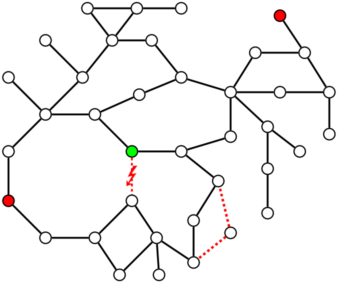

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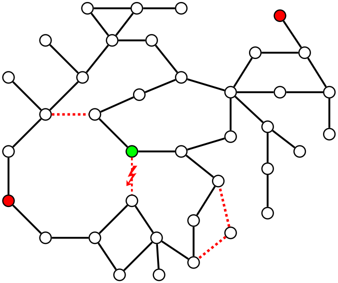

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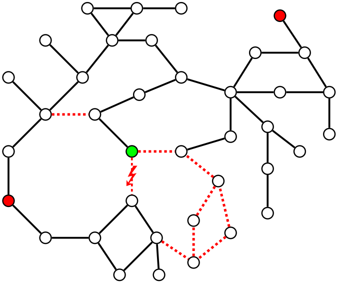

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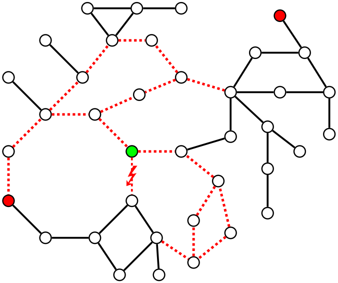

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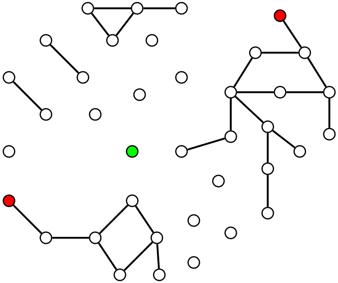

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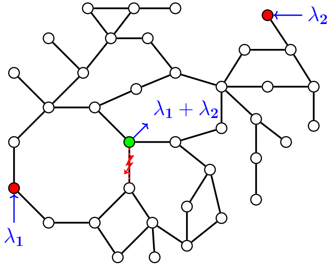

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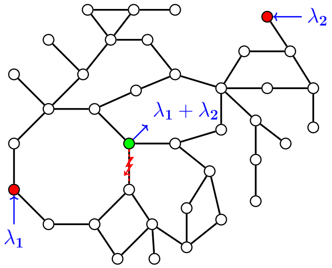

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Cascading Failure in DC Networks

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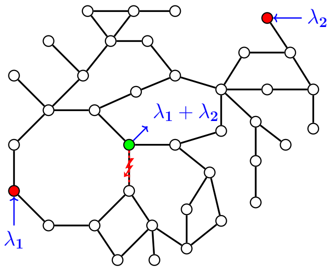

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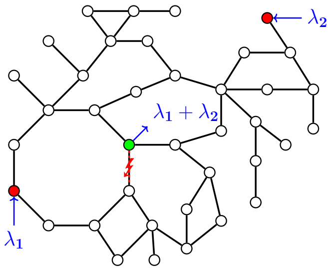

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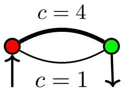
Paradoxes

- infeasible $\xrightarrow{\text{link failure}}$ feasible



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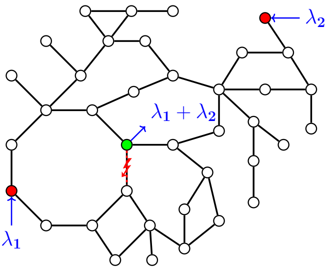

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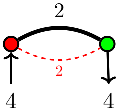

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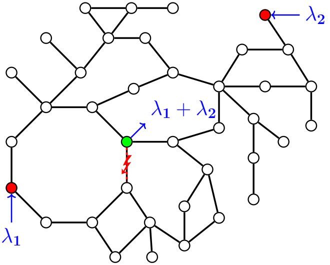

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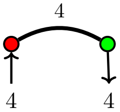

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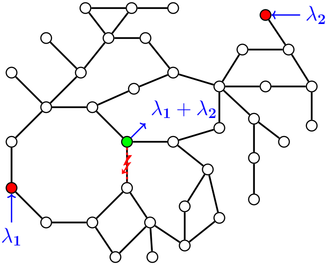

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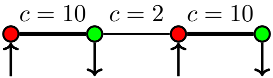

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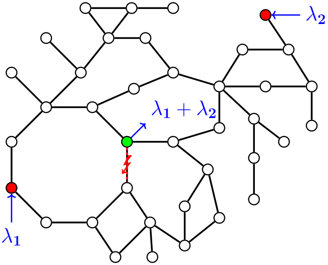

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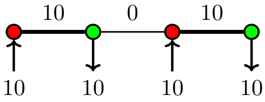

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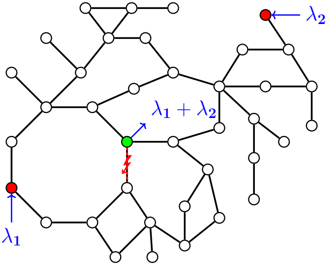

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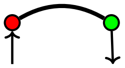

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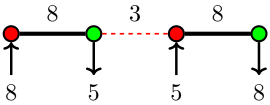

Control of Cascading Failure

$$\max_{\text{load shedding}} |\lambda^N|$$

s.t. cascading dynamics

given λ^0

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$$\text{s.t. cascading dynamics}$$

$$\text{given } \lambda^0$$

- "exponential complexity" in general

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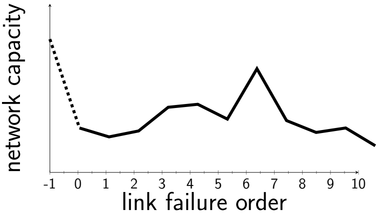

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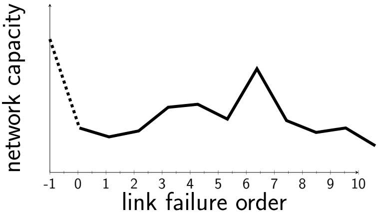

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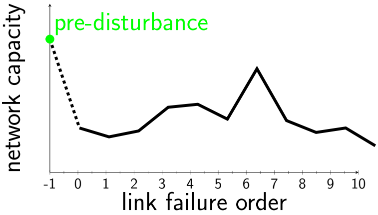

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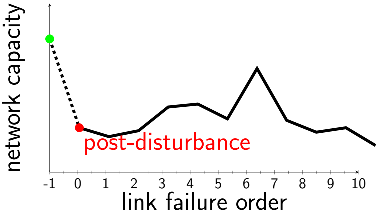

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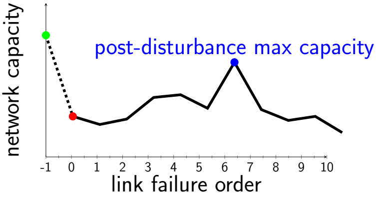
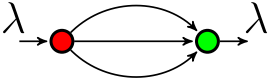
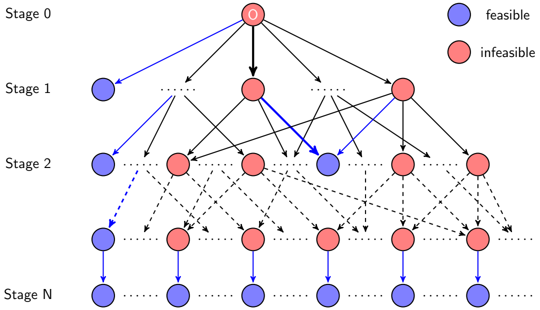




Parallel networks

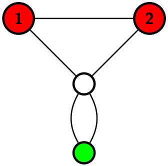
- failure order independent of control

Optimal Control for General Networks



- solution from tree search
- state $\equiv$ (active links, λ)
- uncountably infinite states!

Equivalent Abstraction Synthesis



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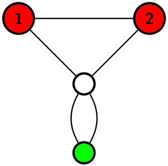

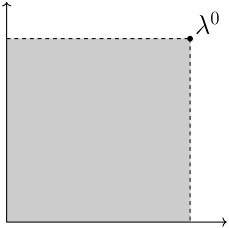

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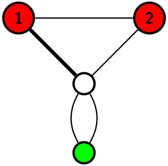

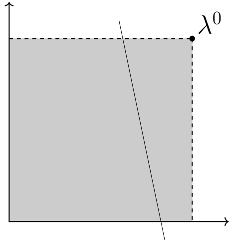

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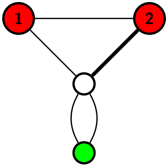

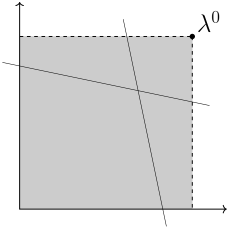

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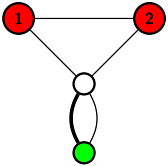

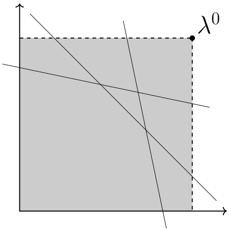

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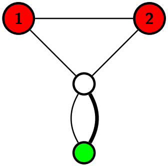

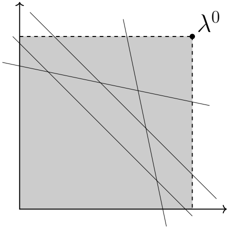

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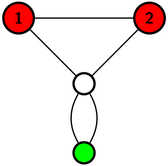
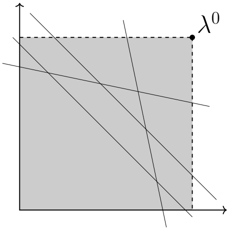
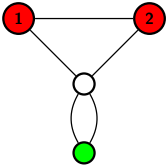
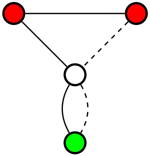


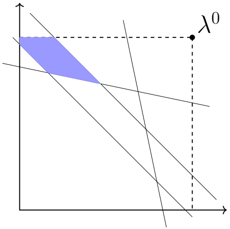


Arrangement of Hyperplanes [TOG04]

Equivalent Abstraction Synthesis







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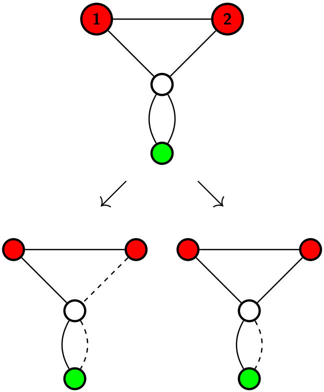
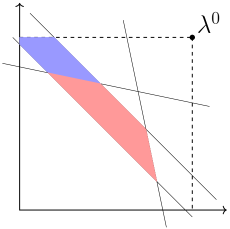
Equivalent Abstraction Synthesis





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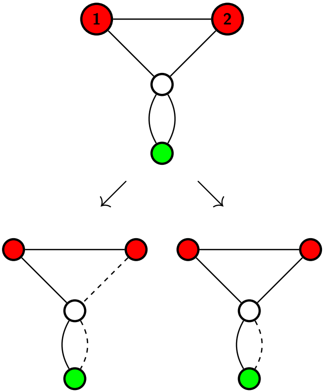

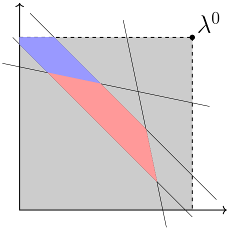

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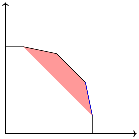
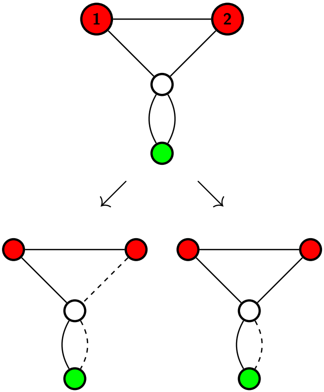
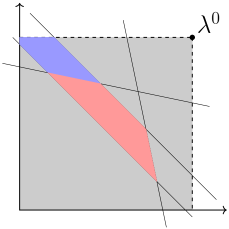
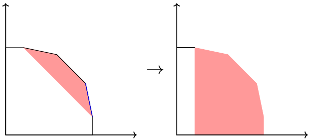


Equivalent Abstraction Synthesis





Arrangement of Hyperplanes [TOG04]



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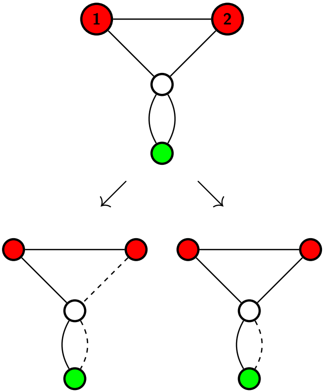

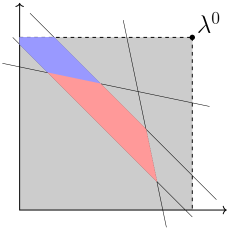

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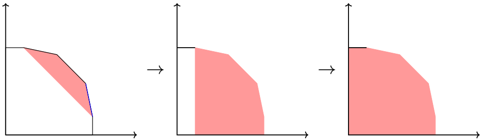

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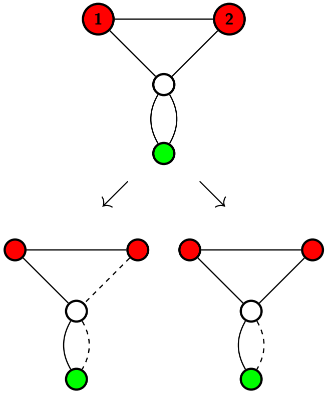

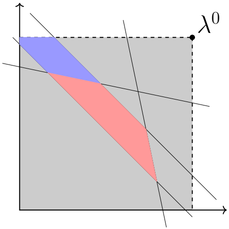

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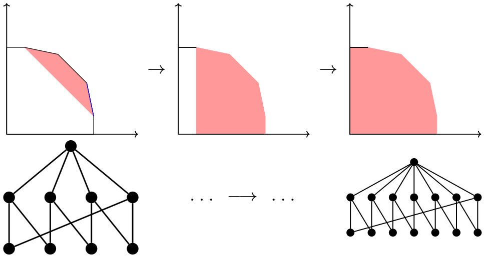
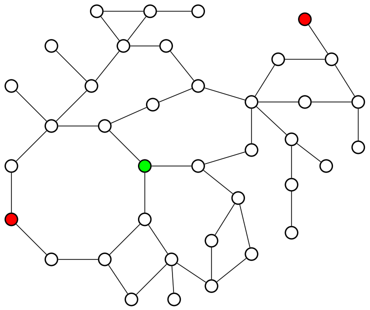


[TOG04]: C. D. Toth, J. O'Rourke and J. E. Goodman, *Handbook of discrete and computational geometry*, CRC press, 2004.

Performance Evaluation of Sub-optimal Controllers



IEEE39 network

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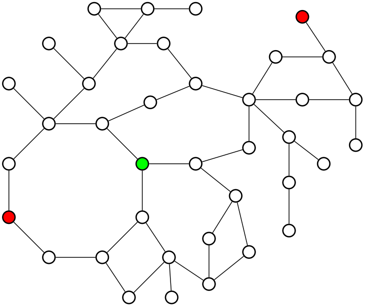

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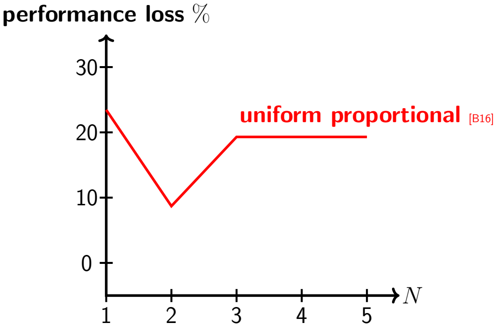

[B16]: D. Bienstock, *Electrical Transmission System Cascades and Vulnerability: An Operations Research Viewpoint*, SIAM 2016.

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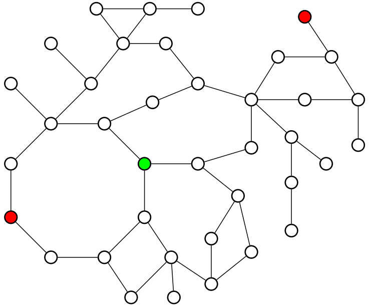

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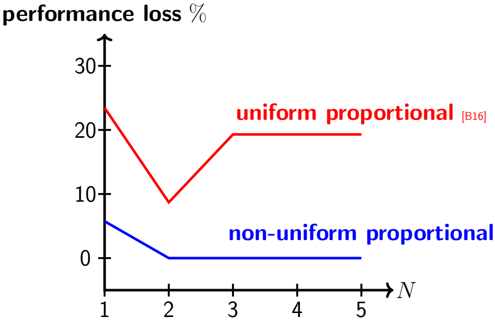

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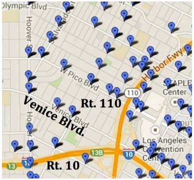
Outline

- Capacity Computation for the Static Case
- Dynamical Case
  - robustness to uncertainty vs. loss in capacity
  - optimal control of cascading failure
- **Lessons From the Field**

Traffic Signal Control Overview



HOME SEARCH The New York Times

To Fight Gridlock, Los Angeles Synchronizes Every Red Light

By IAN LOVETT APRIL 1, 2013

Fixed Time (i.e., Open-loop) Control

$> 90\%$ intersections

COORDINATION *Press [F] key to select Green Factors or Force-Off*

Local Plan (7-1..9)		Cycle	Offset	Perm	φ1	φ2	φ3	φ4	φ5	φ6	φ7	φ8
Plan 1	Green Factor	60	48	0	0	21	0	27	0	21	0	27
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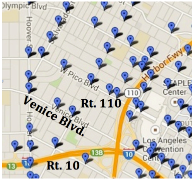

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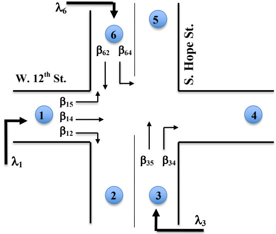

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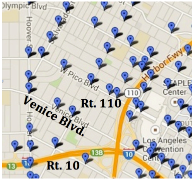

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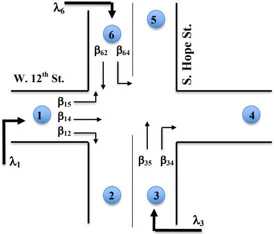

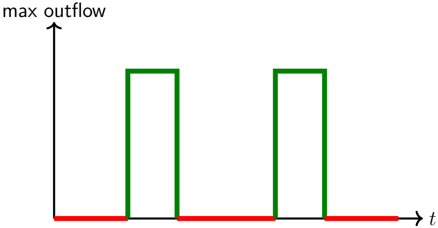

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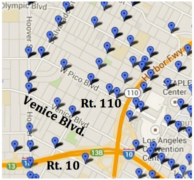

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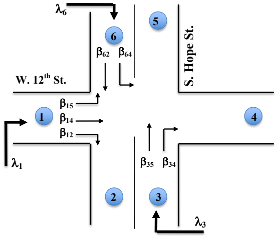

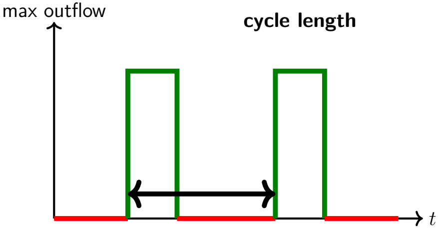

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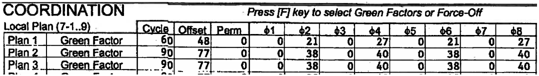

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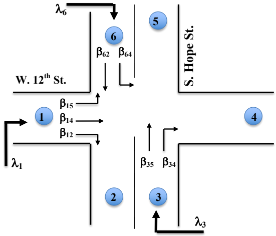

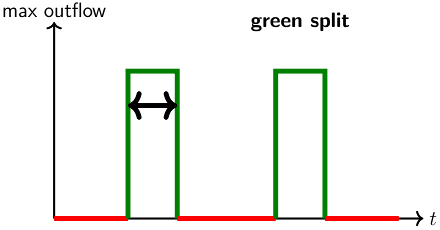

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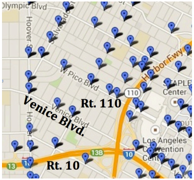

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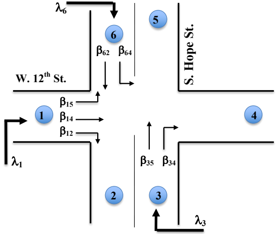

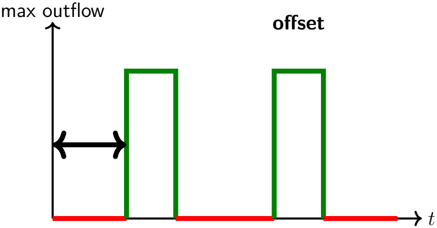

From Averaged to ON-OFF Model

$$\dot{x} = \underbrace{\lambda(t) + R^T z(x,t)}_{\text{inflow}} - \underbrace{z(x,t)}_{\text{outflow}}$$

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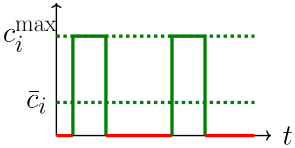

green

$$z_i = \begin{cases} c_i^{\max} & x_i > 0 \\ \text{inflow} & x_i = 0 \end{cases} \xrightarrow{\text{avg}} z_i = \begin{cases} \bar{c}_i & x_i > 0 \\ \text{inflow} & x_i = 0 \end{cases}$$

red

$$z_i = 0$$

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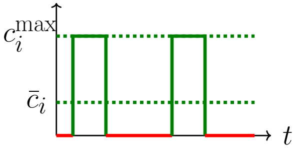

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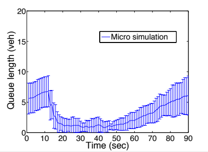
Inconsistency

E.g., isolated link:

$$\dot{x}_i = \begin{cases} \lambda_i - \bar{c}_i & x_i > 0 \\ 0 & x_i = 0 \end{cases}$$

$$\lambda_i < \bar{c}_i \implies x_i(t) \to 0$$



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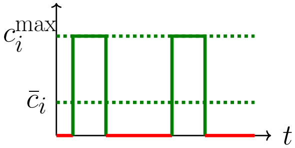

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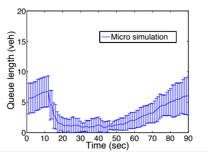
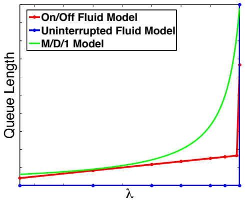
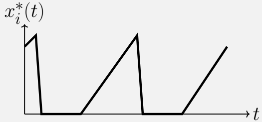




Performance Evaluation for ON-OFF Model



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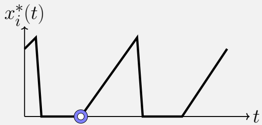

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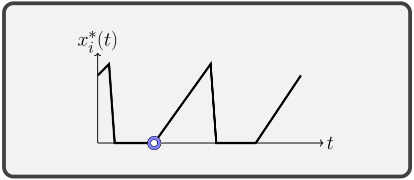

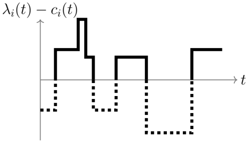

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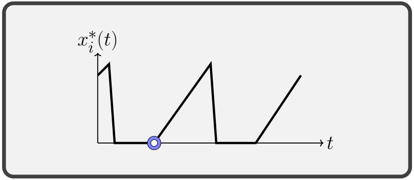

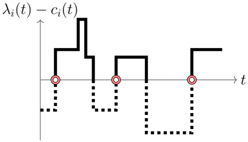

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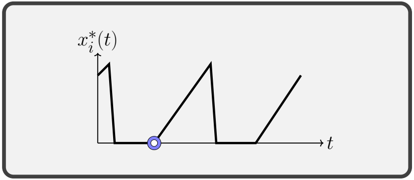

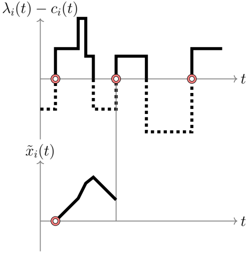

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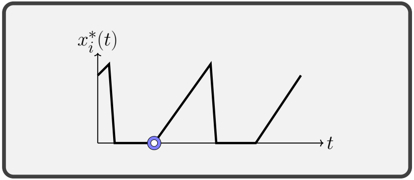

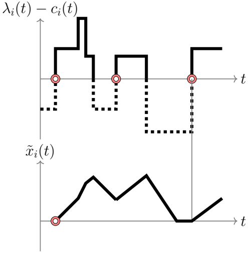

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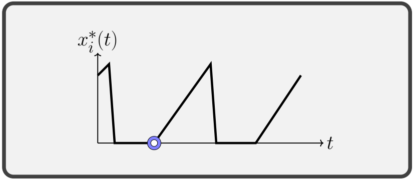

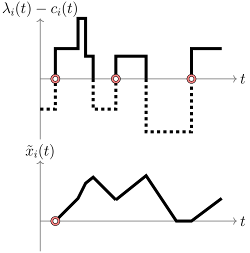

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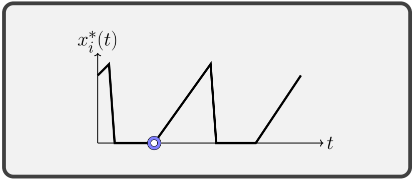

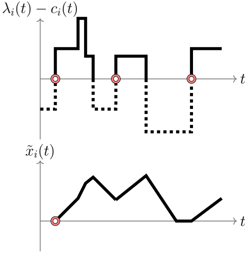

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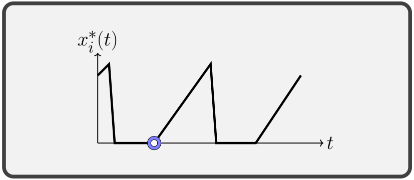

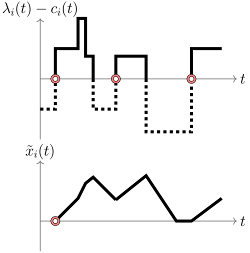

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$$\hat{x} \xrightarrow{\text{monotone}} x^*$$ [Hosseini '19]

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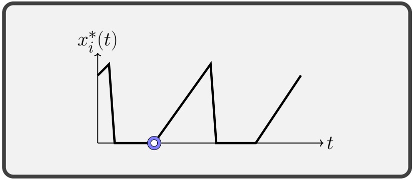

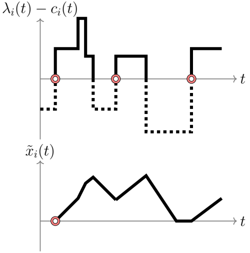

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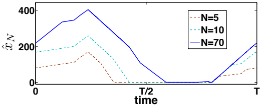
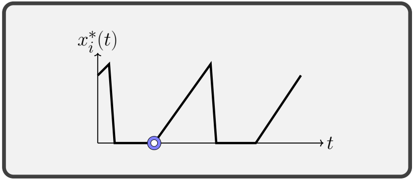
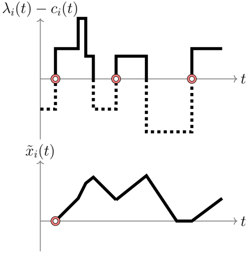


- (◎, . . . , ◎) ⊆ (◎, . . . , ◎)

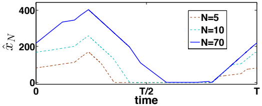
Performance Evaluation for ON-OFF Model

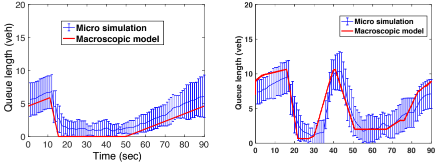




- $\lambda_i \to (\hat{x}_i, \hat{z}_i)$
- $\lambda_i + \sum_j R_{ji}\hat{z}_j \to (\hat{x}_i, \hat{z}_i)$
- ...

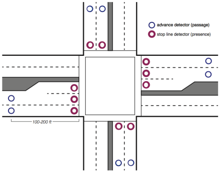
$\hat{x} \xrightarrow{\text{monotone}} x^*$ [Hosseini '19]





- (◉, ..., ◉) ⊆ (◉, ..., ◉)

From State to Output Feedback Control



- direct access to x not available
- y: detector measurement

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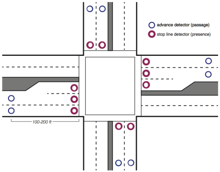

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  $y \to \hat{x} \to u(\hat{x})$

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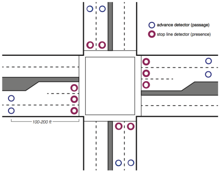

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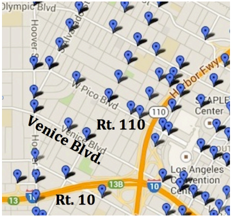

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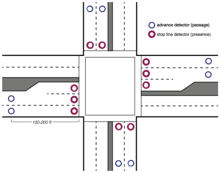

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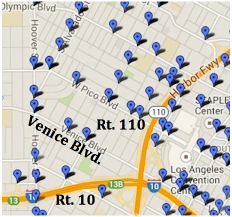

- maximally stabilizing output feedback control [Hosseini '19]

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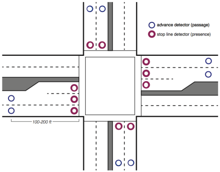

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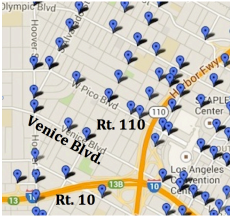
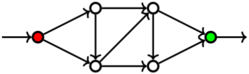
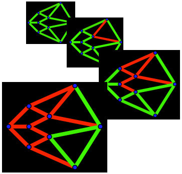


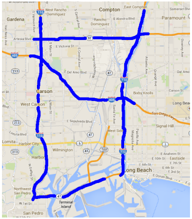
- maximally stabilizing output feedback control [Hosseini '19]
- pilot test: $\sim 20\%$ improvement w.r.t. incumbent

Xtelligent

Concluding Remarks



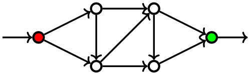


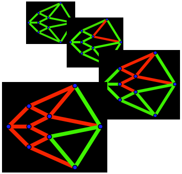


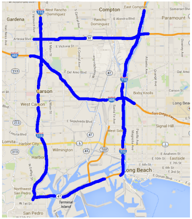
Summary

- {network flow} + {physics, control}
- incremental network reduction, monotonicity, abstraction synthesis

Concluding Remarks







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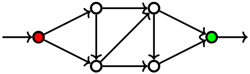
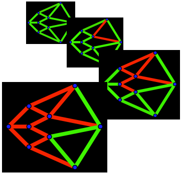
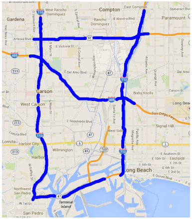
Ongoing and Future Work

- distributed (feedback) optimal control [Jafari, KS '19]

Concluding Remarks







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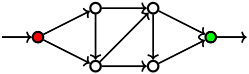

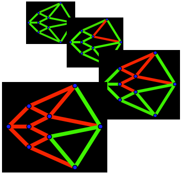

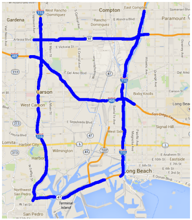

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- connections to existing system theoretic tools

References & Acknowledgments

- K. Savla, J. S. Shamma, and M. A. Dahleh. Network effects on robustness of dynamic systems.
  Annual Review of Control, Robotics, and Autonomous Systems, 2019.
  To appear
- Q. Ba and K. Savla. Robustness of DC networks under controllable link weights.
  IEEE Transactions on Control of Network Systems, 5(3):1479–1491, 2018
- Q. Ba and K. Savla. Computing optimal control of cascading failure in DC networks.
  IEEE Transactions on Automatic Control, June 2020.
  In press. Available at https://arxiv.org/abs/1712.06064
- Q. Ba. *Elements of Robustness and Optimal Control for Infrastructure Networks.*
  PhD thesis, University of Southern California, 2018
- P. Hosseini. *Novel Tools for Analysis and Control of Traffic Flow over Signalized Networks.*
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- S. Jafari and K. Savla. A decentralized optimal feedback flow control approach for transport networks.
  https://arxiv.org/pdf/1805.11271.pdf; preliminary version at ACC 2019

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- K. Savla, J. S. Shamma, and M. A. Dahleh. Network effects on robustness of dynamic systems. *Annual Review of Control, Robotics, and Autonomous Systems*, 2019. To appear
- Q. Ba and K. Savla. Robustness of DC networks under controllable link weights. *IEEE Transactions on Control of Network Systems*, 5(3):1479–1491, 2018
- Q. Ba and K. Savla. Computing optimal control of cascading failure in DC networks. *IEEE Transactions on Automatic Control*, June 2020. In press. Available at `https://arxiv.org/abs/1712.06064`
- Q. Ba. *Elements of Robustness and Optimal Control for Infrastructure Networks.* PhD thesis, University of Southern California, 2018
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