Decentralized Throughput-Optimal Traffic Signal Control

Gustav Nilsson
School of Electrical and Computer Engineering, Georgia Institute of Technology
gustav.nilsson@gatech.edu
http://gustavnilsson.name

Join work with Giacomo Como

September 26th 2019

Resilient Control of Infrastructure Networks, September 24-27, 2019 - Politecnico di Torino

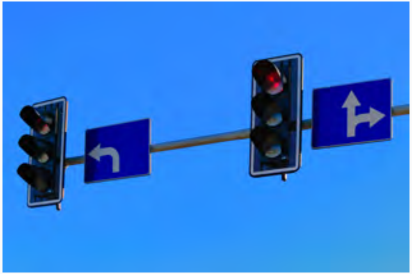
Feedback Control to Improve Transportation Networks

- Growing number of sensors $\rightarrow$ more realtime data $\rightarrow$ faster demand changes
- More possibilities for communication (V2V, V2I, ...)
- But the road network remains (almost) the same
- Autonomous vehicles may not reduce the traffic demand [Spieser et al., 2014]

Aim: Use feedback to improve the utilization of the network resources

Global Optimality by Local Decisions?

Local measurements and control actions



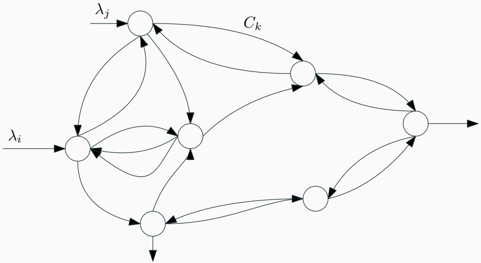
$\Longrightarrow$

Global throughput optimality

Benefits: A decentralized and scalable control policy

Challenge: Use local control policies to optimize global objectives

Traffic Network as a Graph



- Capacited multigraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, C)$
  - $\mathcal{V}$ - set of junctions
  - $\mathcal{E}$ - set of cells
  - $C = \text{diag}(c)$, c_i - outflow capacity of cell i
- External inflows λ_i
- Traffic volume x_i in cell i

Model - Dynamics

- x_i traffic volume in cell i
- λ_i external inflow
- c_i the cells outflow capacity
- R routing matrix – R_{ij} fraction of flow from i to j
- z_i outflow from cell i
- $u_i(x) \leq c_i$ traffic signal control (Assumption: when $x_i > 0$, $z_i = u_i(x)$.)

$$\dot{x}_i = \lambda_i + \sum_{j \in \mathcal{E}} R_{ji} z_j - z_i \,, \quad 0 \leq z_i \leq c_i u_i(x_i) \,, \quad \forall i \in \mathcal{E}$$

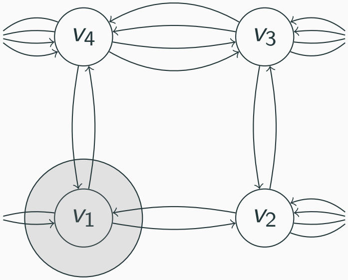
- When $x_i = 0$, z_i chosen s.t. $\dot{x}_i \geq 0$

Theorem
If $u(x)$ is Lipschitz in x, a unique solution exists.

G. Nilsson. "On Robust Distributed Control of Transportation Networks". PhD Thesis.

Model - Phases

- Activation constraints to avoid collisions
- $\mathcal{E}_v = \{\text{cells entering junction } v\}$
- Set of phases $p \in \mathcal{P}_v$: Incoming cells to junction v that can be activated simultaneously
- Local phase matrix $P^{(v)} = \{0,1\}^{\mathcal{E}_v \times \mathcal{P}_v}$



$$P_{ip}^{(v)} = \begin{cases} 1 & \text{cell } i \text{ belongs to } p\text{-th phase} \\ 0 & \text{otherwise} \end{cases}$$

Example

$$P^{(v)} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Model - Control with Phases

- Local control space for a node v, $\mathcal{U}_v = \{\nu^{(v)} \in \mathbb{R}_+^{\mathcal{P}_v} \mid \sum_{p \in \mathcal{P}_v} \nu_p^{(v)} \leq 1\}$
- $\nu^{(v)} \in \mathcal{U}_v$, $\nu_p^{(v)}$ fraction of the cycle to phase $p \in \mathcal{P}_v$
- $T_0^{(v)} = 1 - \sum_p \nu_p^{(v)}$ fraction of the cycle allocated to phase shifts. Cycle length $\propto 1/T_0$

$$\dot{x} = \lambda + (I - R^T)z$$

$$x \geq 0, \quad 0 \leq z \leq u(x)$$

$$u(x) = CP\nu(x)$$

$$P = \begin{bmatrix} P^{(v_1)} & & & \\ & P^{(v_2)} & & \\ & & \ddots & \\ & & & P^{(v_n)} \end{bmatrix} \quad C = \begin{bmatrix} c_1 & & & \\ & c_2 & & \\ & & \ddots & \\ & & & c_e \end{bmatrix} \quad u(x) = \begin{bmatrix} u^{(v_1)}(x) \\ u^{(v_2)}(x) \\ \vdots \\ u^{(v_n)}(x) \end{bmatrix}$$

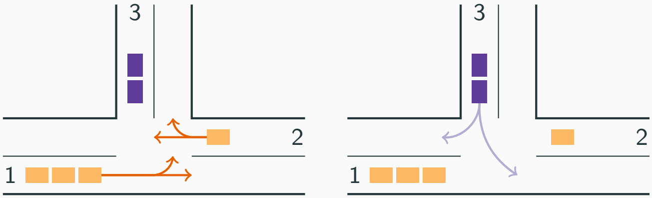
Generalized Proportional Allocation (GPA) - Orthogonal Phases

- For all $p, q \in \mathcal{P}_v$, $p^T q = 0$ – Each lane only belongs to one phase
- Set of local cells $x^{(v)} = \{x_i \mid i \in \mathcal{E}_v\}$
- Green light split in proportion to the queues in each phase

$$\nu_q(x^{(v)}) = \frac{\left(P^T x^{(v)}\right)_q}{\kappa_v + \sum_{r \in \mathcal{P}_v} \left(P^T x\right)_r} = \frac{\sum_i P_{iq} x_i}{\kappa_v + \sum_{i \in \mathcal{E}_v} x_i} \quad q \in \mathcal{P}_v$$

- $\kappa_v > 0$ design parameter

Example



$$\nu_1 = \frac{x_1 + x_2}{\kappa_v + x_1 + x_2 + x_3}$$

$$\nu_2 = \frac{x_3}{\kappa_v + x_1 + x_2 + x_3}$$

GPA - General Case

For non-orthogonal phases, the GPA is a convex optimization problem for each junction v:

$$\nu^{(v)}(x) \in \operatorname{argmax}_{\mu \in \mathcal{U}_v} \sum_{i \in \mathcal{E}_v} x_i \log((P^{(v)}\mu)_i) + \kappa_v \log(1 - \mathbf{1}^T \mu)$$

The previous stability theorem still holds, uniqueness still an open question

Observations:

- The controller only needs to know the incoming queue lengths $\Rightarrow$ Decentralized, Scalability
- The controller needs no information about R, λ or the network topology $\Rightarrow$ Resilience, Scalability
- When x_i grows large, $1 - \mathbf{1}^T \nu$ will be small $\Rightarrow$ Longer cycles during higher loads

GPA - Stability

Theorem (Stability)
For an exogenous arrival vector λ *and a both inflow connected and outflow connected* R *satisfying* $a_i = (I - R^T)^{-1}\lambda \in int(\{z \in \mathbb{R}_+^{\mathcal{E}} \mid 0 \leq z \leq CP\nu \text{ where } \nu \in \mathcal{U}\})$ *the dynamical flow network with GPA control is stable, i.e., the queue lengths* $x(t)$ *remains bounded in time. Moreover, every solution* $x(t)$ *approaches the set*

$$\mathcal{X} = \{x \in \mathbb{R}_+^{\mathcal{E}} \mid c_i(P\nu(x))_i = a_i \text{ for all } i \text{ such that } x_i > 0\}\,.$$

Observe: The GPA controller is able to stabilize the traffic network, whenever any controller is able to do so. This without any knowledge about R or λ.

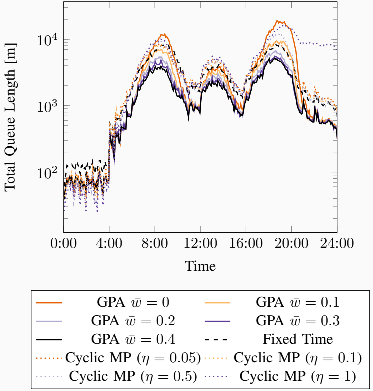
G. Nilsson and G. Como. "Generalized Proportional Allocation Policies for Robust Control of Dynamical Flow Networks". (arXiv:1907.02045)

Micro-Simulations – LuST - Luxembourg Scenario

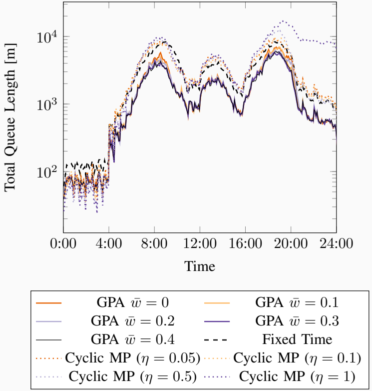
- Simulates all traffic in Luxembourg during a full day
- About 200 signalized junctions
- We added sensors to measure queues (50 m)
- Non-orthogonal phases ⇒ convex optimization solved in real-time by CVXPY
- Results compared to the standard fixed-time plan that comes with the scenario and a cyclic version of the MaxPressure controller

Luxembourg Scenario - Results

$\kappa = 5$



$\kappa = 10$



Total Travel Time: 54 100 h $\Rightarrow$ 48 500 h

G. Nilsson and G. Como, “A Micro-Simulation Study of the Generalized Proportional Allocation Traffic Signal Control”. (arXiv:1901.09976)

Conclusions and Future Work

Presented a traffic light controller that:

- is able to *stabilize* the traffic network whenever it is possible
- does *not require any routing information*
- is *decentralized* and *scalable*
- is *validated* to work in micro simulator

Future work:

- Finite storage capacities, spill-back
- Saturation in measurements of queue-lengths
- Tuning of the parameter κ
- Dynamic route choice behavior, i.e., R depends on the state of the network
- Coordination between different junctions, green-waves

Gustav Nilsson
School of Electrical and Computer Engineering, Georgia Institute of Technology
gustav.nilsson@gatech.edu
http://gustavnilsson.name