How to Achieve Fast Spread in Controlled Evolutionary Dynamics

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Evolutionary dynamics on graphs

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Evolutionary dynamics on graphs

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Genetically modified mosquitoes to help control dengue, malaria

By M. Sai Gopal | Published: 27th Jun 2017 11:44 pm Updated: 28th Jun 2017 12:29 am

Our goals

Define a novel framework to **model** evolutionary dynamics, which allows for including **control**

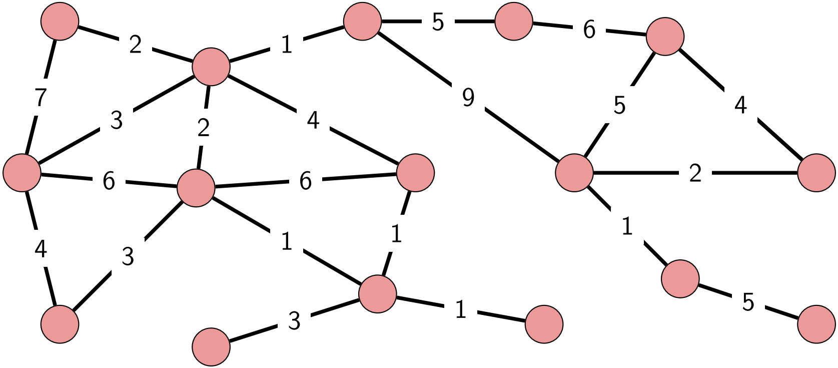
Understand how the **spreading time** is influenced by the **graph topology** and the **control policy**

Design a **feedback control** policy to **speed up** the spreading process and test it on a real-world **case study**

Model

Weighted graph

- **Connected** graph
- **Node set** $\mathcal{V} = \{1, \dots, n\}$
- Undirected **links**, symmetric **weight matrix** $W \in \mathbb{R}_{\geq 0}^{n \times n}$

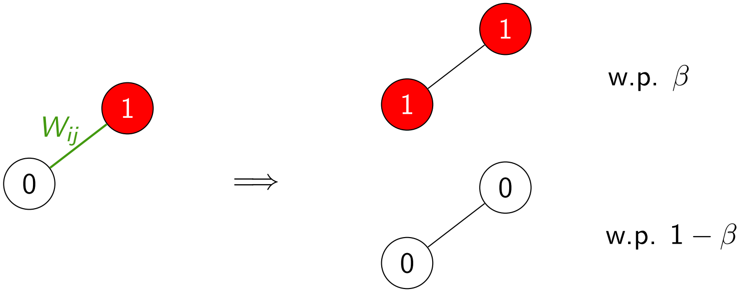


Evolutionary dynamics

- $X_i(t) \in \{0,1\}$ **state** of node i at time $t \in \mathbb{R}_{\geq 0}$:

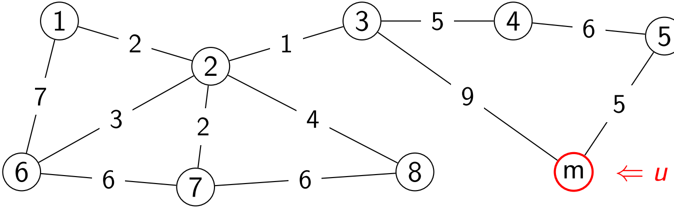
$$X_i(t) = \begin{cases} 1 & \text{if } i \text{ has the novel state at time } t \\ 0 & \text{if } i \text{ has the old state at time } t \end{cases}$$

- Link $\{i,j\}$ is activated by a **Poisson clocks** with rate W_{ij}
- If $X_i(t) \neq X_j(t) \implies$ **conflict**: novel state wins w.p. $\beta > 1/2$



External control

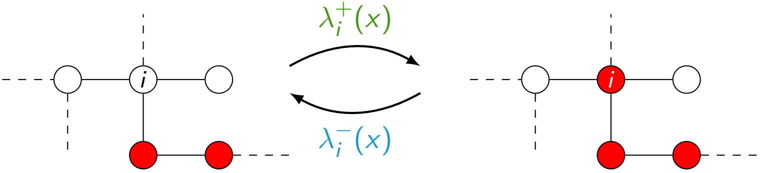
- A **target node** $m(t) \subset \mathcal{V}$ is selected[1]
- Novel state is **introduced** in $m(t)$ with rate $u(t) \geq 0$
- Simplest choice: **constant control** $m(t) = m$, $u(t) = u$.
- **Feedback control** $m(t) = m(X(t))$, $u(t) = u(X(t))$



[1]We can generalize it to a target set of nodes

Markov jump process

- $X(t)$ **Markov jump process** with $X(0) = 0\mathbb{1}$.



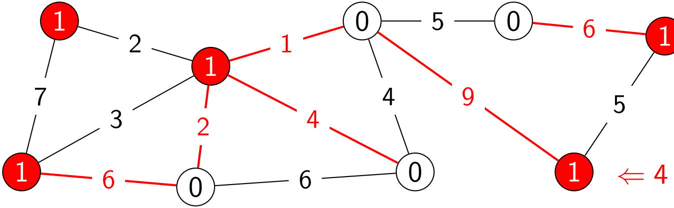
$$\begin{cases} \lambda_i^+(x) & = & (1 - x_i)\left[\beta \sum_j W_{ij} x_j + u(t)\delta_{i=m(t)}\right] \\ \lambda_i^-(x) & = & x_i(1-\beta)\sum_j W_{ij}(1 - x_j) \end{cases}$$

- $X = \mathbb{1}$ unique absorbing state $\Longrightarrow$ novel state will spread
- Two performance indexes: expected **spreading time** and **cost**:

$$\tau = \mathbb{E}\left[\inf t : X(t) = \mathbb{1}\right] \qquad \upsilon = \mathbb{E}\left[\int_0^\infty u(t)\mathrm{d}t\right]$$

Three observables

- $A(t) = \sum_i X_i(t)$ **number** of nodes with the novel state
- $B(t) = \sum_i \sum_j X_i(t)(1 - X_j(t))W_{ij}$ **boundary** between the two states
- $C(t) = (1 - X_{m(t)}(t))u(t)$ **effective control** in nodes with state 0



$A(t) = 5 \qquad B(t) = 28 \qquad C(t) = 0$

General results

Performance guarantees (PG)

If $B(t) + C(t) \geq f(A(t)) > 0$ for any $t \geq 0$, then

$$\tau \leq \frac{\beta}{(2\beta - 1)f(0)} + \frac{1}{2\beta - 1} \sum_{a=1}^{n-1} \frac{1}{f(a)}$$

Fundamental limitation (FL)

Called T_h and J_h the contributions to τ and υ each time $A(t) = h$, it holds

$$\mathbb{E}[T_h] \geq \frac{1 - \mathbb{E}[J_h]}{B(t)}.$$

Constant Control

Constant control

Upper bound on the expected spreading time (UB)

Let $\phi(a) : 1, \ldots n-1 \to \mathbb{R}$ be the minimum conductance. Then,

$$\tau \leq \frac{\beta}{(2\beta-1)u} + \frac{1}{2\beta-1} \sum_{a=1}^{n-1} \frac{1}{\phi(a)}$$

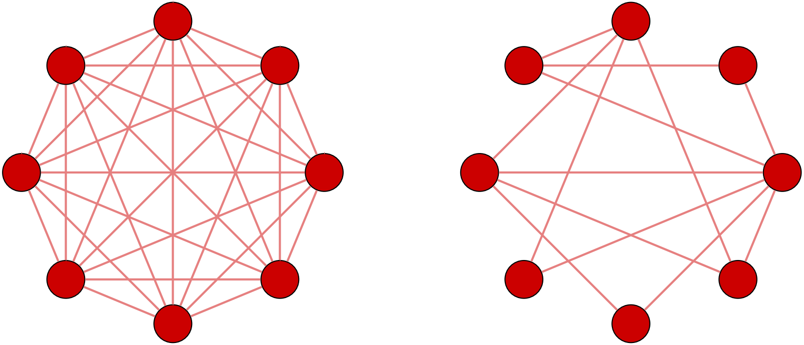
Lower bounds on the expected spreading time (LB)

I: Let $\eta(a) : 1, \ldots n-1 \to \mathbb{R}$ be the maximum expansiveness. Then,

$$\tau \geq \frac{1}{u} + \sum_{a=1}^{n-1} \frac{1}{\eta(a)}$$

II: Let ξ be the (weighted) bottleneck of the graph. Then, $\tau \geq \xi^{-1}$.

Example I: fast spread on expander graphs

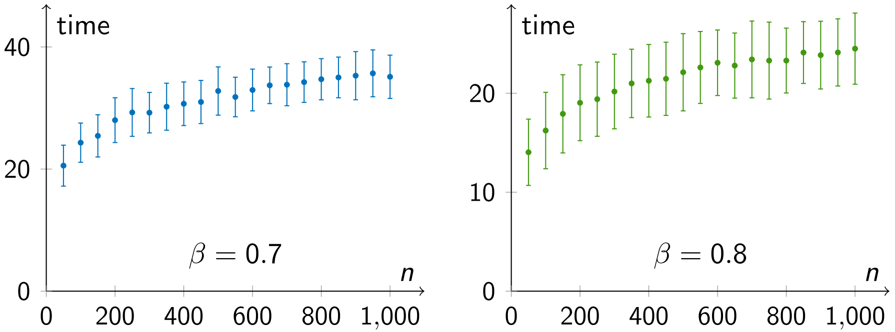


Complete

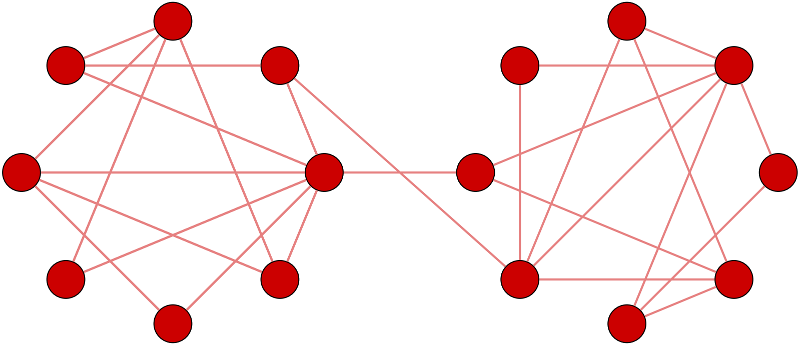
Erdős-Rényi

Example I: fast spread on expander graphs

☺ UB + LB I $\implies$ **fast spread**: $\tau = \Theta(\ln n)$



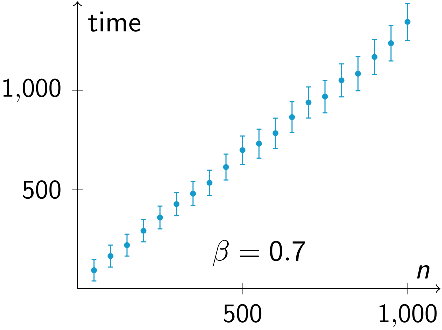
Example II: slow spread on stochastic block models

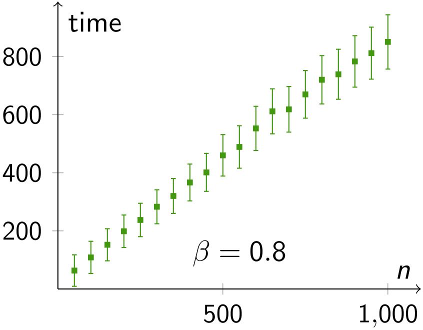


Example II: slow spread on stochastic block models

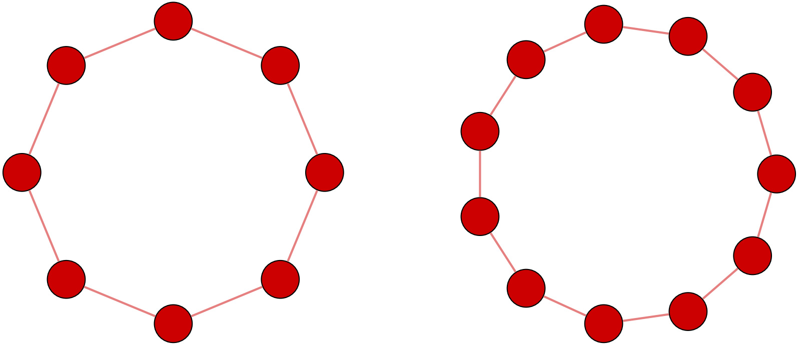
☺ The fundamental limit allows fast spread

☹ Constant control: UB + LB II $\Longrightarrow$ **slow spread**: $\tau = \Theta(n)$





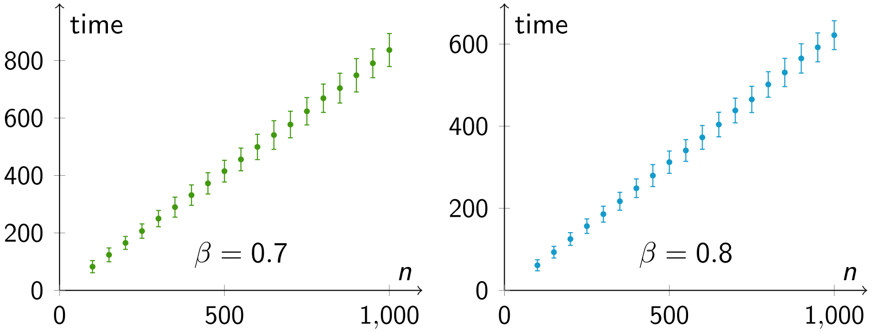
Example III: slow spread on rings



Example III: slow spread on rings

☹ FL $\Longrightarrow$ **slow spread** for any control policy

$$\tau \geq \frac{n - \upsilon}{2\upsilon} \in \Theta(n)$$



To sum up...

☺ If the topology ensures fast spread with constant control...

...we reached our goal!

☹ If the fundamental limitation does not allow fast spread...

...there is no solution!

⇒ If the fundamental limitation allows fast spread, but constant control fails...

...we need to improve the control!

Feedback Control

Feedback control policy

- **Avoid waste**: $m(t)$ moved in a random node with state 0
  $\Longrightarrow$ no optimization on $m(t)$ (☺ computationally good!)

- **Contrast slowdowns:** velocity of the process proportional to $B(t)$
  $\Longrightarrow$ $u(t)$ should compensate when $B(t)$ is small

$$u(t) = u(A(t), B(t)) = \begin{cases} C - B(t) & \text{if } A(t) \neq n,\ B(t) < C \\ 0 & \text{else} \end{cases}$$

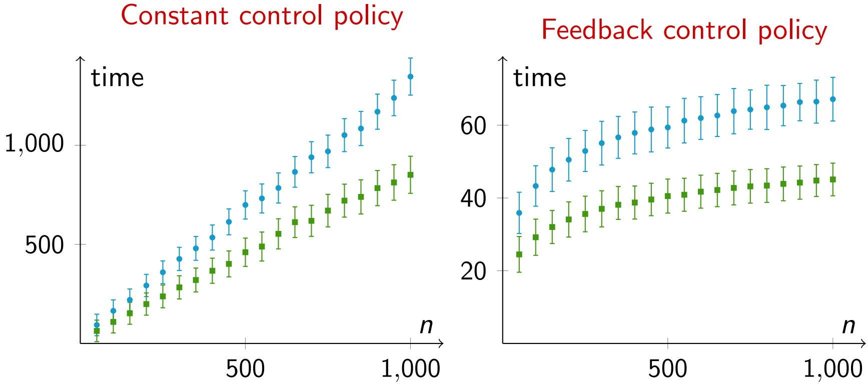
Upper bounds on spreading time and cost under feedback control

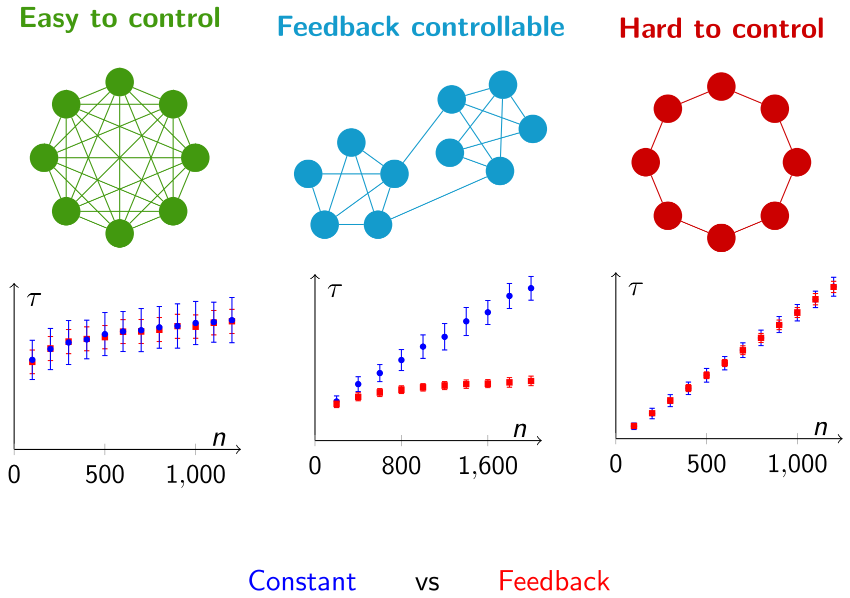
$$\tau \leq \frac{\beta}{(2\beta - 1)C} + \frac{1}{2\beta - 1} \sum_{a=1}^{n-1} \frac{1}{\max\{\phi(a), C\}}$$

$$\upsilon \leq \frac{\beta}{(2\beta - 1)} + \frac{1}{2\beta - 1} \left|\{a : \phi(a) < C\}\right|$$

Application of feedback control policy to SBMs

- $C < \frac{1}{2} - \frac{1}{n} \implies$ control activates only at **bottleneck**
- ☺ Upper bounds $\implies$ **fast spread**: $\tau \in \Theta(\ln n)$, $\upsilon \in \Theta(1)$





Easy to control
Feedback controllable
Hard to control
τ
n
0
500
1,000
τ
n
0
800
1,600
τ
n
0
500
1,000
Constant
vs
Feedback


CNN
Health » Diet + Fitness | Living Well | Parenting + Family | Vital Signs
FDA approves Zika-fighting genetically modified mosquito
By Sandee LaMotte, CNN
Updated 2126 GMT (0526 HKT) August 5, 2016
Genetically modified mosquitoes to help control dengue, malaria
By M. Sai Gopal | Published: 27th Jun 2017 11:44 pm Updated: 28th Jun 2017 12:29 am
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U.S. One Step Closer to Releasing Engineered Mosquito to Fight Zika
Community support for a field trial, required before any releases, is not guaranteed
Cases of Dengue Drop 91 Percent Due to Genetically Modified Mosquitoes
When Will The Keys Begin Using Genetically Modified Mosquitoes To Fight Zika?
March 28, 2017 Environment

Case study: Zika in Rwanda

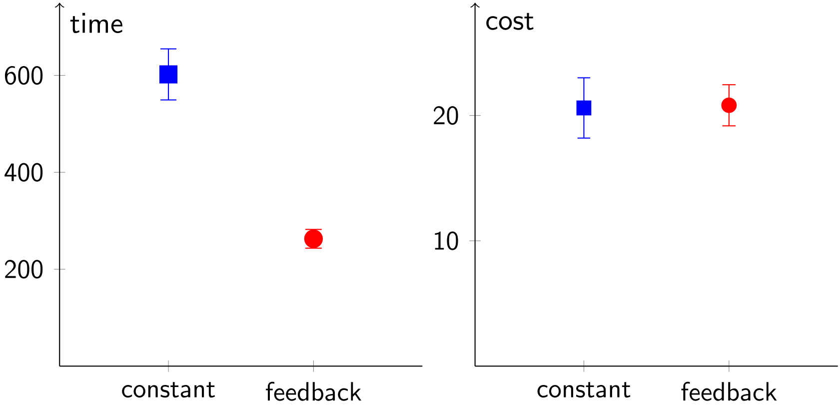
- Zika Alert in Rwanda since 2016 [CDC, accessed online September 26, 2019]

Case study: model parameters

- Location connected within a certain distance. Threshold set to 11.7 km: max distance traveled by mosquitoes to lay eggs [Bogojevic et al., J.Amer.Mosq.Cont.Ass., 2007]
- Activation rate $w = 1/10$. 10 days life-cycle of *Aedes aegypti* [CDC Centers for Disease Control and Prevention, accessed online September 26, 2019]

Parameter	Meaning	Value
n	Number of locations	1621
W_{ij}	Activation rate	0.1
β	Evolutionary advantage	0.53
u	Control rate (constant)	2
C	Control parameter (feedback)	1.5
t	Time unit	day

Case study: results of numerical simulations



☺ Same cost, **performance improved**: $\tau \searrow -56\%$, p-value$<< 0.001$

Conclusions and future works

Analytical tractable model for controlled evolutionary dynamics

- ☺ **General results** to bound spreading time and control cost
- ☺ For some networks, **constant control** guarantees fast spread
- ☺ **Feedback control** can strongly improve the performance

Current/future work

- Look for an **optimal control** strategy
- Use our tools to tackle **different problems** (e.g., slow the spread)

More details can be found in...

- *Fast Diffusion of a Mutant in Controlled Evolutionary Dynamics*, IFAC Papers OnLine 50-1, pp. 11908–11913, 2017
- *Controlling Evolutionary Dynamics in Networks: A Case Study*, IFAC Papers OnLine 51-23, pp. 349–354, 2018
- *Fast Spread in Controlled Evolutionary Dynamics*, Working Paper

Thank you for your attention!

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