Distributed generalized Nash equilibrium seeking in aggregative games on time-varying networks

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EINDHOVEN UNIVERSITY OF TECHNOLOGY

Outline

Mathematical setup

Part I - GNE seeking under partial-decision information

Part II - Convergence analysis: Monotone operator/Fixed-point theory

Conclusion and outlook

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Conclusion and outlook

We focus on Generalized Aggregative Games

- $N \gg 1$ agents/players, each with cost function and constraint set
- Game $=$ { inter-dependent optimization problems }

$$\forall i : \quad \begin{cases} \underset{x_i \in \mathbb{R}^n}{\operatorname{argmin}} \quad J_i(x_i, \boldsymbol{x}_{-i}) & \longleftarrow \quad \text{cost function} \\ \\ \\ \end{cases}$$

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TU/e

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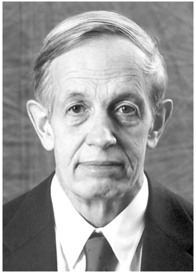
$$\forall i: \quad \begin{cases} \underset{x_i \in \mathbb{R}^n}{\operatorname{argmin}} & J_i(x_i, \frac{1}{N}\sum_{j=1}^N x_j) & \longleftarrow \text{convex in } x_i \\ \text{s.t.} & x_i \in \Omega_i \subset \mathbb{R}^n & \longleftarrow \text{closed, convex} \\ & \sum_{j=1}^N A_j x_j - b_j \leq 0 & \longleftarrow \text{affine coupling con.} \end{cases}$$

Generalized Nash equilibrium (GNE) is desired

A GNE is a feasible set of strategies $\mathrm{col}(x_1^*, \ldots, x_N^*)$ such that

$$\forall i: \quad J_i(x_i^*, \boldsymbol{x}_{-i}^*) \leq J_i(y, \boldsymbol{x}_{-i}^*), \quad \forall y \in \mathcal{X}(x_{-i}^*),$$

Everyone's choice is optimal given the choices of others.



John F. Nash Jr.
Nobel Econ. 94

J.Nash, *Nash, Equilibrium points in N-person games*, PNAS, 1950.

Literature Overview: GNE seeking in full-decision information

- via (primal-dual) projected pseudo-gradient dynamics

$$\forall i: \quad x_i^{k+1} = \mathrm{proj}_{\Omega_i}\Big(x_i - \alpha_i \nabla_{x_i} J_i(x_i^k, x_{-i}^k)\Big)$$

Facchinei, Kanzow, *Generalized Nash equilibrium problems*, 4OR, 2007.

Kannan, Shanbhag, *Distributed computation of equilibria in monotone Nash games via iterative regularization techniques* SICON, 2012.

Paccagnan, Gentile, Parise, Kamgarpour, Lygeros *Distributed computation of generalized Nash equilibria in quadratic aggregative games with affine coupling constraints*, IEEE CDC, 2016.

Belgioioso, Grammatico, *Semi-decentralized Nash equilibrium seeking in aggregative games with separable coupling constraints*, IEEE L-CSS, 2017.

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GNE in Aggregative Games - full-decision information

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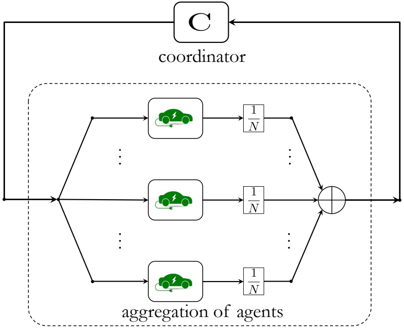

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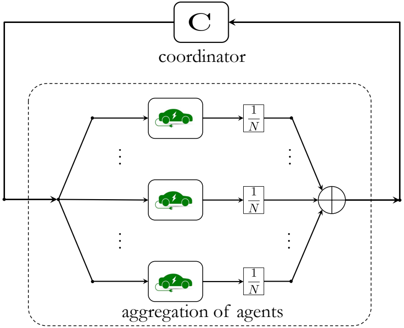
(S.1) Agents: local strategy update

$$x_i^{k+1} = \mathrm{proj}_{\Omega_i}\left[x_i^k - \gamma(\nabla_{x_i} J_i(x_i^k, \sigma^k) + A_i^\top \lambda^k)\right]$$

(S.2) Central: multiplier and average updates

$$\sigma^{k+1} = \frac{1}{N}\sum_{j=1}^N x_j^{k+1}$$

$$\lambda^{k+1} = \mathrm{proj}_{\mathbb{R}_{\geq 0}}\left[\lambda^k + \gamma(2A\mathbf{x}^{k+1} - A\mathbf{x}^k - b)\right]$$



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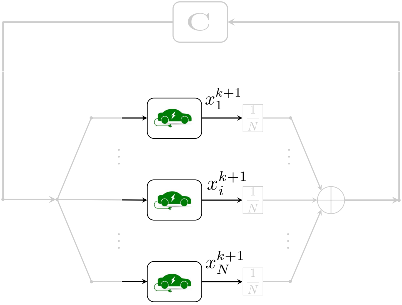

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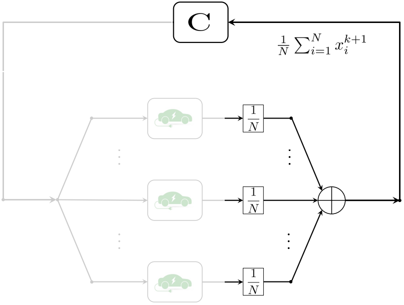

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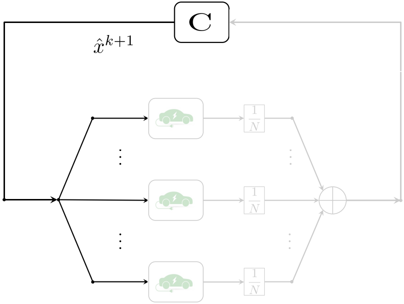

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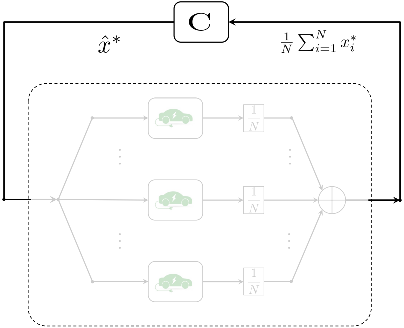

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Outline

Literature Overview: GNE seeking in partial-decision information

- Each agent is endowed with estimates of each other strategy

$$\forall i: \quad \hat{\boldsymbol{x}}_i = \mathrm{col}(\hat{x}_{i,1}, \ldots, \hat{x}_{i,N}), \quad \hat{x}_{i,i} = x_i,$$

- via (primal-dual) projected pseudo-gradient dynamics + consensus dynamics

$$\forall i: \quad \begin{cases} x_i^{k+1} = \mathrm{proj}_{\Omega_i}\Big(x_i - \alpha_i \left(\nabla_{x_i} J_i(x_i^k, \hat{x}_{i,-i}^k) + \beta_i \sum_{j=1}^N w_{ij}(x_i^k - \hat{x}_{j,i}^k)\right)\Big) \\ \hat{x}_{i,-i}^{k+1} = x_{i,-i}^k - \beta_i \sum_{j=1}^N w_{ij}(\hat{x}_{i,-i}^k - \hat{x}_{j,-i}^k) \end{cases}$$

T. Tatarenko, A. Nedić, IEEE CDC, 2018.

L. Pavel, IEEE TAC, 2019.

M. Bianchi, G. Belgioioso, S. Grammatico, arXiv, 2019.

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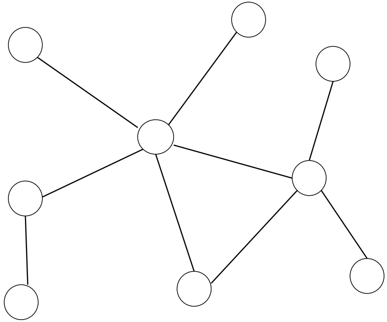

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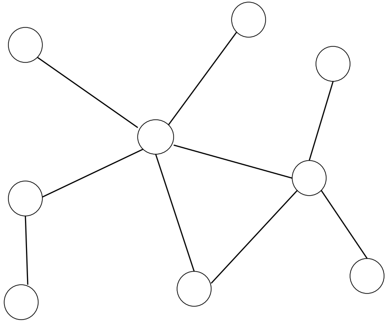

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- Local variables of agent i:
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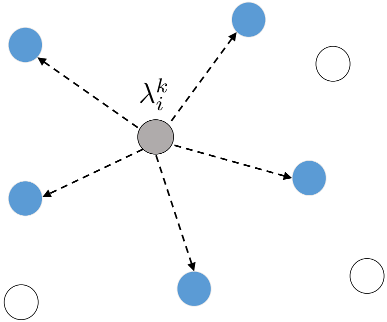

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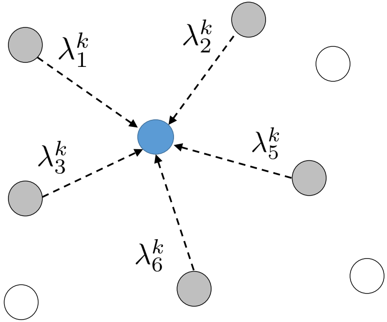

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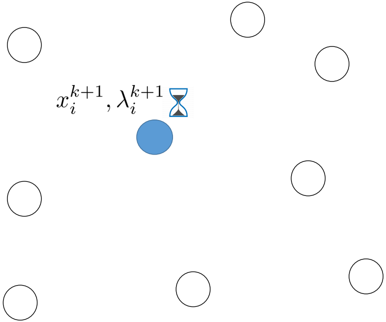

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- Iteration k ...

TU/e

(S.1) Communication over network

$$\hat{\sigma}_i^k = \sum_{j=1}^N w_{ij}\sigma_j^k, \quad \hat{z}_i^k = \sum_{j=1}^N w_{ij} z_j^k, \quad \hat{y}_i^k = \sum_{j=1}^N w_{ij} y_j^k,$$

(S.2) Local Primal-Dual update

$$\tilde{x}_i^k = \mathrm{Proj}_{\Omega_i}\left[x_i^k - \alpha_i\left(\nabla_{x_i} J_i(x_i^k, \hat{\sigma}_i^k) + A_i^\top \hat{z}_i^k\right)\right],$$
$$\tilde{\lambda}_i^k = \mathrm{proj}_{\mathbb{R}_+}\left(\lambda_i^k + \beta_i(y_i^k - \lambda_i^k + \hat{z}_i^k)\right),$$

(S.3) Local Krasnosel'skii–Mann Process

$$x_i^{k+1} = x_i^k + \gamma^k(\tilde{x}_i^k - x_i^k),$$
$$\lambda_i^{k+1} = \lambda_i^k + \gamma^k(\tilde{\lambda}_i^k - \lambda_i^k),$$

(S.4) Local Dynamic Tracking

$$\sigma_i^{k+1} = \hat{\sigma}_i^k + x_i^{k+1} - x_i^k,$$
$$z_i^{k+1} = \hat{z}_i^k + \lambda_i^{k+1} - \lambda_i^k.$$
$$y_i^{k+1} = \hat{y}_i^k + C_i(2\tilde{x}_i^k - x_i^k) - C_i(2\tilde{x}_i^{k-1} - x_i^{k-1}).$$

Algorithm. Local updating rules:

1. Communication and distributed averaging
2. Inexact projected-pseudogradient dynamics
3. Krasnosels'kii–Mann process
4. Dynamic Tracking of the aggregation quantities

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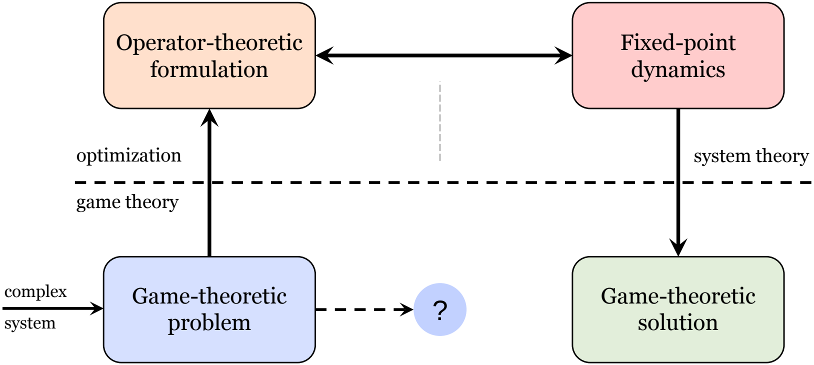
Operator theoretic approach: schematic outline



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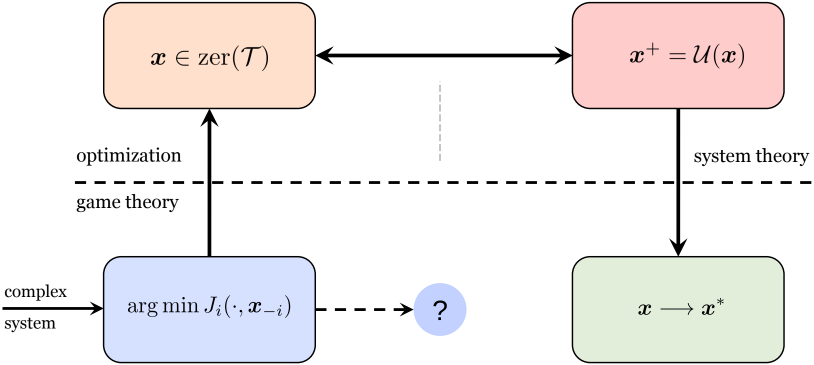

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How to estimate on-line aggregative quantities?

Operator theoretic approach - partial information

How to estimate on-line aggregative quantities?

Idea: combine splitting methods with gradient tracking techniques

Nedić et al., SIAM, 2017. Xu, CDC, 2015. Zanella, CDC, 2011. Xi, TAC, 2018.

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$$x_i^{k+1} = \sum_{j=1}^{N} w_{ij} x_i^k - \alpha_i \nabla f_i(x_i^k)$$

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Ideal (non-distributed) scheme

$$x_i^{k+1} = \sum_{j=1}^{N} w_{ij} x_i^k - \alpha_i \frac{1}{N} \sum_{j=1}^{N} \nabla f_i(x_i^k)$$

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DIGing (Nedić et al., *SIAM*, 2017)

$$x_i^{k+1} = \sum_{j=1}^{N} w_{ij} x_i^k - \alpha_i y_i(k),$$

$$y_i^{k+1} = \sum_{j=1}^{N} w_{ij} y_i^k + \nabla f_i(x_i^{k+1}) - \nabla f_i(x_i^k),$$

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$$x_i^{k+1} = \sum_{j=1}^{N} w_{ij} x_i^k - \alpha_i y_i(k),$$

$$y^\infty = \frac{1}{N} \sum_i \nabla f_i(x^\infty)$$

How to deal with the inexactness of the estimates?

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Krasnosel'skii–Mann fixed-point iteration with errors

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k + \gamma^k (T(\boldsymbol{x}^k) - \boldsymbol{x}^k + e^k)$$
$$e^k = T(\hat{\boldsymbol{x}}^k) - T(\boldsymbol{x}^k)$$

P. Combettes, *Quasi-Fejerian Analysis of some optimization algorithms*, ELSEVIER, 2001

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How to deal with the inexactness of the estimates?

Krasnosel'skii–Mann fixed-point iteration with errors

$$\boxed{\begin{aligned} \boldsymbol{x}^{k+1} &= \boldsymbol{x}^k + \gamma^k (T(\boldsymbol{x}^k) - \boldsymbol{x}^k + e^k) \\ e^k &= T(\hat{\boldsymbol{x}}^k) - T(\boldsymbol{x}^k) \end{aligned}}$$

P. Combettes, *Quasi-Fejerian Analysis of some optimization algorithms*, Elsevier, 2001

Theorem. If $\sum_{k=0}^{\infty} \gamma^k \|e^k\| < \infty$, T non-expansive, then $\boldsymbol{x}^k \to \boldsymbol{x}^* \in \mathrm{fix}(T)$.

Outline

Mathematical setup

Part I - GNE seeking under partial-decision information

Part II - Convergence analysis: Monotone operator/Fixed-point theory

Conclusion and outlook

Conclusion and outlook

- Problem: GNE seeking in aggregative games under partial decision-information
- Framework: Monotone operator/fixed-point theory
- Our result: Full-distributed algorithm on time-varying networks

G. Belgioioso, A. Nedić, S. Grammatico, *Distributed generalized Nash equilibrium seeking in aggregative games on time-varying networks*, *arXiv* (available at https://arxiv.org/abs/1907.00191), 2019.

Conclusion and outlook

- Same Framework: Monotone operator/fixed-point theory
- Different Viewpoint: The network dynamics is fixed

$$\boldsymbol{x}(k+1) = \begin{bmatrix} \mathrm{prox}_{f_1} & & \\ & \ddots & \\ & & \mathrm{prox}_{f_N} \end{bmatrix} W(k)\boldsymbol{x}(k)$$

- Problem: Under what conditions on W and f_i's does the system converge?

C. Cenedese, G. Belgioioso, Y.Kawano, S. Grammatico, M. Cao, *Asynchronous and time-varying proximal type dynamics multi-agent network games*, *arXiv* (available at `https://arxiv.org/abs/1909.11203`), 2019.

Thank you for your attention!

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