Charging plug-in electric vehicles as a mixed-integer aggregative game

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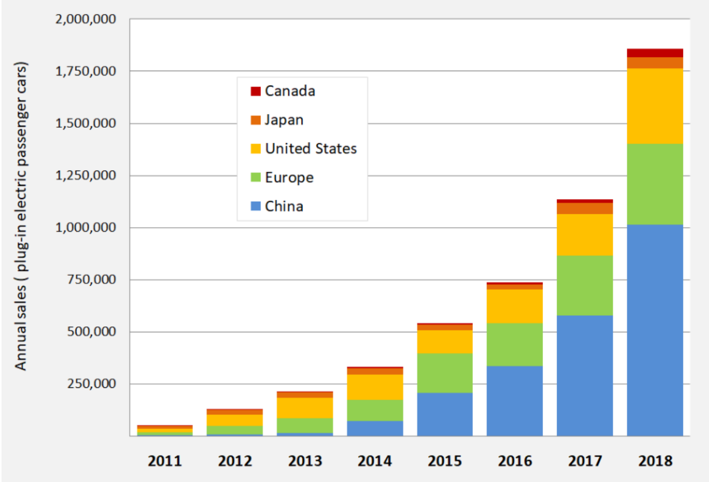
Charging of EV

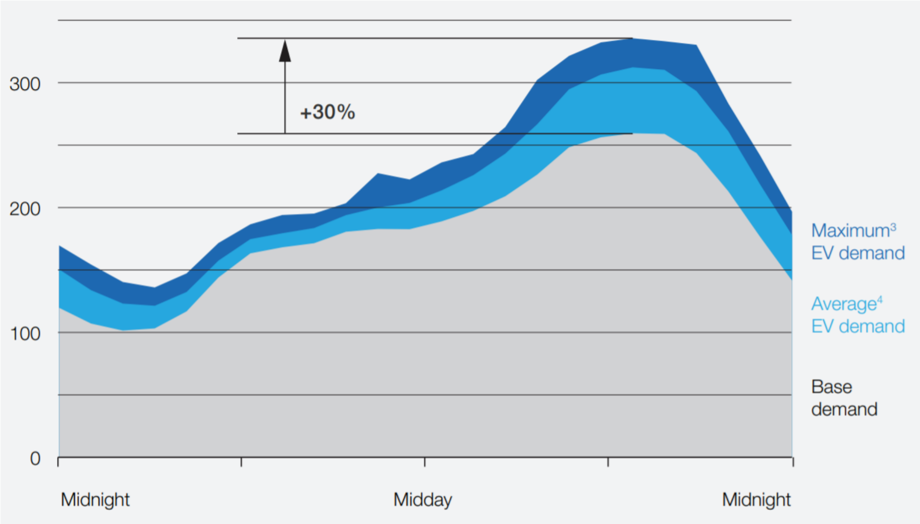
- The sales of EV are skyrocketing in the past years
- Increment in the overall **energy demand and peaks**

Can we create a charging scheduling that **decreases the peak** and **satisfies all the users**?

Global annual sales (2011-2018)[1]



Estimated daily energy demand[2]

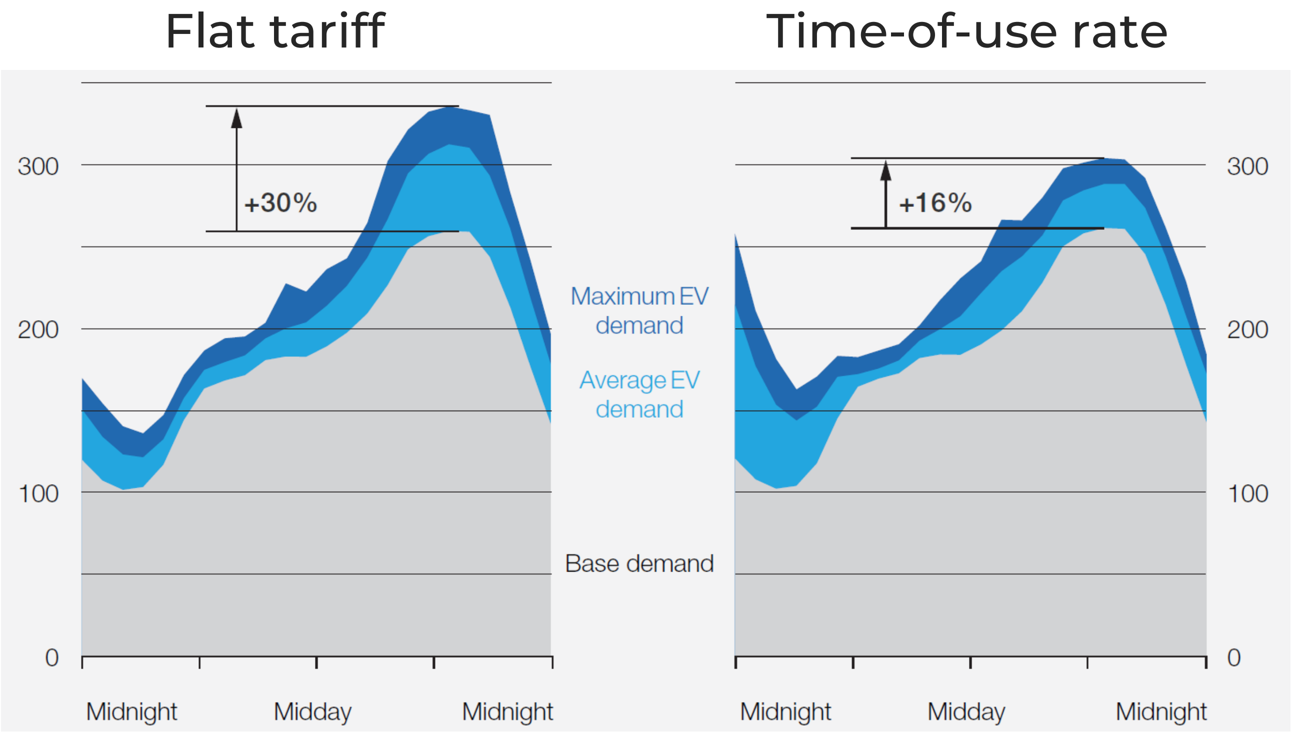


1: IEA analysis based on country submissions, complemented by ACEA (2019); EAFO (2019); EV Volumes (2019); Marklines (2019); OICA (2019).
2: OpenEI; McKinsey analysis

Time-of-use rates

- Shaping the price can lead to a decrement of the demand peak
- Simple rule that can halve the demand peak

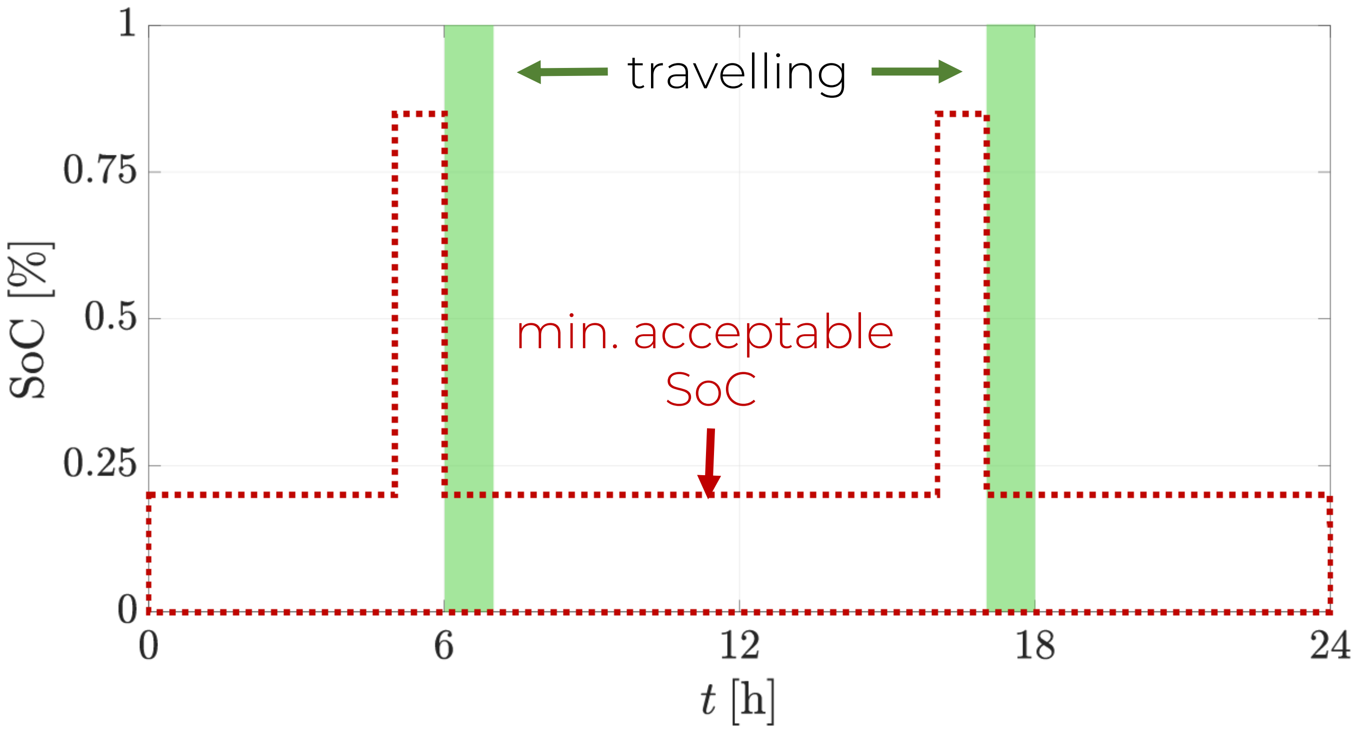
<u>Intuition:</u> a **dynamic energy** price can achieve **valley filling**



Source: OpenEI; McKinsey analysis

The user point of view

- Working from 7 a.m. to 5,30 p.m.
- Distance from work ~30 km

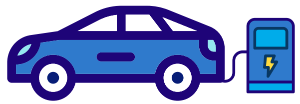


Goal: minimize the cost while satisfy the SoC constraints

State of Charge (SoC) dynamics

- Binary variable to model the EV connection to the grid

$$\delta(t) := \begin{cases} 1 & \text{if} \\ 0 & \text{otherwise} \end{cases}$$



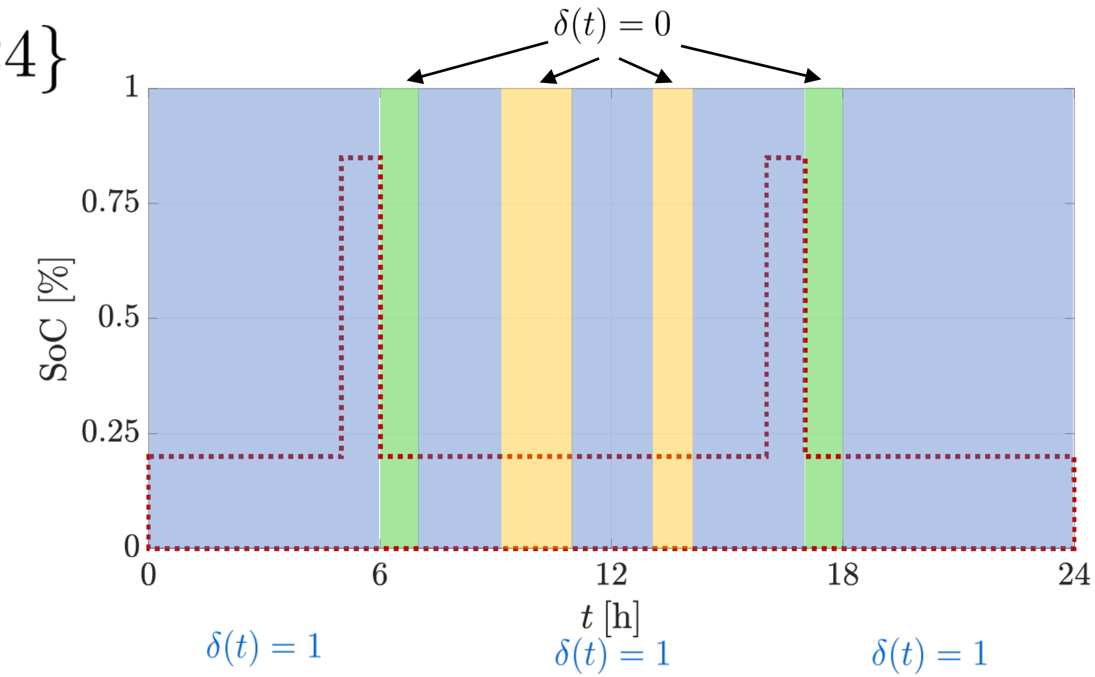
- SoC dynamics $x(t)$ for $t \in \mathcal{T} := \{0, \dots, 24\}$

$$x(t+1) = x(t) + b\,\delta(t)\,u(t) - (1 - \delta(t))\,\mu(t)$$

charging : $u(t) > 0$
discharging : $u(t) < 0$

discharge due to travel

$\mu(t)$ → function of battery capacity and km travelled



Energy price

- Consider a fleet of EVs $\mathcal{I} = \{1, \ldots, N\}$
- Price of purchasing the energy for agent $i \in \mathcal{I}$ in the interval t

$$p_i(t) = c(d(t) + \sum_{j \in \mathcal{I} \setminus \{i\}} u_j^+(t)) u_i(t) \quad t \in \mathcal{T}$$

cost per energy unit

demant not from EV

$$\begin{cases} u_j(t) & \text{if} \quad u_j(t) > 0 \\ 0 & \text{otherwise} \end{cases}$$

- Different price for every time interval

Cost function (1/2)

- **Grid to EV :** cost to charging

$$J_i^{\mathrm{g2v}}(u_i(t), \boldsymbol{u}_{-i}(t)) := p_i(t) + \rho_i^{+}(u_i(t) - u_i(t-1))^2$$

degradation cost

- **EV to Grid :** reward to discharge

$$J_i^{\mathrm{v2g}}(u_i(t)) := r_i(t)u_i(t) + \rho_i^{-}(u_i(t) - u_i(t-1))^2$$

reward for discharging

- Possibe reward function → $r_i(t) = r_i\, d(t)$

Cost function (2/2)

- **Binary** variables to identify

not discharging $[\delta_i^{\mathrm{c}}(t) = 1] \iff [u_i(t) \geq 0]$

not charging $[\delta_i^{\mathrm{d}}(t) = 1] \iff [u_i(t) \leq 0]$

- **Final cost funciton** over the whole day

$$J_i := \sum_{t \in \mathcal{T}} J_i^{\mathrm{g2v}}(u_i(t), \boldsymbol{u}_{-i}(t))\left(1 - \delta_i^{\mathrm{d}}(t)\right) + J_i^{\mathrm{v2g}}(u_i(t))\left(1 - \delta_i^{\mathrm{c}}(t)\right)$$

all time intervals

Game formulation

- Each agent i chooses the charging schedule that **minimizes J_i** while satisfying the **local/coupling constraints**
- A **noncooperative game** arises

$$\forall i \in \mathcal{I}: \begin{cases} \min\limits_{u_i, x_i, \delta_i, \delta_i^{\mathrm{d}}, \delta_i^{\mathrm{c}}} & J_i(u_i, \delta_i^{\mathrm{d}}, \delta_i^{\mathrm{c}}, \boldsymbol{u}_{-i}) \\ \text{s.t.} & x_i(t) \in [0, 1],\ u_i(t) \in [\underline{u}\delta_i(t), \overline{u}\delta_i(t)], \\ & \delta_i(t) \in \mathcal{B}(t),\ \delta_i^{\mathrm{c}}(t), \delta_i^{\mathrm{d}}(t) \in \{0,\ 1\} \\ & x_i(t) \geq x_{i,\mathrm{ref}}(t) \\ & \text{SoC dynamics} \\ & \delta_i^{\mathrm{c}}(t), \delta_i^{\mathrm{d}}(t) \to \text{charging}\backslash\text{discharging cond.} \end{cases}$$

$$\mathcal{B}(t) := \begin{cases} 0 & \mu_i(t) > 0 \\ \{0,\ 1\} & \text{otherwise} \end{cases}$$

Additional constraints

$$\forall i \in \mathcal{I}: \begin{cases} \min\limits_{u_i, x_i, \delta_i, \delta_i^{\mathrm{d}}, \delta_i^{\mathrm{c}}} & J_i(u_i, \delta_i^{\mathrm{d}}, \delta_i^{\mathrm{c}}, \boldsymbol{u}_{-i}) \\ \text{s.t.} & x_i(t) \in [0,1],\ u_i(t) \in [\underline{u}\delta_i(t), \overline{u}\delta_i(t)], \\ & \delta_i(t) \in \mathcal{B}(t),\ \delta_i^{\mathrm{c}}(t), \delta_i^{\mathrm{d}}(t) \in \{0,\ 1\} \\ & x_i(t) \geq x_{i,\mathrm{ref}}(t) \\ & \text{SoC dynamics} \\ & \delta_i^{\mathrm{c}}(t), \delta_i^{\mathrm{d}}(t) \rightarrow \text{charging}\backslash\text{discharging} \\ & h_i \quad \text{consecutive connected interval} \end{cases}$$

Local constraints:

- Avoid persistent switch between connected and unconnected.
- EV remain connected to the charging station for at least h_i intervals

$$[\delta_i(t-1) = 0] \wedge [\delta_i(t) = 1]$$

$$\Downarrow$$

$$[\delta_i(t+h) = 1,\ \forall h \leq h_i]$$

Additional constraints

$$\forall i \in \mathcal{I}: \begin{cases} \min\limits_{u_i, x_i, \delta_i, \delta_i^{\mathrm{d}}, \delta_i^{\mathrm{c}}} & J_i(u_i, \delta_i^{\mathrm{d}}, \delta_i^{\mathrm{c}}, \boldsymbol{u}_{-i}) \\ \text{s.t.} & x_i(t) \in [0,1],\ u_i(t) \in [\underline{u}\delta_i(t), \overline{u}\delta_i(t)], \\ & \delta_i(t) \in \mathcal{B}(t),\ \delta_i^{\mathrm{c}}(t), \delta_i^{\mathrm{d}}(t) \in \{0,\ 1\} \\ & x_i(t) \geq x_{i,\mathrm{ref}}(t) \\ & \text{SoC dynamics} \\ & \delta_i^{\mathrm{c}}(t), \delta_i^{\mathrm{d}}(t) \rightarrow \text{charging}\backslash\text{discharging} \\ & h_i \quad \text{consecutive connected interval} \\ & d(t) + \sum_{j \in \mathcal{I}} u_j(t) \in [0, \overline{d}] \\ & \sum_{j \in \mathcal{I}} \delta_j(t) \leq \overline{v} \end{cases}$$

Coupling constraints:

- Grid capacity

$$d(t) + \sum_{j \in \mathcal{I}} u_j(t) \in [0, \overline{d}]$$

- # of EV connected simultaneously $\leq \overline{v}$

$$\sum_{j \in \mathcal{I}} \delta_j(t) \leq \overline{v}$$

Note: the logical constraints can be translated in affine ones introducing auxiliaries variables .
[Cenedese et al., «Charging plug-in electric vehicles as a mixed-integer aggregative game»

Additional constraints

$$
\forall i \in \mathcal{I}: \begin{cases}
\min\limits_{u_i, x_i, \delta_i, \delta_i^{\mathrm{d}}, \delta_i^{\mathrm{c}}} & J_i(u_i, \delta_i^{\mathrm{d}}, \delta_i^{\mathrm{c}}, \boldsymbol{u}_{-i}) \\
\text{s.t.} & x_i(t) \in [0,1],\ u_i(t) \in [\underline{u}\delta_i(t), \overline{u}\delta_i(t)], \\
& \delta_i(t) \in \mathcal{B}(t),\ \delta_i^{\mathrm{c}}(t), \delta_i^{\mathrm{d}}(t) \in \{0,\ 1\} \\
& x_i(t) \geq x_{i,\mathrm{ref}}(t) \\
& \text{SoC dynamics} \\
& \delta_i^{\mathrm{c}}(t), \delta_i^{\mathrm{d}}(t) \to \text{charging}\backslash\text{discharging} \\
& h_i \quad \text{consecutive connected interval} \\
& d(t) + \sum_{j \in \mathcal{I}} u_j(t) \in [0, \overline{d}] \\
& \sum_{j \in \mathcal{I}} \delta_j(t) \leq \overline{v}
\end{cases}
$$

- Formulate the problem as a **mixed-integer noncooperative game (Γ)** with **rational players**

$$
\forall i \in \mathcal{I}: \begin{cases}
\min\limits_{z_i} J_i(z_i, \boldsymbol{z}_{-i}) \\
\text{s.t.} \quad (z_i, \boldsymbol{z}_{-i}) \in \boldsymbol{\mathcal{Z}}
\end{cases}
$$

Note: the logical constraints can be translated in affine ones introducing auxiliaries variables .
[Cenedese et al., «Charging plug-in electric vehicles as a mixed-integer aggregative game»

ε-Mixed-Integer Nash equilibrium

- No user can decrease its cost by choosing a different schedule → user satisfaction

Definition: Let $\boldsymbol{\varepsilon} > \mathbf{0}$, then $\boldsymbol{z}^* \in \mathcal{Z}$ is an ε-MINE of the game Γ if for all $i \in \mathcal{I}$ it holds

$$J_i(z_i^*, \boldsymbol{z}^*_{-i}) \leq \inf_{z_i \in \mathcal{Z}_i(\boldsymbol{z}^*_{-i})} J_i(z_i, \boldsymbol{z}^*_{-i}) + \varepsilon$$

Potential game

- Let the decision variables be $z_i(t)$ we reorganize the cost function

$$J_i(z_i, \boldsymbol{z}_{-i}) = \phi_i(z_i) + \sum_{j \in \mathcal{I} \setminus \{i\}} \omega_{ij}(z_i, z_j)$$

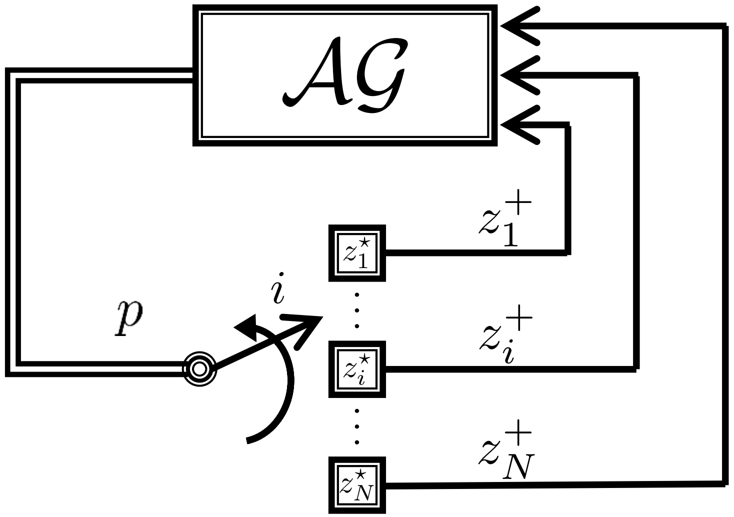
- The game is a **mixed-integer generalized potential game** with exact potential function

$$P(z_i, \boldsymbol{z}_{-i}) - P(y_i, \boldsymbol{z}_{-i}) = J_i(z_i, \boldsymbol{z}_{-i}) - J_i(y_i, \boldsymbol{z}_{-i})$$

$$P(\boldsymbol{z}) := \sum_{i \in \mathcal{I}} \left(\phi_i(z_i) + \sum_{j \in \mathcal{I}, j < i} \omega_{ij}(z_i, z_j) \right)$$

Update and convergence

Initialization: Choose $\boldsymbol{z}(0) \in \boldsymbol{\mathcal{Z}}$, set $k := 0$
while $\boldsymbol{z}(k)$ is not an ε-Mixed-Integer Nash Equilibrium (ε-MINE) **do**
$\quad\mathcal{AG}$ **do**
$\quad\quad$Chooses $i := i(k) \in \mathcal{I}$
$\quad\quad$Sets p_i (function of $\boldsymbol{u}(k)$) for all $t \in \mathcal{T}$
$\quad\quad$Sends $p_i(\boldsymbol{u}(k))$, and the other aggregate info
$\quad$**end**
$\quad$Player i **do**
$\quad\quad$Compute $z_i^*(k)$ via Best Response dynamics
$\quad\quad$**if**
$\quad\quad$ $J_i(z_i(k), p_i(\boldsymbol{u}(k))) - J_i(z_i^*(k), p_i(\boldsymbol{u}(k))) \geq \varepsilon$
$\quad\quad\quad$ $z_i(k+1) := z_i^*(k)$
$\quad\quad$**else**
$\quad\quad\quad$ $z_i(k+1) := z_i(k)$
$\quad\quad$**end**
$\quad$**end**
$\quad\mathcal{AG}$ collects $z_i(k+1)$
$\quad$Set $z_j(k+1) := z_j(k)\ \forall j \neq i,\ k := k+1$
end



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  $\mathcal{AG}$ **do**
    Chooses $i := i(k) \in \mathcal{I}$ → One agent updates each iteration k
    Sets p_i (function of $\boldsymbol{u}(k)$) for all $t \in \mathcal{T}$
    Sends $p_i(\boldsymbol{u}(k))$, and the other aggregate info
  end
  Player i **do**
    Compute $z_i^*(k)$ via Best Response dynamics
    if $J_i(z_i(k), p_i(\boldsymbol{u}(k))) - J_i(z_i^*(k), p_i(\boldsymbol{u}(k))) \geq \varepsilon$
      $z_i(k+1) := z_i^*(k)$
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    end
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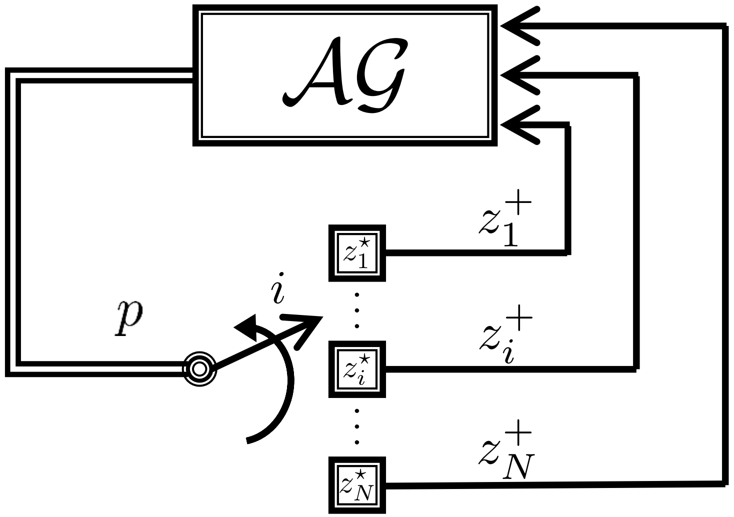
end



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  $\mathcal{AG}$ **do**
    Chooses $i := i(k) \in \mathcal{I}$
    Sets p_i (function of $\boldsymbol{u}(k)$) for all $t \in \mathcal{T}$ ⟶ Aggregator computes price
    Sends $p_i(\boldsymbol{u}(k))$, and the other aggregate info
  end
  Player i **do**
    Compute $z_i^*(k)$ via Best Response dynamics
    if $J_i(z_i(k), p_i(\boldsymbol{u}(k))) - J_i(z_i^*(k), p_i(\boldsymbol{u}(k))) \geq \varepsilon$
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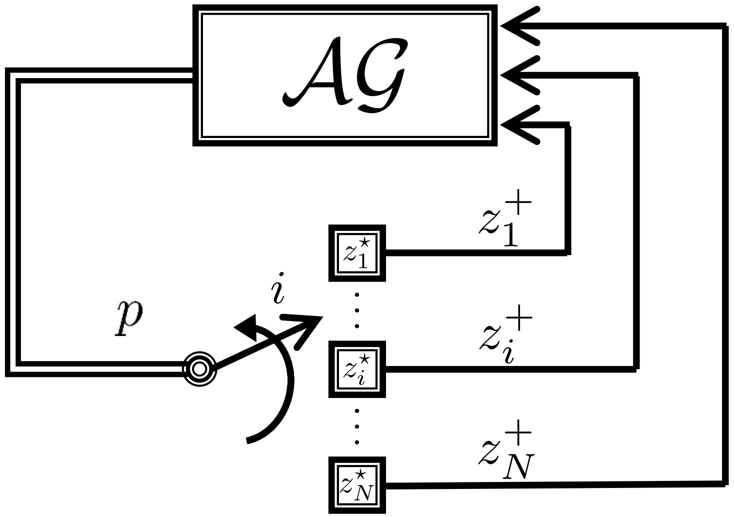

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    end
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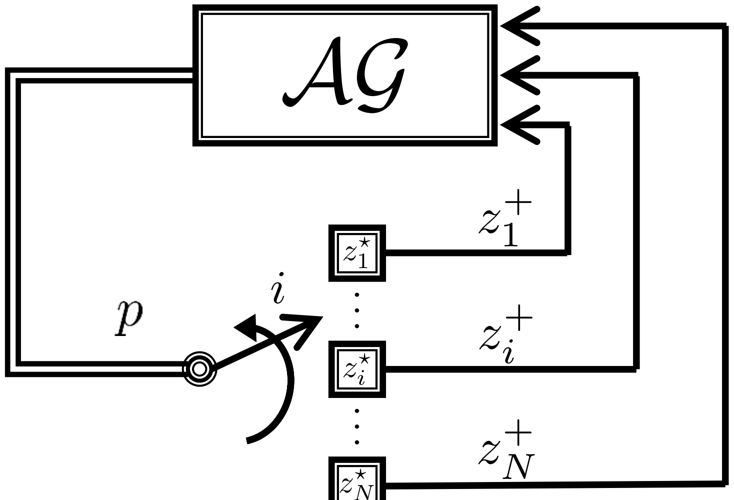
  $\mathcal{AG}$ collects $z_i(k+1)$
  Set $z_j(k+1) := z_j(k)$ $\forall j \neq i$, $k := k+1$
end

$$\min_{z_i} J_i(z_i, \boldsymbol{z}_{-i})$$
$$\text{s.t.} \quad (z_i, \boldsymbol{z}_{-i}) \in \boldsymbol{\mathcal{Z}}$$



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- $\mathcal{AG}$ **do**
  - Chooses $i := i(k) \in \mathcal{I}$
  - Sets p_i (function of $\boldsymbol{u}(k)$) for all $t \in \mathcal{T}$
  - Sends $p_i(\boldsymbol{u}(k))$, and the other aggregate info
- **end**
- Player i **do**
  - Compute $z_i^*(k)$ via Best Response dynamics
  - **if** $J_i(z_i(k), p_i(\boldsymbol{u}(k))) - J_i(z_i^*(k), p_i(\boldsymbol{u}(k))) \geq \varepsilon$
    - $z_i(k+1) := z_i^*(k)$
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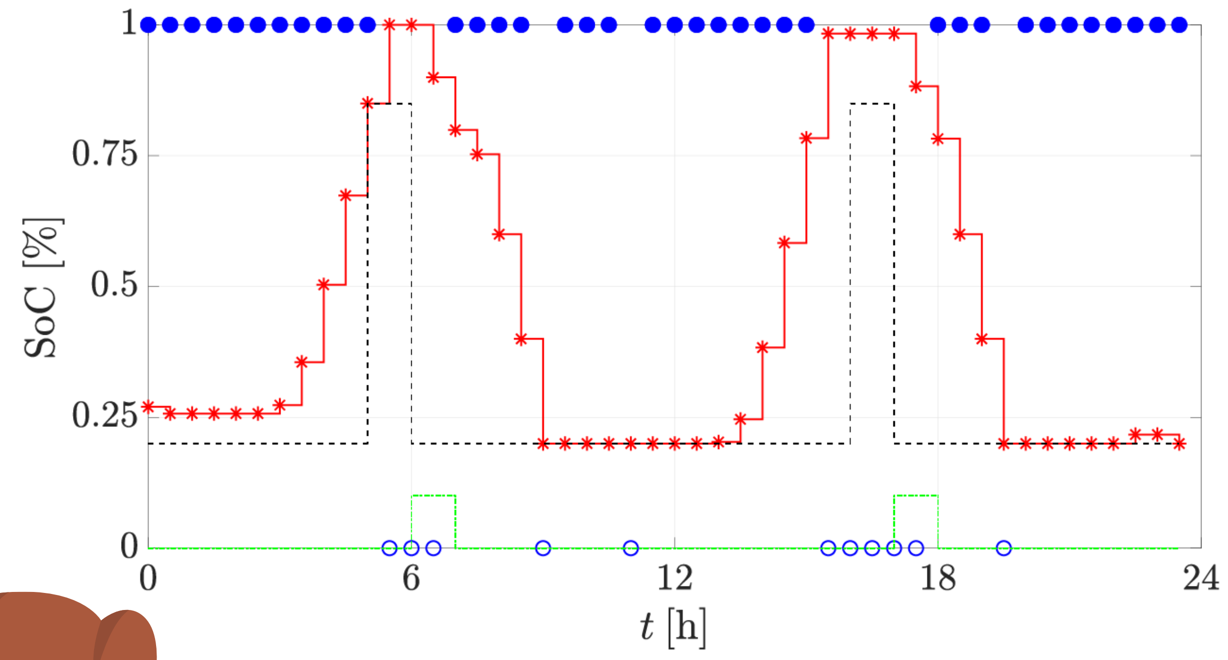
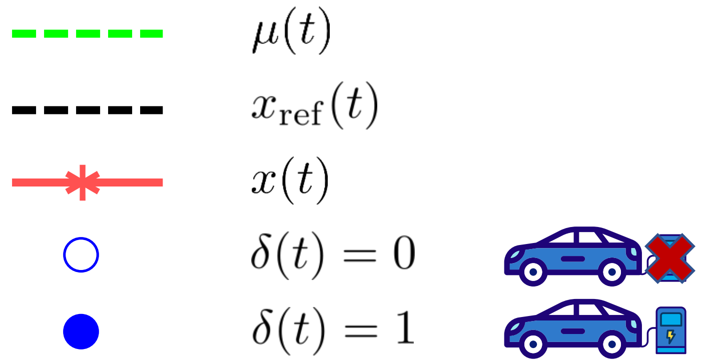
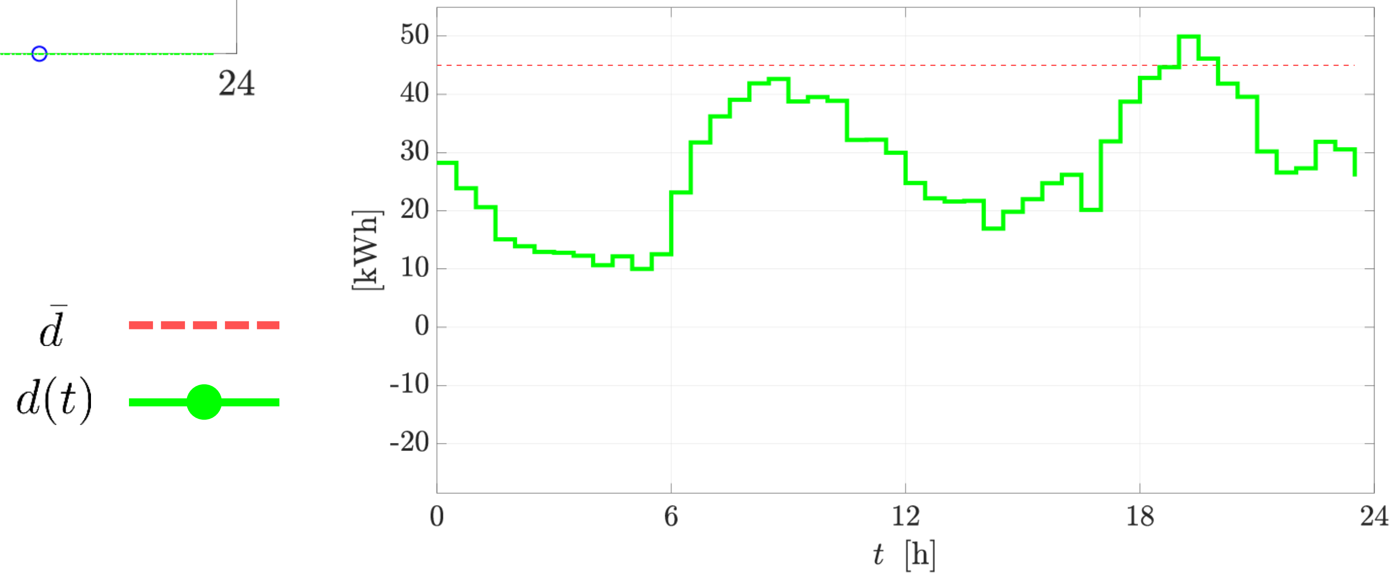
end

Theorem: The sequence $(\boldsymbol{z}(k))_{k=0}^{\infty}$ generated from the algorithm converges to a ε-MINE of the game Γ.

Experimental results







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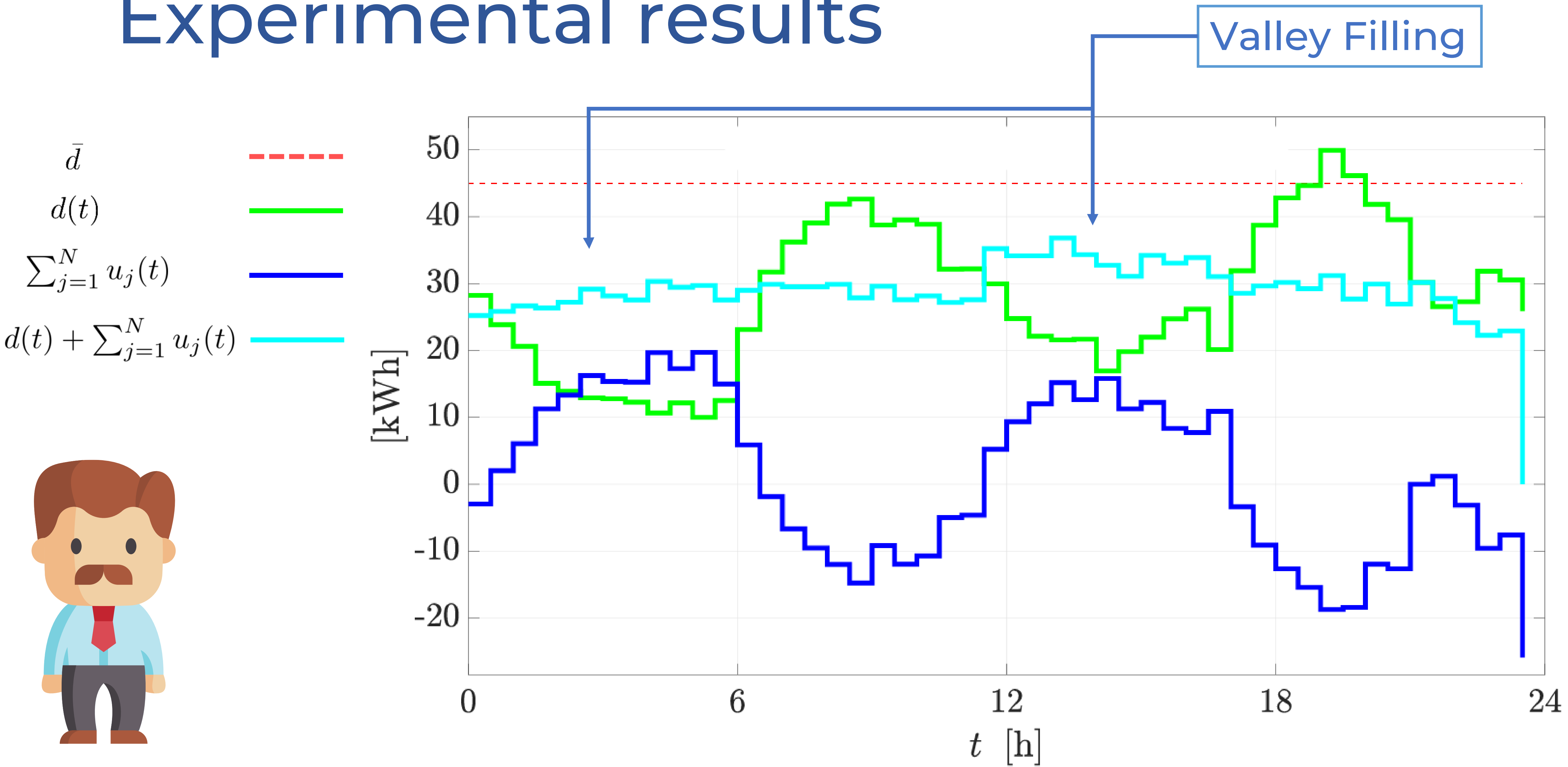

Conclusions

- Formalize the scheduling of EV as a mixed-integer generalized potential game
- Taken into consideration both users and grid constraints
- Asynchronous constrained best response dynamics converges to a ε-MINE of the game

Thanks

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Paper ref.: Cenedese et al., «Charging plug-in electric vehicles as a mixed-integer aggregative game»