



POLITECNICO  
DI TORINO

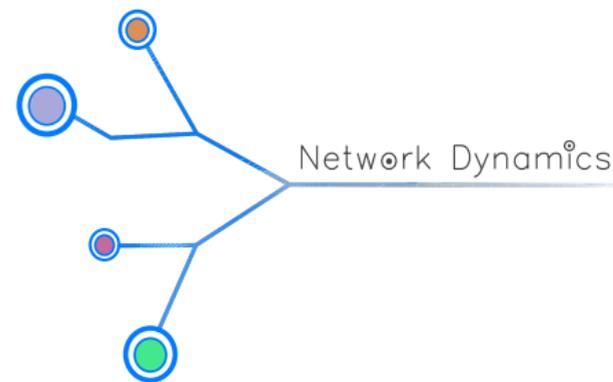


# Contagion in Financial Networks

Leonardo Massai, Fabio Fagnani, Giacomo Como

leonardo.massai@polito.it

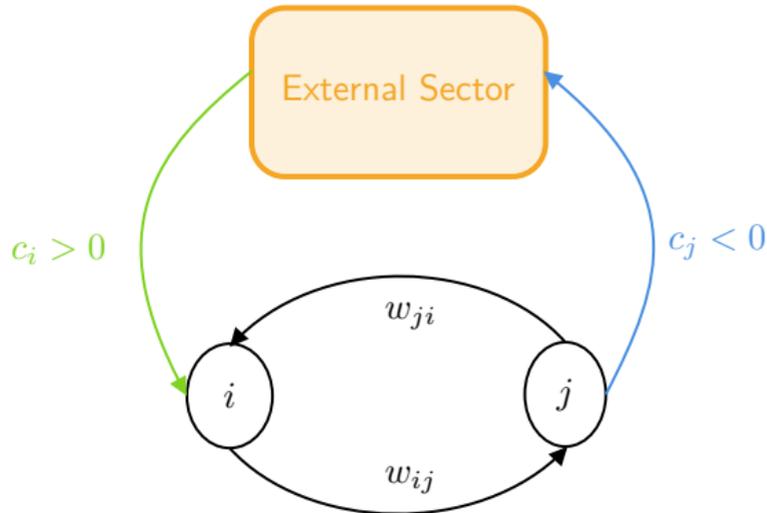
Department of Mathematical Sciences (DISMA) , Politecnico di Torino



# Outline



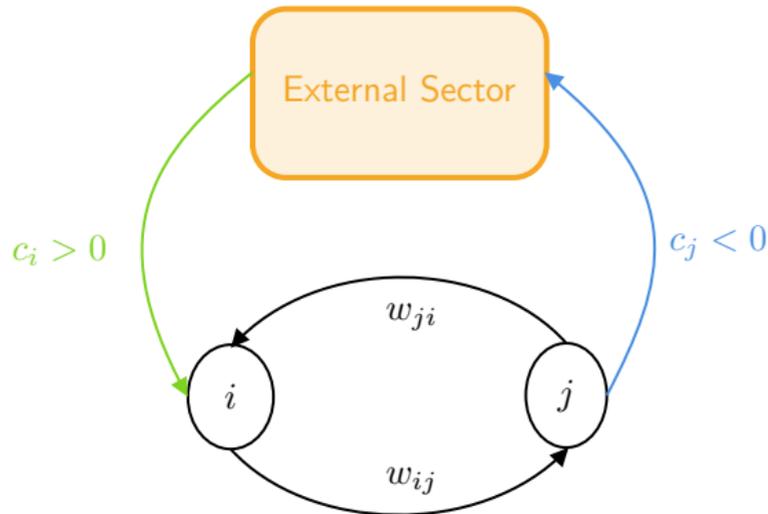
- The Financial Network Model
  - Introduction
- Uniqueness of Clearing Vectors
  - Existence and Uniqueness
  - The Out-Connected Case
  - The Stochastic Irreducible Case
  - The General Case
- Critical Transitions
  - The Dependence of Clearing Vectors on the Shock
  - Jump Discontinuity
- Results and Ongoing Research



- $w_{ij}$  inter-bank liability;
- $c_i > 0$  positive money inflow;
- $c_j < 0$  outside debt.

### Everything is fine

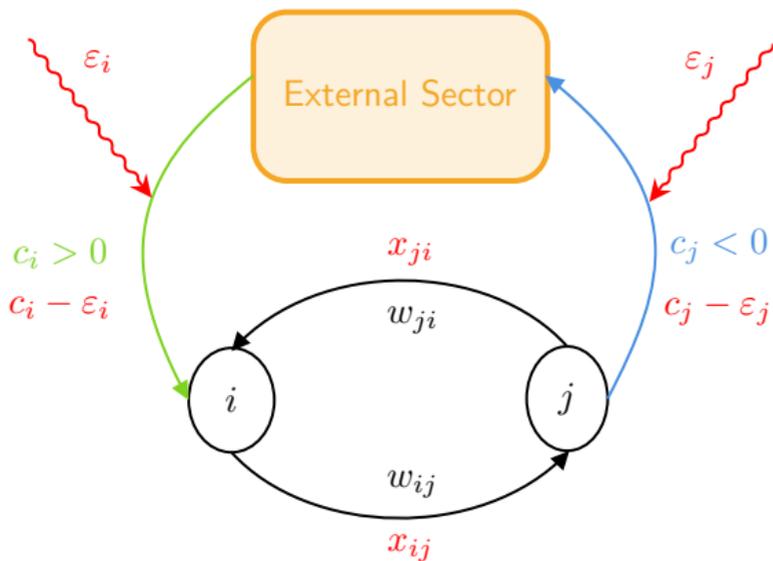
In normal conditions, every bank  $i$  can meet its total liability:  $w_i = \sum_j w_{ij}$ .



- $w_{ij}$  inter-bank liability;
- $c_i > 0$  positive money inflow;
- $c_j < 0$  outside debt.

### Everything is fine

In normal conditions, every bank  $i$  can meet its total liability:  $w_i = \sum_j w_{ij}$ .



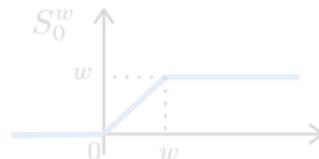
- Shocks  $\varepsilon$  hit the network by reducing  $c$ ;
- Nodes may default and not be able to pay their liabilities (direct effect);
- Shocks propagate across the network because of reduced payments (indirect effect).

### Clearing Vectors

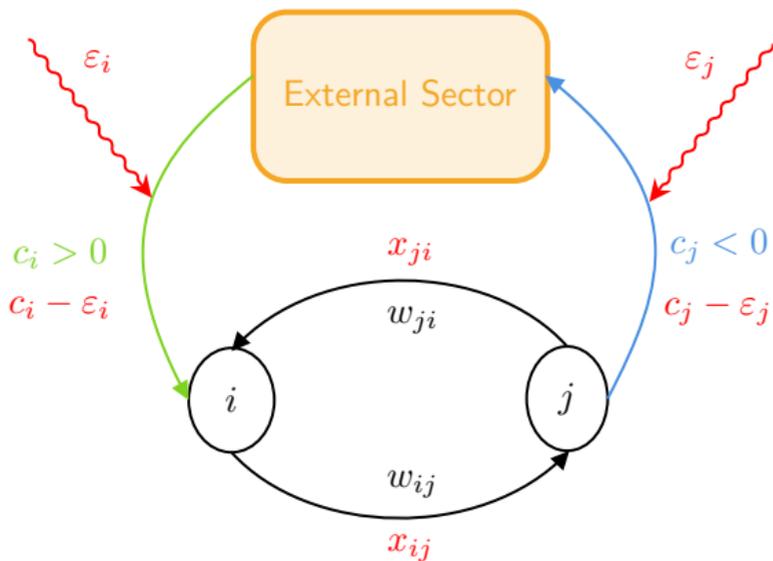
$x$  is a set of consistent payments after the shock:

$$x = S_0^w (P'x + c - \varepsilon)$$

where  $(P)_{ij} = \frac{w_{ij}}{w_i}$  and  $S_0^w$  is a saturation:



- Notice that any solution is such that  $x \in \mathcal{L}_0^w := \{x \in \mathbb{R}^n : 0 \leq x \leq w\}$



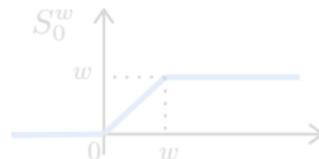
- Shocks  $\varepsilon$  hit the network by reducing  $c$ ;
- Nodes may default and not be able to pay their liabilities (direct effect);
- Shocks propagate across the network because of reduced payments (indirect effect).

### Clearing Vectors

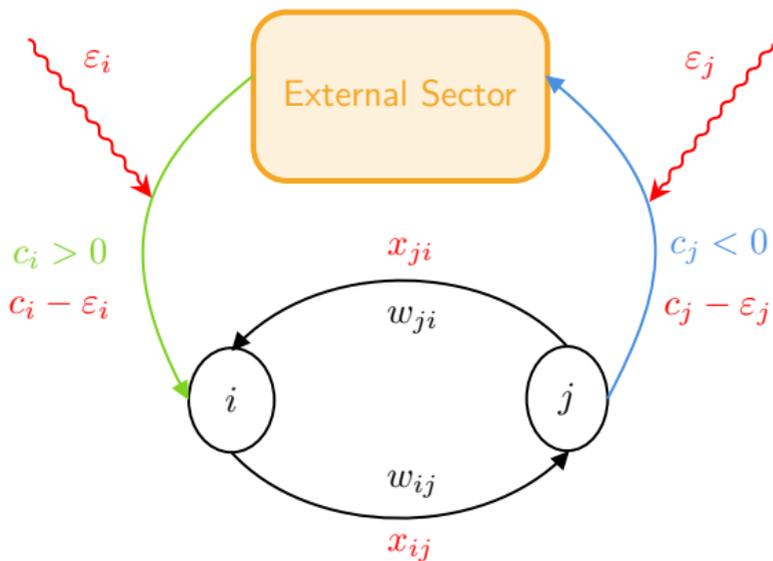
$x$  is a set of consistent payments after the shock:

$$x = S_0^w (P'x + c - \varepsilon)$$

where  $(P)_{ij} = \frac{w_{ij}}{w_i}$  and  $S_0^w$  is a saturation:



- Notice that any solution is such that  $x \in \mathcal{L}_0^w := \{x \in \mathbb{R}^n : 0 \leq x \leq w\}$



- Shocks  $\varepsilon$  hit the network by reducing  $c$ ;
- Nodes may default and not be able to pay their liabilities (direct effect);
- Shocks propagate across the network because of reduced payments (indirect effect).

### Clearing Vectors

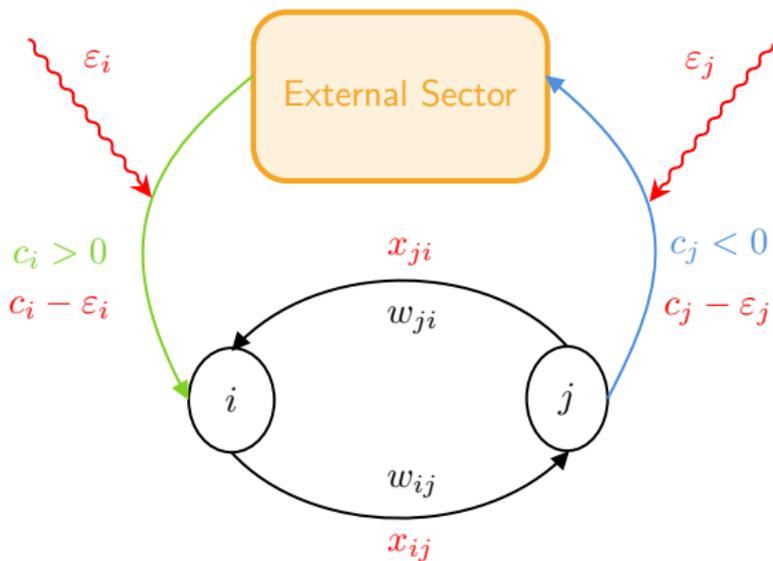
$x$  is a set of consistent payments after the shock:

$$x = S_0^w (P'x + c - \varepsilon)$$

where  $(P)_{ij} = \frac{w_{ij}}{w_i}$  and  $S_0^w$  is a saturation:



- Notice that any solution is such that  $x \in \mathcal{L}_0^w := \{x \in \mathbb{R}^n : 0 \leq x \leq w\}$



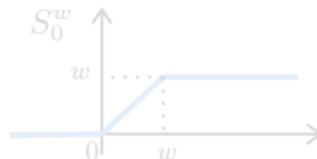
- Shocks  $\varepsilon$  hit the network by reducing  $c$ ;
- Nodes may default and not be able to pay their liabilities (direct effect);
- Shocks propagate across the network because of reduced payments (indirect effect).

### Clearing Vectors

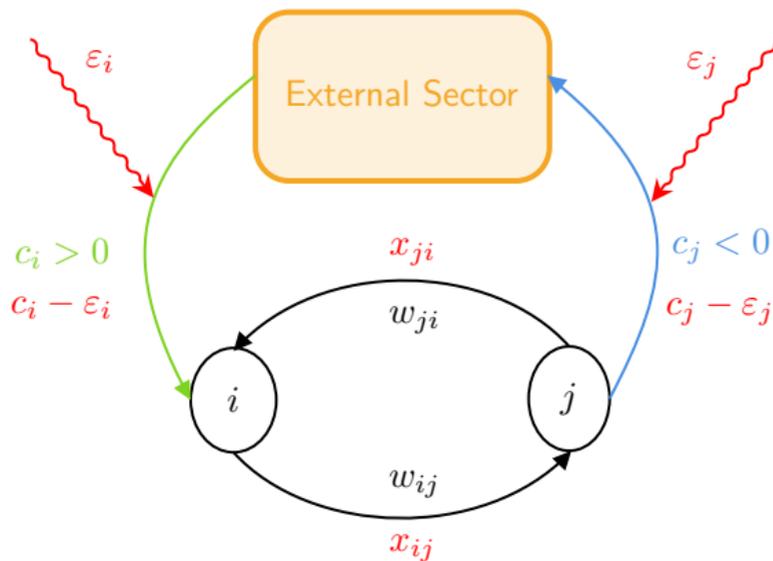
$x$  is a set of consistent payments after the shock:

$$x = S_0^w (P'x + c - \varepsilon)$$

where  $(P)_{ij} = \frac{w_{ij}}{w_i}$  and  $S_0^w$  is a saturation:



- Notice that any solution is such that  $x \in \mathcal{L}_0^w := \{x \in \mathbb{R}^n : 0 \leq x \leq w\}$



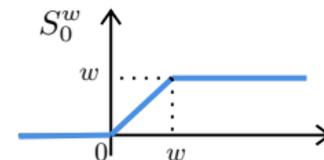
- Shocks  $\varepsilon$  hit the network by reducing  $c$ ;
- Nodes may default and not be able to pay their liabilities (direct effect);
- Shocks propagate across the network because of reduced payments (indirect effect).

### Clearing Vectors

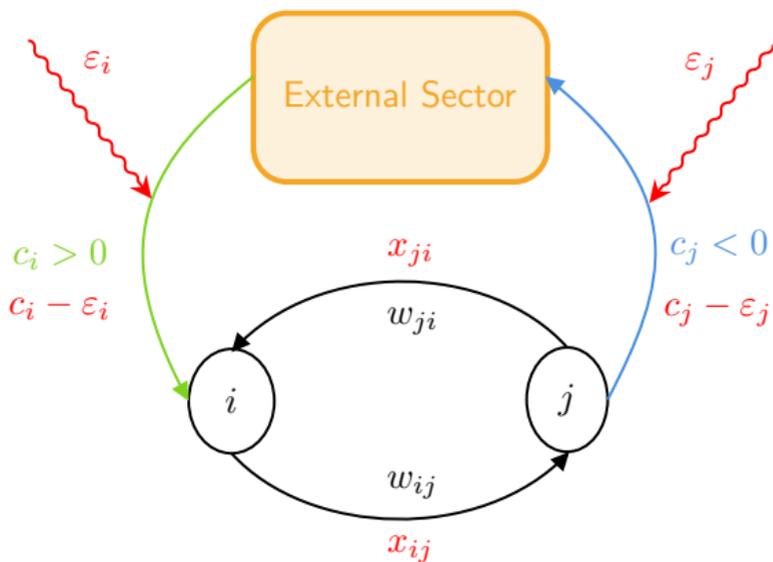
$x$  is a set of consistent payments after the shock:

$$x = S_0^w (P'x + c - \varepsilon)$$

where  $(P)_{ij} = \frac{w_{ij}}{w_i}$  and  $S_0^w$  is a saturation:



- Notice that any solution is such that  $x \in \mathcal{L}_0^w := \{x \in \mathbb{R}^n : 0 \leq x \leq w\}$



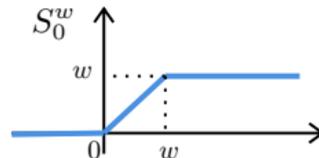
- Shocks  $\varepsilon$  hit the network by reducing  $c$ ;
- Nodes may default and not be able to pay their liabilities (direct effect);
- Shocks propagate across the network because of reduced payments (indirect effect).

### Clearing Vectors

$x$  is a set of consistent payments after the shock:

$$x = S_0^w (P'x + c - \varepsilon)$$

where  $(P)_{ij} = \frac{w_{ij}}{w_i}$  and  $S_0^w$  is a saturation:



- Notice that any solution is such that  $x \in \mathcal{L}_0^w := \{x \in \mathbb{R}^n : 0 \leq x \leq w\}$

## Existence and Uniqueness of Clearing Vectors

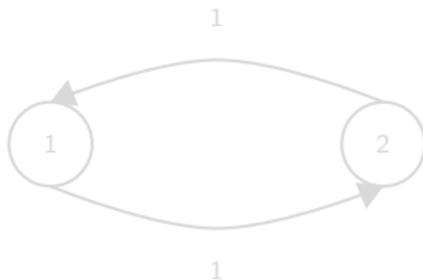


- Existence of clearing vectors follows from Brouwer fixed point Theorem.
- One can prove that it always exist a maximal and a minimal solution  $\bar{x}$  and  $\underline{x}$  respectively.

In general however the solution will not be unique:

## Example

Consider the network consisting of two nodes only depicted below with  $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ,  $w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $c = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ .



It is immediate to check that any vector of the form  $x = \begin{bmatrix} t \\ t \end{bmatrix}$ ,  $t \in [0, 1]$  is a clearing vector.

## Existence and Uniqueness of Clearing Vectors

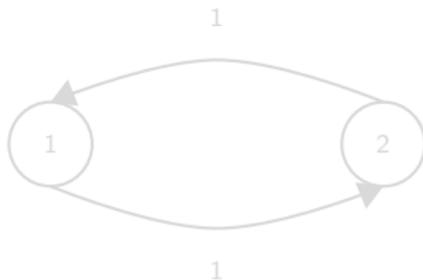


- Existence of clearing vectors follows from Brouwer fixed point Theorem.
- One can prove that it always exist a maximal and a minimal solution  $\bar{x}$  and  $\underline{x}$  respectively.

In general however the solution will not be unique:

## Example

Consider the network consisting of two nodes only depicted below with  $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ,  $w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $c = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ .



It is immediate to check that any vector of the form  $x = \begin{bmatrix} t \\ t \end{bmatrix}$ ,  $t \in [0, 1]$  is a clearing vector.

## Existence and Uniqueness of Clearing Vectors

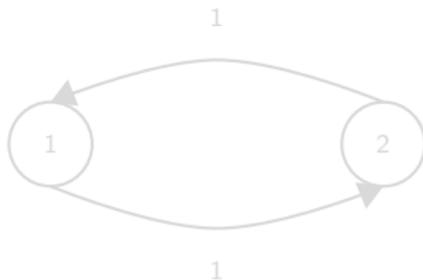


- Existence of clearing vectors follows from Brouwer fixed point Theorem.
- One can prove that it always exist a maximal and a minimal solution  $\bar{x}$  and  $\underline{x}$  respectively.

In general however the solution will not be unique:

## Example

Consider the network consisting of two nodes only depicted below with  $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ,  $w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $c = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ .



It is immediate to check that any vector of the form  $x = \begin{bmatrix} t \\ t \end{bmatrix}$ ,  $t \in [0, 1]$  is a clearing vector.

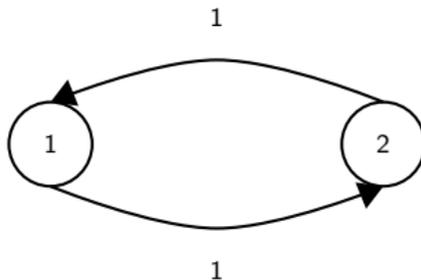
## Existence and Uniqueness of Clearing Vectors

- Existence of clearing vectors follows from Brouwer fixed point Theorem.
- One can prove that it always exist a maximal and a minimal solution  $\bar{x}$  and  $\underline{x}$  respectively.

In general however the solution will not be unique:

### Example

Consider the network consisting of two nodes only depicted below with  $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ,  $w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $c = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ .

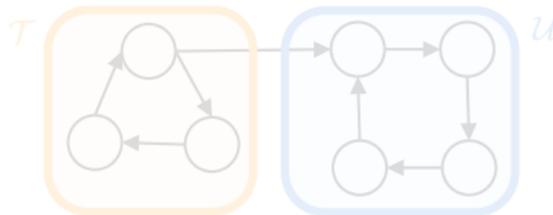


It is immediate to check that any vector of the form  $x = \begin{bmatrix} t \\ t \end{bmatrix}$ ,  $t \in [0, 1]$  is a clearing vector.

### Theorem (Uniqueness for out-connected graphs)

Let  $P$  be an out-connected matrix, then the clearing vector is unique.

- We can partition any graph in a transient part  $\mathcal{T}$  and trapping sets  $\mathcal{U}$ . I.e.  $\mathcal{V} = \mathcal{T} \cup (\cup_k \mathcal{U}_k)$ ;
- $P_{\mathcal{T}}$  is out-connected  $\implies$  the solution  $x_{\mathcal{T}}$  is unique.

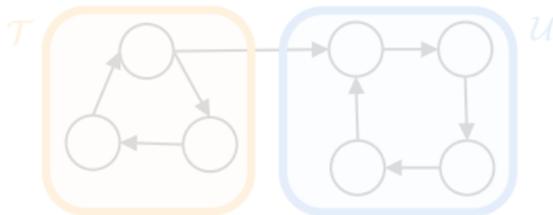


- What about the solution on  $\mathcal{U}$ ? Notice that  $P_{\mathcal{U}}$  is stochastic and irreducible.

### Theorem (Uniqueness for out-connected graphs)

Let  $P$  be an out-connected matrix, then the clearing vector is unique.

- We can partition any graph in a transient part  $\mathcal{T}$  and trapping sets  $\mathcal{U}$ . I.e.  $\mathcal{V} = \mathcal{T} \cup (\cup_k \mathcal{U}_k)$ ;
- $P_{\mathcal{T}}$  is out-connected  $\implies$  the solution  $x_{\mathcal{T}}$  is unique.

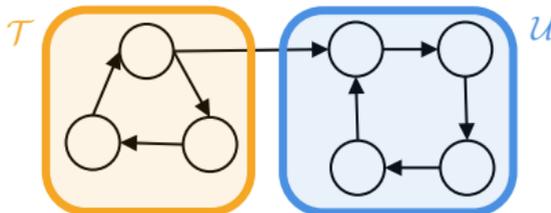


- What about the solution on  $\mathcal{U}$ ? Notice that  $P_{\mathcal{U}}$  is stochastic and irreducible.

### Theorem (Uniqueness for out-connected graphs)

Let  $P$  be an out-connected matrix, then the clearing vector is unique.

- We can partition any graph in a transient part  $\mathcal{T}$  and trapping sets  $\mathcal{U}$ . I.e.  $\mathcal{V} = \mathcal{T} \cup (\cup_k \mathcal{U}_k)$ ;
- $P_{\mathcal{T}}$  is out-connected  $\implies$  the solution  $x_{\mathcal{T}}$  is unique.

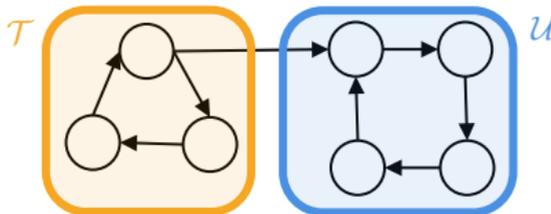


- What about the solution on  $\mathcal{U}$ ? Notice that  $P_{\mathcal{U}}$  is stochastic and irreducible.

### Theorem (Uniqueness for out-connected graphs)

Let  $P$  be an out-connected matrix, then the clearing vector is unique.

- We can partition any graph in a transient part  $\mathcal{T}$  and trapping sets  $\mathcal{U}$ . I.e.  $\mathcal{V} = \mathcal{T} \cup (\cup_k \mathcal{U}_k)$ ;
- $P_{\mathcal{T}}$  is out-connected  $\implies$  the solution  $x_{\mathcal{T}}$  is unique.

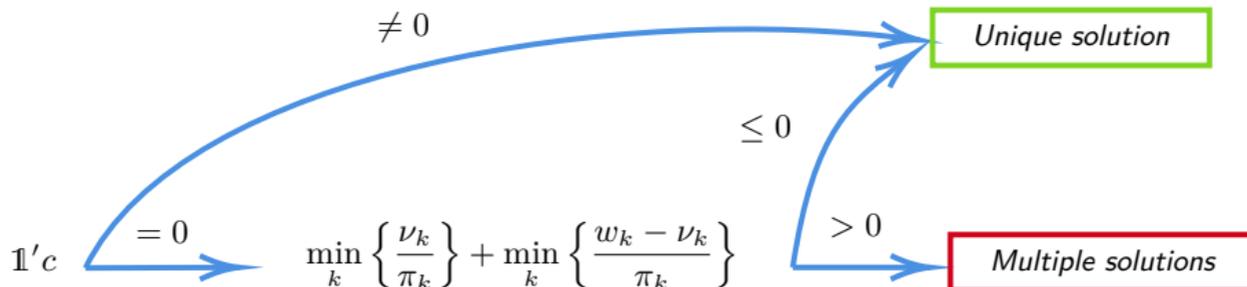


- What about the solution on  $\mathcal{U}$ ? Notice that  $P_{\mathcal{U}}$  is stochastic and irreducible.

**Theorem (Uniqueness for the stochastic irreducible case)**

Let  $P$  be an irreducible stochastic matrix; let  $\pi$  be its unique invariant probability measure and

$$\nu = \frac{1}{2} \sum_{k \geq 0} \left( \frac{I + P^k}{2} \right) c. \text{ Then it holds:}$$



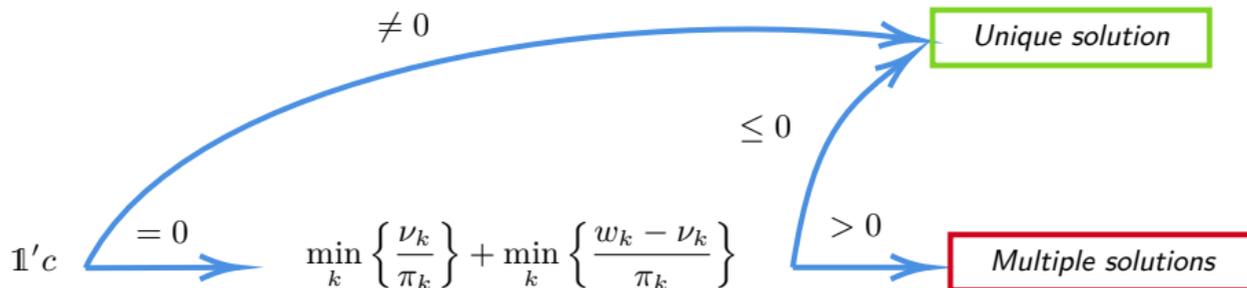
- In case we have multiple solutions, we have that:

$$\mathcal{X} = \left\{ x = \nu + \alpha \pi : - \min_k \left\{ \frac{\nu_k}{\pi_k} \right\} \leq \alpha \leq \min_k \left\{ \frac{w_k - \nu_k}{\pi_k} \right\} \right\}$$

**Theorem (Uniqueness for the stochastic irreducible case)**

Let  $P$  be an irreducible stochastic matrix; let  $\pi$  be its unique invariant probability measure and

$$\nu = \frac{1}{2} \sum_{k \geq 0} \left( \frac{I + P^k}{2} \right) c. \text{ Then it holds:}$$

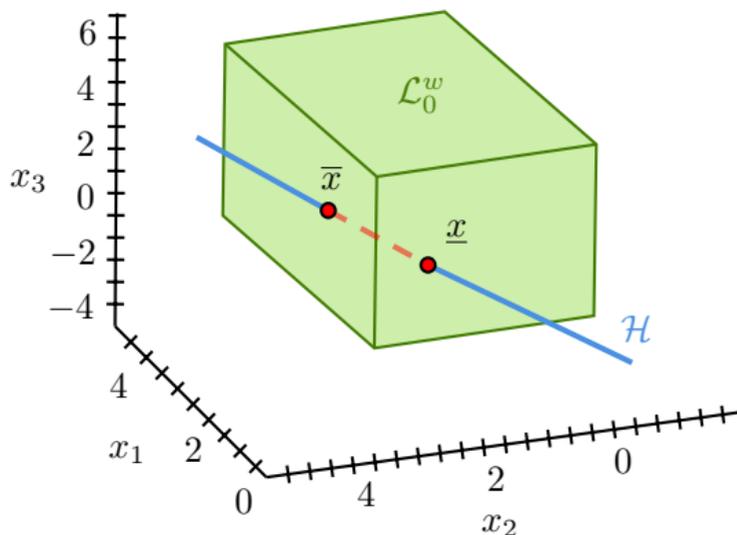


- In case we have multiple solutions, we have that:

$$\mathcal{X} = \left\{ x = \nu + \alpha \pi : - \min_k \left\{ \frac{\nu_k}{\pi_k} \right\} \leq \alpha \leq \min_k \left\{ \frac{w_k - \nu_k}{\pi_k} \right\} \right\}$$

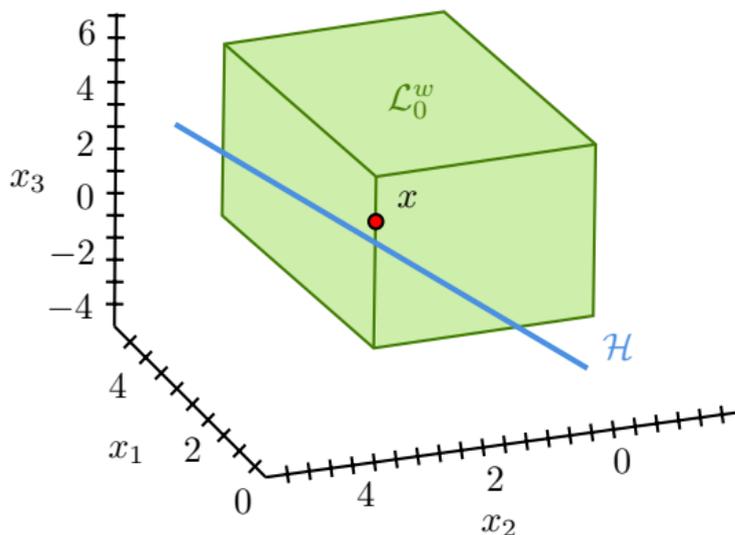
## A geometrical Interpretation

When  $\mathbf{1}'c = 0$ , we have multiple solutions when the line  $\mathcal{H} = \{x \in \mathbb{R}^n : x = \nu + \alpha\pi\}$  intersects non trivially the lattice  $\mathcal{L}_0^w$ .



(a) Multiple solutions (the red dots and segment).

$$\min_k \left\{ \frac{\nu_k}{\pi_k} \right\} + \min_k \left\{ \frac{w_k - \nu_k}{\pi_k} \right\} > 0$$

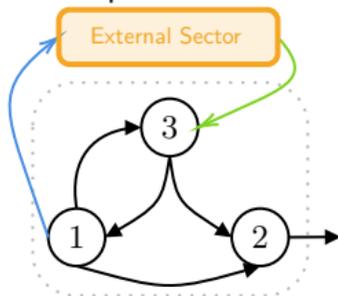


(b) Unique solution (the red dot).

$$\min_k \left\{ \frac{\nu_k}{\pi_k} \right\} + \min_k \left\{ \frac{w_k - \nu_k}{\pi_k} \right\} \leq 0$$

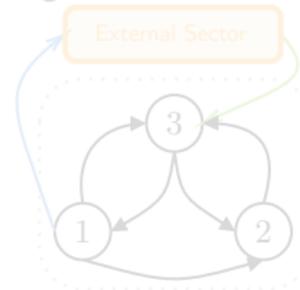
### The Out-Connected Case

Unique solution.

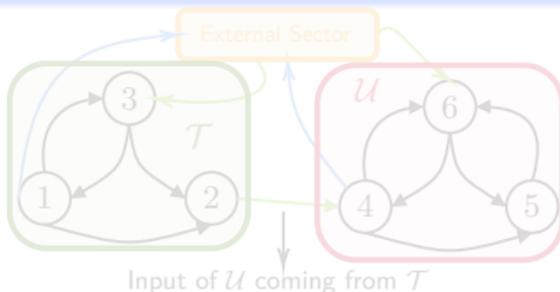


### The Stochastic-Irreducible Case

Uniqueness depends on  $c$ , i.e. on what is coming from and going to the external environment.



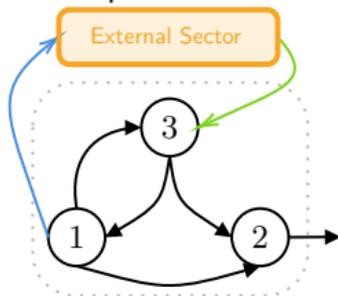
### The General Case



- $x_{\mathcal{T}}$  is unique;
- For every trapping set  $\mathcal{U}$ , we use the Theorem;
- To do so, we also need to consider the input coming from  $\mathcal{T}$ :  $h_{\mathcal{U}} := c_{\mathcal{U}} + P_{\mathcal{U}\mathcal{T}}x_{\mathcal{T}}$

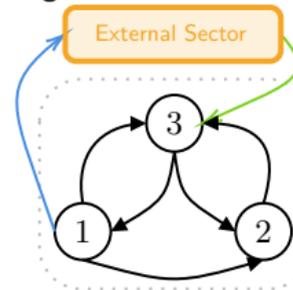
### The Out-Connected Case

Unique solution.

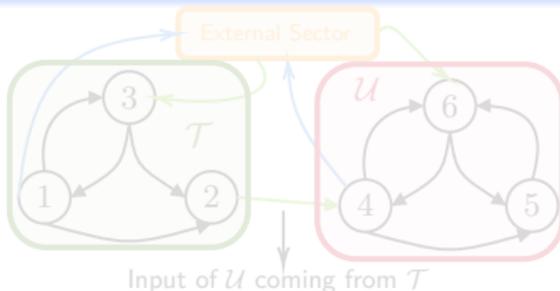


### The Stochastic-Irreducible Case

Uniqueness depends on  $c$ , i.e. on what is coming from and going to the external environment.



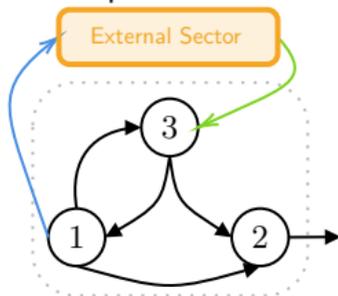
### The General Case



- $x_{\mathcal{T}}$  is unique;
- For every trapping set  $\mathcal{U}$ , we use the Theorem;
- To do so, we also need to consider the input coming from  $\mathcal{T}$ :  $h_{\mathcal{U}} := c_{\mathcal{U}} + P_{\mathcal{U}\mathcal{T}}x_{\mathcal{T}}$

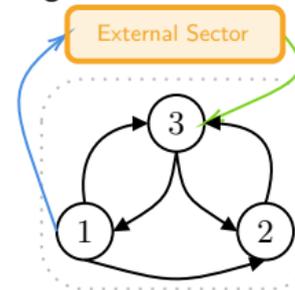
### The Out-Connected Case

Unique solution.

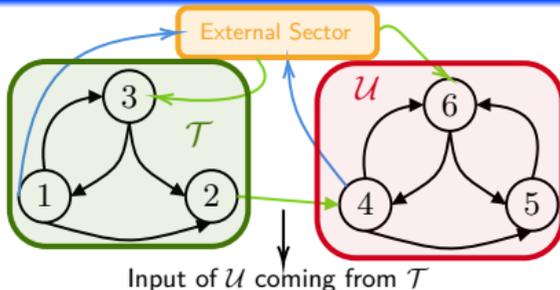


### The Stochastic-Irreducible Case

Uniqueness depends on  $c$ , i.e. on what is coming from and going to the external environment.



### The General Case



- $x_{\mathcal{T}}$  is unique;
- For every trapping set  $\mathcal{U}$ , we use the Theorem;
- To do so, we also need to consider the input coming from  $\mathcal{T}$ :  $h_{\mathcal{U}} := c_{\mathcal{U}} + P_{\mathcal{U}\mathcal{T}}x_{\mathcal{T}}$

Dependence of  $x$  on  $c$ 

- The uniqueness ultimately depends on the input \ output vector  $c$ .
- There exists a set of critical vectors  $c^*$  such that we have multiple solutions, namely:

$$\mathcal{M} = \left\{ c \in \mathbb{R}^n : \mathbf{1}'c = 0, \min_k \left\{ \frac{\nu_k}{\pi_k} \right\} + \min_k \left\{ \frac{w_k - \nu_k}{\pi_k} \right\} > 0 \right\}$$

What happens to the solutions when  $c$  approaches a critical  $c^* \in \mathcal{M}$  ?

Let  $\mathcal{A} = \mathbb{R}^n \setminus \mathcal{M}$  be the set where the solution is unique. Then:

- The map  $c \mapsto x(c)$  is continuous on  $\mathcal{A}$ .
- One can prove that for every  $c^* \in \mathcal{M}$ ,

$$\liminf_{\substack{c \in \mathcal{A} \\ c \rightarrow c^*}} x(c) = \underline{x}(c^*), \quad \limsup_{\substack{c \in \mathcal{A} \\ c \rightarrow c^*}} x(c) = \bar{x}(c^*).$$

- This means that the clearing vector undergoes a jump discontinuity at  $c^*$ .

Dependence of  $x$  on  $c$ 

- The uniqueness ultimately depends on the input \ output vector  $c$ .
- There exists a set of critical vectors  $c^*$  such that we have multiple solutions, namely:

$$\mathcal{M} = \left\{ c \in \mathbb{R}^n : \mathbf{1}'c = 0, \min_k \left\{ \frac{\nu_k}{\pi_k} \right\} + \min_k \left\{ \frac{w_k - \nu_k}{\pi_k} \right\} > 0 \right\}$$

What happens to the solutions when  $c$  approaches a critical  $c^* \in \mathcal{M}$  ?

Let  $\mathcal{A} = \mathbb{R}^n \setminus \mathcal{M}$  be the set where the solution is unique. Then:

- The map  $c \mapsto x(c)$  is continuous on  $\mathcal{A}$ .
- One can prove that for every  $c^* \in \mathcal{M}$ ,

$$\liminf_{\substack{c \in \mathcal{A} \\ c \rightarrow c^*}} x(c) = \underline{x}(c^*), \quad \limsup_{\substack{c \in \mathcal{A} \\ c \rightarrow c^*}} x(c) = \bar{x}(c^*).$$

- This means that the clearing vector undergoes a jump discontinuity at  $c^*$ .

### Jump discontinuity as a financial breakdown

A jump discontinuity means that even a slight change in the asset/shock value  $c$  may lead to a catastrophic aggregated loss and to sudden defaults of several nodes.

#### Loss function

- Consider shock  $\varepsilon$  that lowers the value of the external asset from  $c$  to  $c - \varepsilon$ ;
- Loss function is:  $l = \mathbb{1}'(\varepsilon + w - x)$

#### Jump size of the loss function at $c^* \in \mathcal{M}$

$$\Delta l(c^*) = \liminf_{\substack{c \in \mathcal{A} \\ c \rightarrow c^*}} l(c) - \limsup_{\substack{c \in \mathcal{A} \\ c \rightarrow c^*}} l(c) = \mathbb{1}'(\bar{x}(c^*) - \underline{x}(c^*)) = \min_k \left\{ \frac{\nu_k}{\pi_k} \right\} + \min_k \left\{ \frac{w_k - \nu_k}{\pi_k} \right\}$$

#### Maximal norm of a jump discontinuity

$$\max_{c \in \mathbb{R}^n} \|\bar{x}(c) - \underline{x}(c)\|_p^p = \left( \min_k \frac{w_k}{\pi_k} \right)^p \|\pi\|_p^p$$

### Jump discontinuity as a financial breakdown

A jump discontinuity means that even a slight change in the asset/shock value  $c$  may lead to a catastrophic aggregated loss and to sudden defaults of several nodes.

#### Loss function

- Consider shock  $\varepsilon$  that lowers the value of the external asset from  $c$  to  $c - \varepsilon$ ;
- Loss function is:  $l = \mathbb{1}'(\varepsilon + w - x)$

#### Jump size of the loss function at $c^* \in \mathcal{M}$

$$\Delta l(c^*) = \liminf_{\substack{c \in \mathcal{A} \\ c \rightarrow c^*}} l(c) - \limsup_{\substack{c \in \mathcal{A} \\ c \rightarrow c^*}} l(c) = \mathbb{1}'(\bar{x}(c^*) - \underline{x}(c^*)) = \min_k \left\{ \frac{\nu_k}{\pi_k} \right\} + \min_k \left\{ \frac{w_k - \nu_k}{\pi_k} \right\}$$

#### Maximal norm of a jump discontinuity

$$\max_{c \in \mathbb{R}^n} \|\bar{x}(c) - \underline{x}(c)\|_p^p = \left( \min_k \frac{w_k}{\pi_k} \right)^p \|\pi\|_p^p$$

### Jump discontinuity as a financial breakdown

A jump discontinuity means that even a slight change in the asset/shock value  $c$  may lead to a catastrophic aggregated loss and to sudden defaults of several nodes.

#### Loss function

- Consider shock  $\varepsilon$  that lowers the value of the external asset from  $c$  to  $c - \varepsilon$ ;
- Loss function is:  $l = \mathbb{1}'(\varepsilon + w - x)$

#### Jump size of the loss function at $c^* \in \mathcal{M}$

$$\Delta l(c^*) = \liminf_{\substack{c \in \mathcal{A} \\ c \rightarrow c^*}} l(c) - \limsup_{\substack{c \in \mathcal{A} \\ c \rightarrow c^*}} l(c) = \mathbb{1}'(\bar{x}(c^*) - \underline{x}(c^*)) = \min_k \left\{ \frac{\nu_k}{\pi_k} \right\} + \min_k \left\{ \frac{w_k - \nu_k}{\pi_k} \right\}$$

#### Maximal norm of a jump discontinuity

$$\max_{c \in \mathbb{R}^n} \|\bar{x}(c) - \underline{x}(c)\|_p^p = \left( \min_k \frac{w_k}{\pi_k} \right)^p \|\pi\|_p^p$$

### Jump discontinuity as a financial breakdown

A jump discontinuity means that even a slight change in the asset/shock value  $c$  may lead to a catastrophic aggregated loss and to sudden defaults of several nodes.

#### Loss function

- Consider shock  $\varepsilon$  that lowers the value of the external asset from  $c$  to  $c - \varepsilon$ ;
- Loss function is:  $l = \mathbb{1}'(\varepsilon + w - x)$

#### Jump size of the loss function at $c^* \in \mathcal{M}$

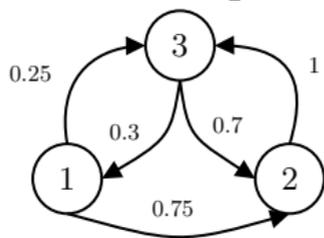
$$\Delta l(c^*) = \liminf_{\substack{c \in \mathcal{A} \\ c \rightarrow c^*}} l(c) - \limsup_{\substack{c \in \mathcal{A} \\ c \rightarrow c^*}} l(c) = \mathbb{1}'(\bar{x}(c^*) - \underline{x}(c^*)) = \min_k \left\{ \frac{\nu_k}{\pi_k} \right\} + \min_k \left\{ \frac{w_k - \nu_k}{\pi_k} \right\}$$

#### Maximal norm of a jump discontinuity

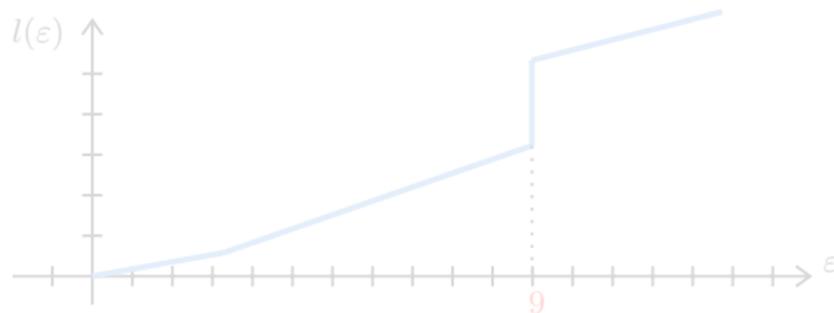
$$\max_{c \in \mathbb{R}^n} \|\bar{x}(c) - \underline{x}(c)\|_p^p = \left( \min_k \frac{w_k}{\pi_k} \right)^p \|\pi\|_p^p$$

**Example**

Consider the network below with  $P = \begin{bmatrix} 0 & 0.75 & 0.25 \\ 0 & 0 & 1 \\ 0.3 & 0.7 & 0 \end{bmatrix}$  and  $w = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$ .

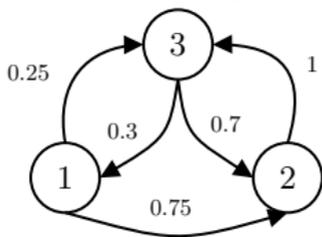


Consider an initial asset  $c = [5, 2, 2]'$  and a total shock magnitude  $\varepsilon \in [0, 12]$  that hits all nodes uniformly, i.e.  $c(\varepsilon) = c - \varepsilon[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]'$ . We expect a jump discontinuity when  $\mathbf{1}'c(\varepsilon) = 0 \implies \varepsilon = 9$ .

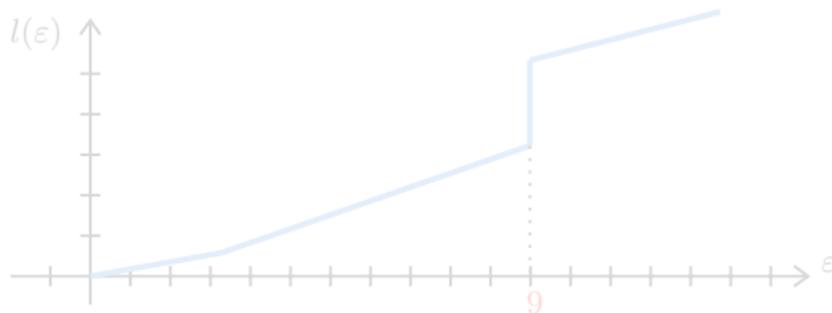


**Example**

Consider the network below with  $P = \begin{bmatrix} 0 & 0.75 & 0.25 \\ 0 & 0 & 1 \\ 0.3 & 0.7 & 0 \end{bmatrix}$  and  $w = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$ .

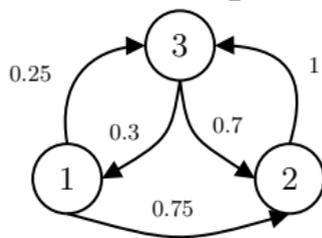


Consider an initial asset  $c = [5, 2, 2]'$  and a total shock magnitude  $\varepsilon \in [0, 12]$  that hits all nodes uniformly, i.e.  $c(\varepsilon) = c - \varepsilon[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]'$ . We expect a jump discontinuity when  $\mathbf{1}'c(\varepsilon) = 0 \implies \varepsilon = 9$ .

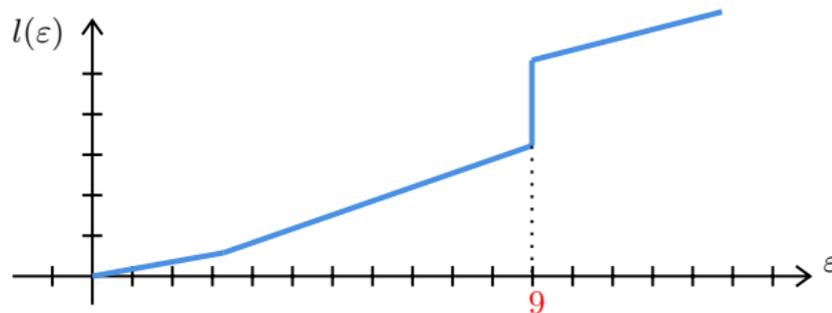


**Example**

Consider the network below with  $P = \begin{bmatrix} 0 & 0.75 & 0.25 \\ 0 & 0 & 1 \\ 0.3 & 0.7 & 0 \end{bmatrix}$  and  $w = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$ .



Consider an initial asset  $c = [5, 2, 2]'$  and a total shock magnitude  $\varepsilon \in [0, 12]$  that hits all nodes uniformly, i.e.  $c(\varepsilon) = c - \varepsilon[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]'$ . We expect a jump discontinuity when  $\mathbf{1}'c(\varepsilon) = 0 \implies \varepsilon = 9$ .



## Main Results

- Sufficient and necessary condition for Uniqueness of clearing vectors;
- Systemic risk measures and existence of critical shocks;
- Structure of solutions with respect to the topological properties of the network.

## Ongoing Research

- Optimal policies for risk reduction;
- Analytical results on particular topologies and random graphs;
- Continuous Model.
- Model extensions (fire sales, bankruptcy costs, cross holdings, etc...);

## Main Results

- Sufficient and necessary condition for Uniqueness of clearing vectors;
- Systemic risk measures and existence of critical shocks;
- Structure of solutions with respect to the topological properties of the network.

## Ongoing Research

- Optimal policies for risk reduction;
- Analytical results on particular topologies and random graphs;
- Continuous Model.
- Model extensions (fire sales, bankruptcy costs, cross holdings, etc...);

## Main Results

- Sufficient and necessary condition for Uniqueness of clearing vectors;
- Systemic risk measures and existence of critical shocks;
- Structure of solutions with respect to the topological properties of the network.

## Ongoing Research

- Optimal policies for risk reduction;
- Analytical results on particular topologies and random graphs;
- Continuous Model.
- Model extensions (fire sales, bankruptcy costs, cross holdings, etc...);



## Main Results

- Sufficient and necessary condition for Uniqueness of clearing vectors;
- Systemic risk measures and existence of critical shocks;
- Structure of solutions with respect to the topological properties of the network.

## Ongoing Research

- Optimal policies for risk reduction;
- Analytical results on particular topologies and random graphs;
- Continuous Model.
- Model extensions (fire sales, bankruptcy costs, cross holdings, etc...);



## Main Results

- Sufficient and necessary condition for Uniqueness of clearing vectors;
- Systemic risk measures and existence of critical shocks;
- Structure of solutions with respect to the topological properties of the network.

## Ongoing Research

- Optimal policies for risk reduction;
- Analytical results on particular topologies and random graphs;
- Continuous Model.
- Model extensions (fire sales, bankruptcy costs, cross holdings, etc...);



## Main Results

- Sufficient and necessary condition for Uniqueness of clearing vectors;
- Systemic risk measures and existence of critical shocks;
- Structure of solutions with respect to the topological properties of the network.

## Ongoing Research

- Optimal policies for risk reduction;
- Analytical results on particular topologies and random graphs;
- Continuous Model.
- Model extensions (fire sales, bankruptcy costs, cross holdings, etc...);



## Main Results

- Sufficient and necessary condition for Uniqueness of clearing vectors;
- Systemic risk measures and existence of critical shocks;
- Structure of solutions with respect to the topological properties of the network.

## Ongoing Research

- Optimal policies for risk reduction;
- Analytical results on particular topologies and random graphs;
- Continuous Model.
- Model extensions (fire sales, bankruptcy costs, cross holdings, etc...);



*Thank you!*