GAME THEORETICAL INFERENCE OF HUMAN BEHAVIOUR IN SOCIAL NETWORKS

[N. Pagan & F. Dörfler, *"Game theoretical inference of human behaviour in social networks"*. Nature Communications (forthcoming).]

Workshop on "Network Dynamics in the Social, Economic, and Financial Sciences"

Torino, 07.11.2019

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ETH zürich

OBSERVATIONS

Actors decide with whom they want to interact.

01

OBSERVATIONS

Actors decide with whom they want to interact.

Network positions provide benefits to the actors.

Forbes

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Using Social Networks To Advance Your Career

Adi Gaskell Contributor

Shutterstock

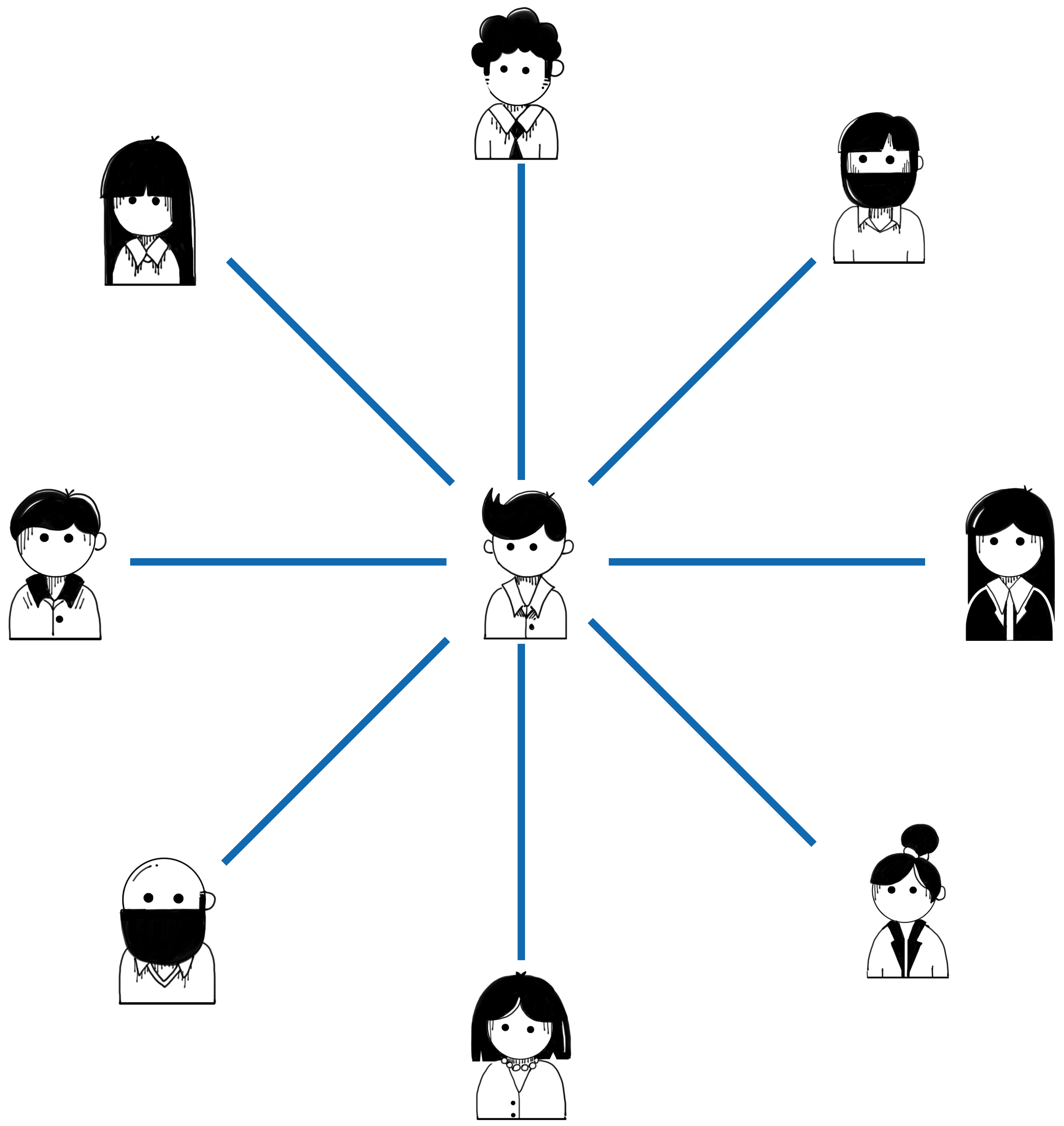
"It's not what you know, it's who you know" is one of those phrases

01

SOCIAL NETWORK POSITIONS' BENEFITS

Social Influence

The more people we are connected to, the more we can influence them.



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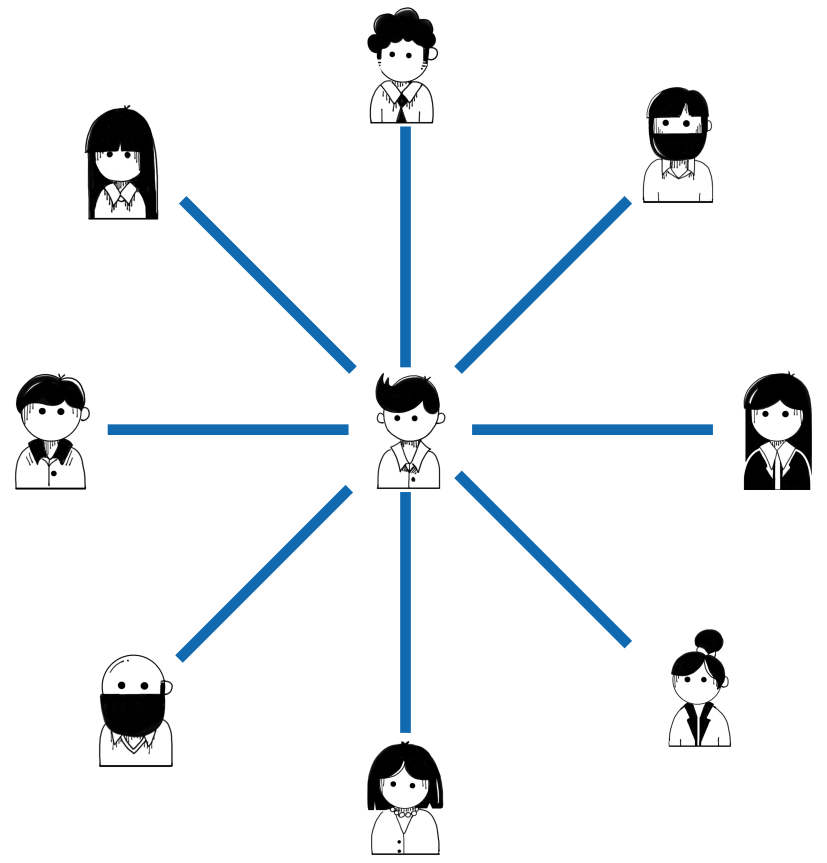

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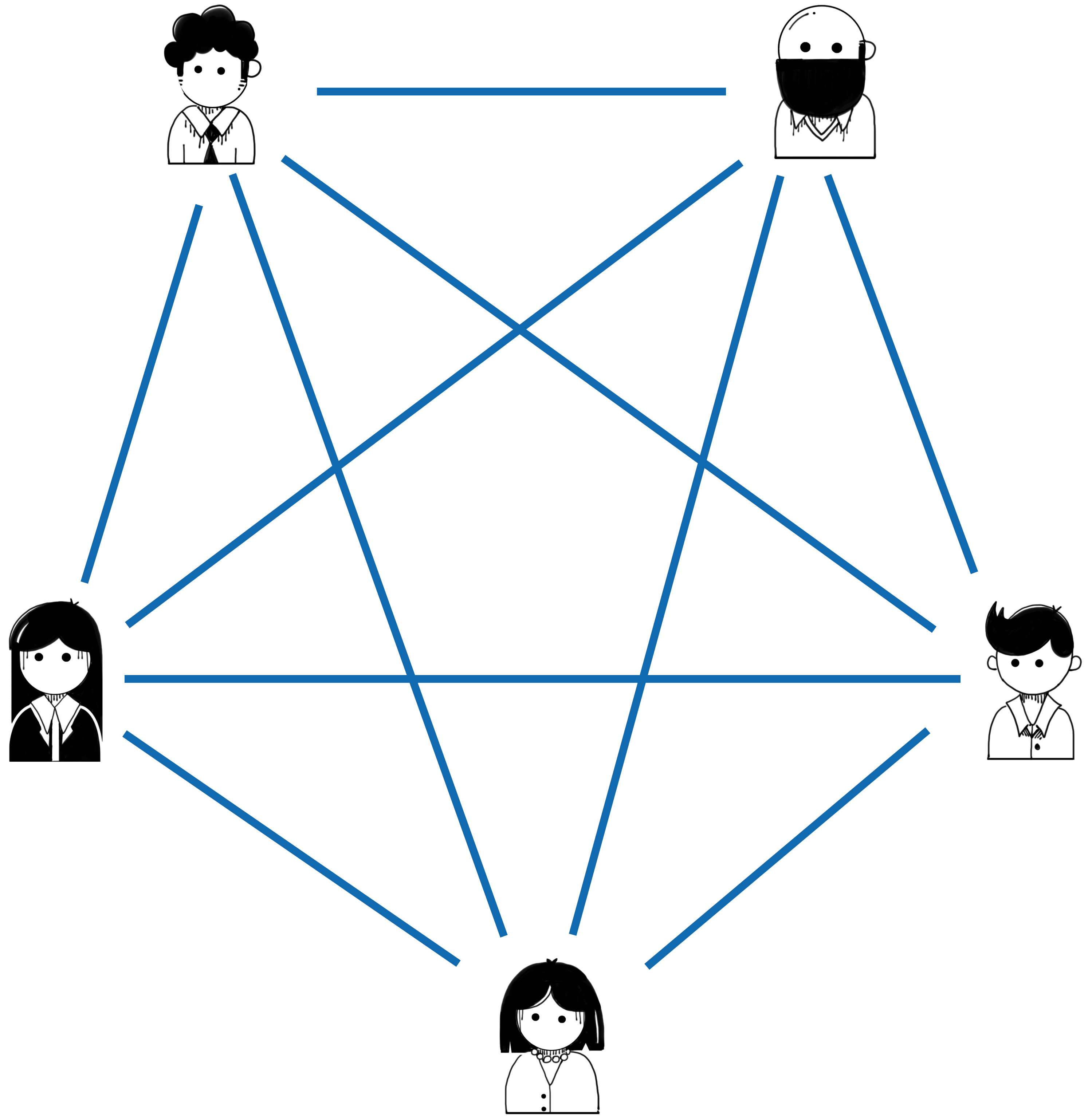
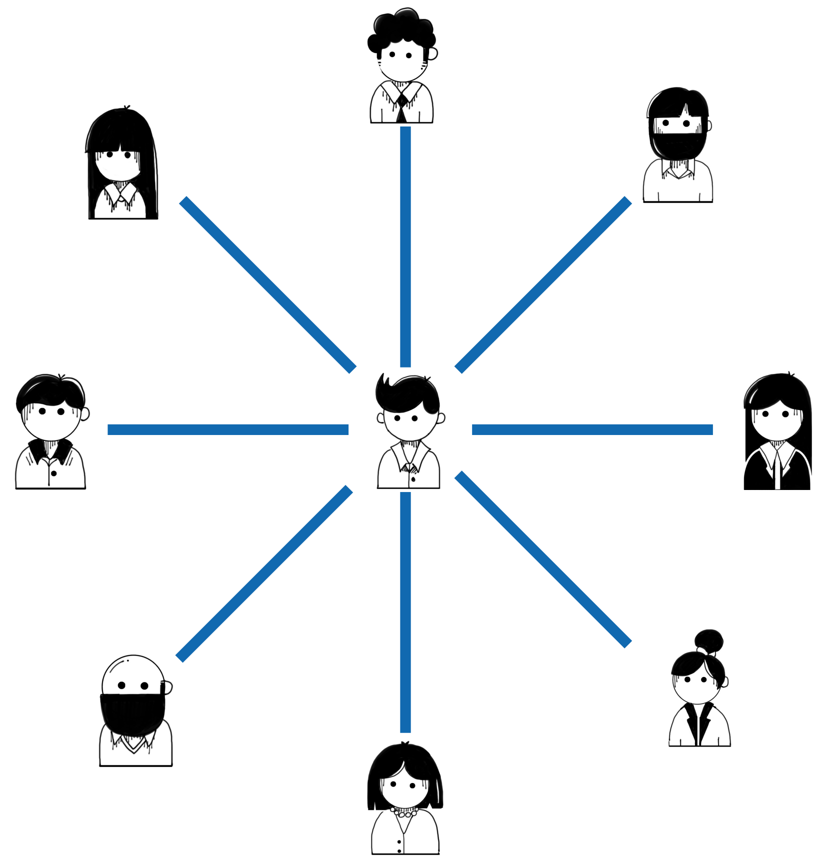


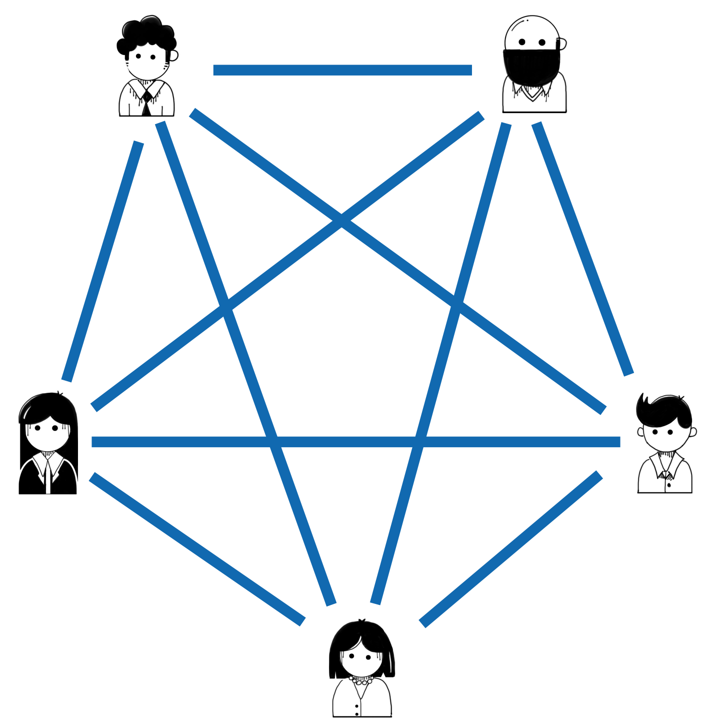
Social Support

The more our friends' friends are our friends, the safer we feel.

02

SOCIAL NETWORK POSITIONS' BENEFITS





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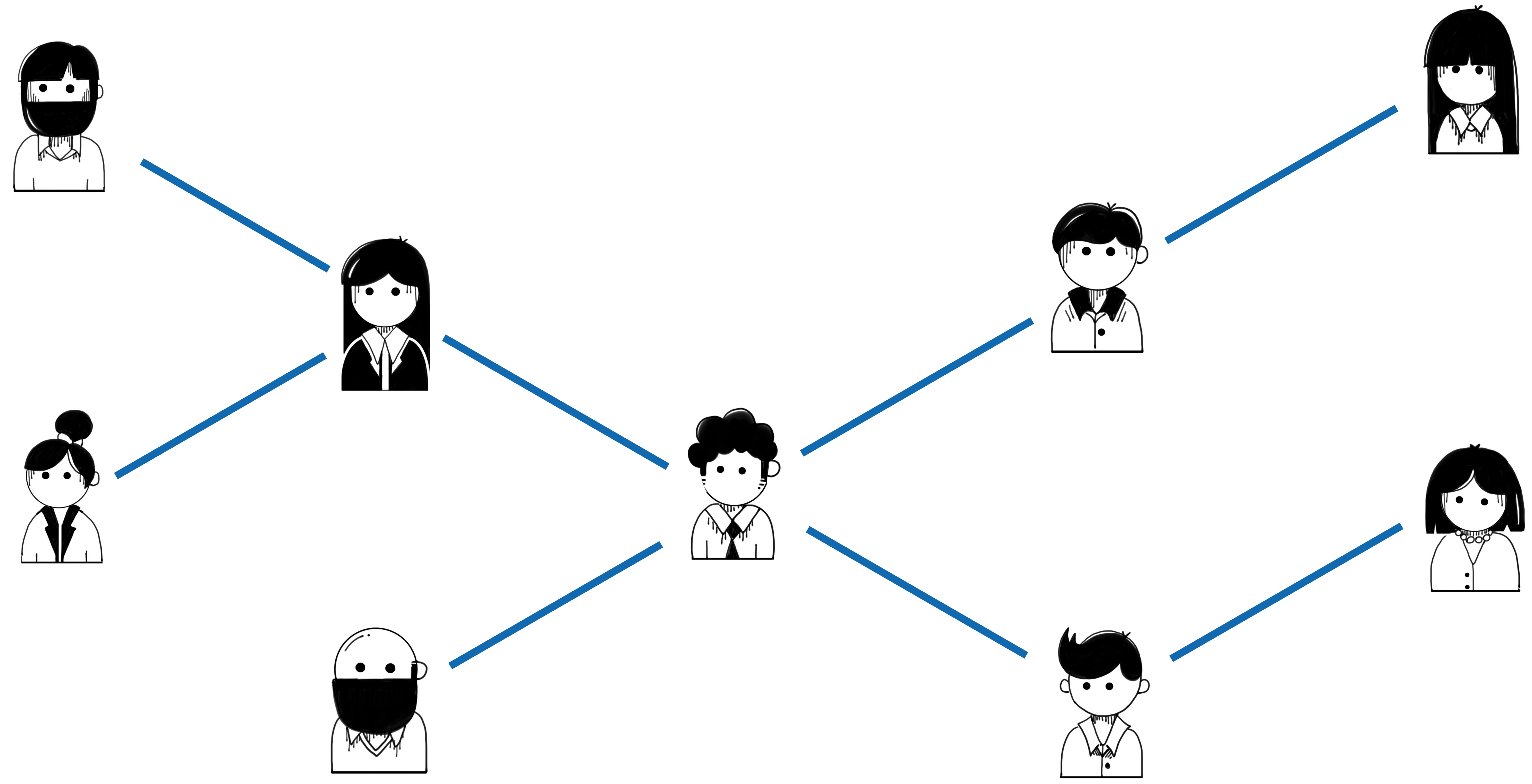
Brokerage

The more we are on the path between people, the more we can control.



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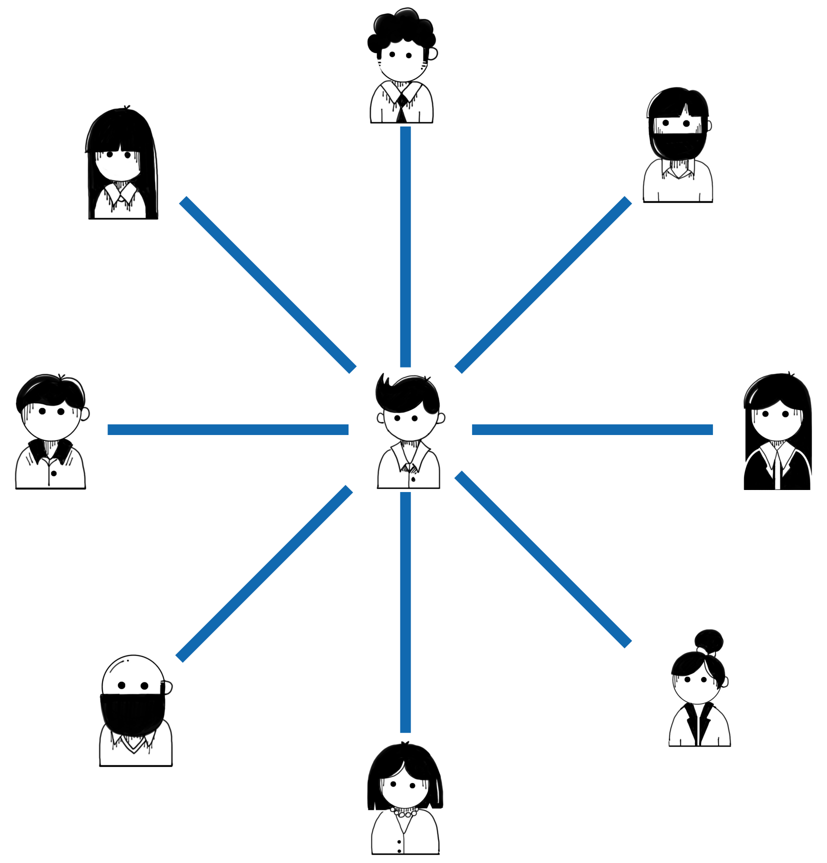
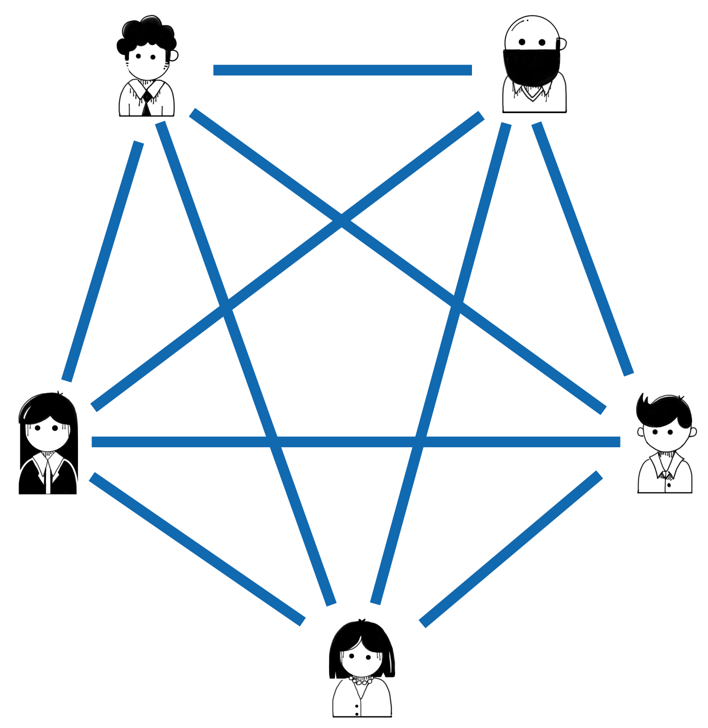
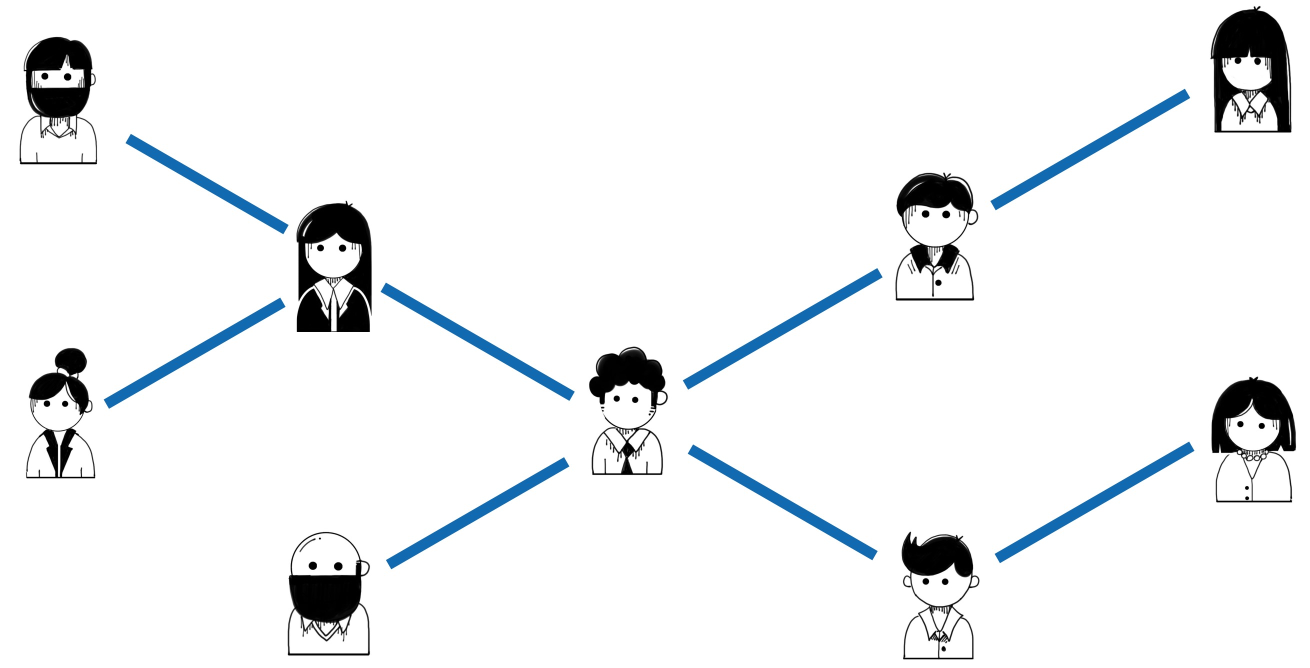


Degree Centrality



Clustering coefficient



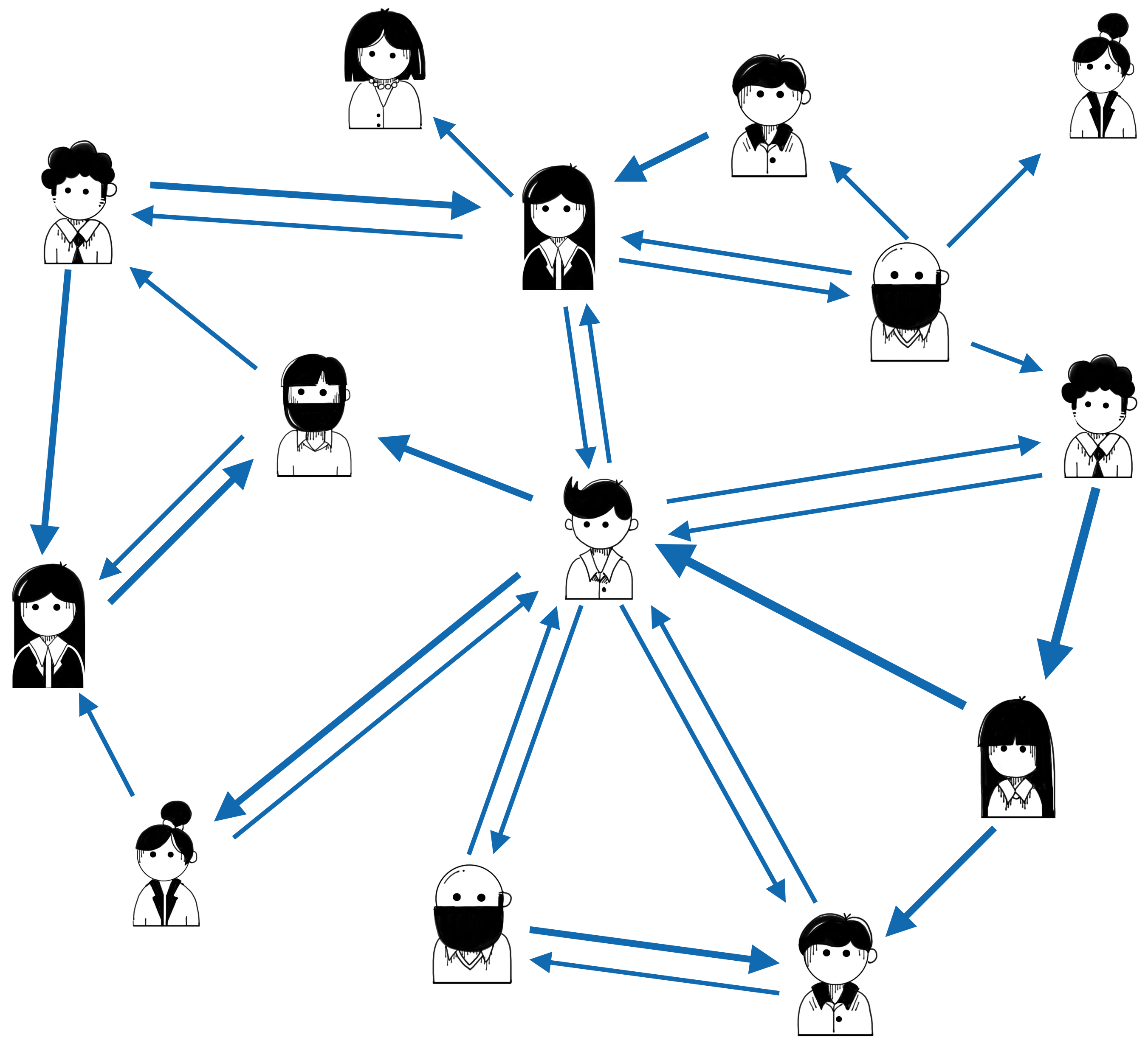
Betwenness Centrality

02

SOCIAL NETWORK FORMATION MODEL

Directed weighted network $\mathscr{G}$ with $\mathscr{N} = \{1,\ldots,N\}$ agents.

$a_{ij} \in [0,1]$ quantifies the importance of the friendship among i and j from i's point of view.



03

SOCIAL NETWORK FORMATION MODEL

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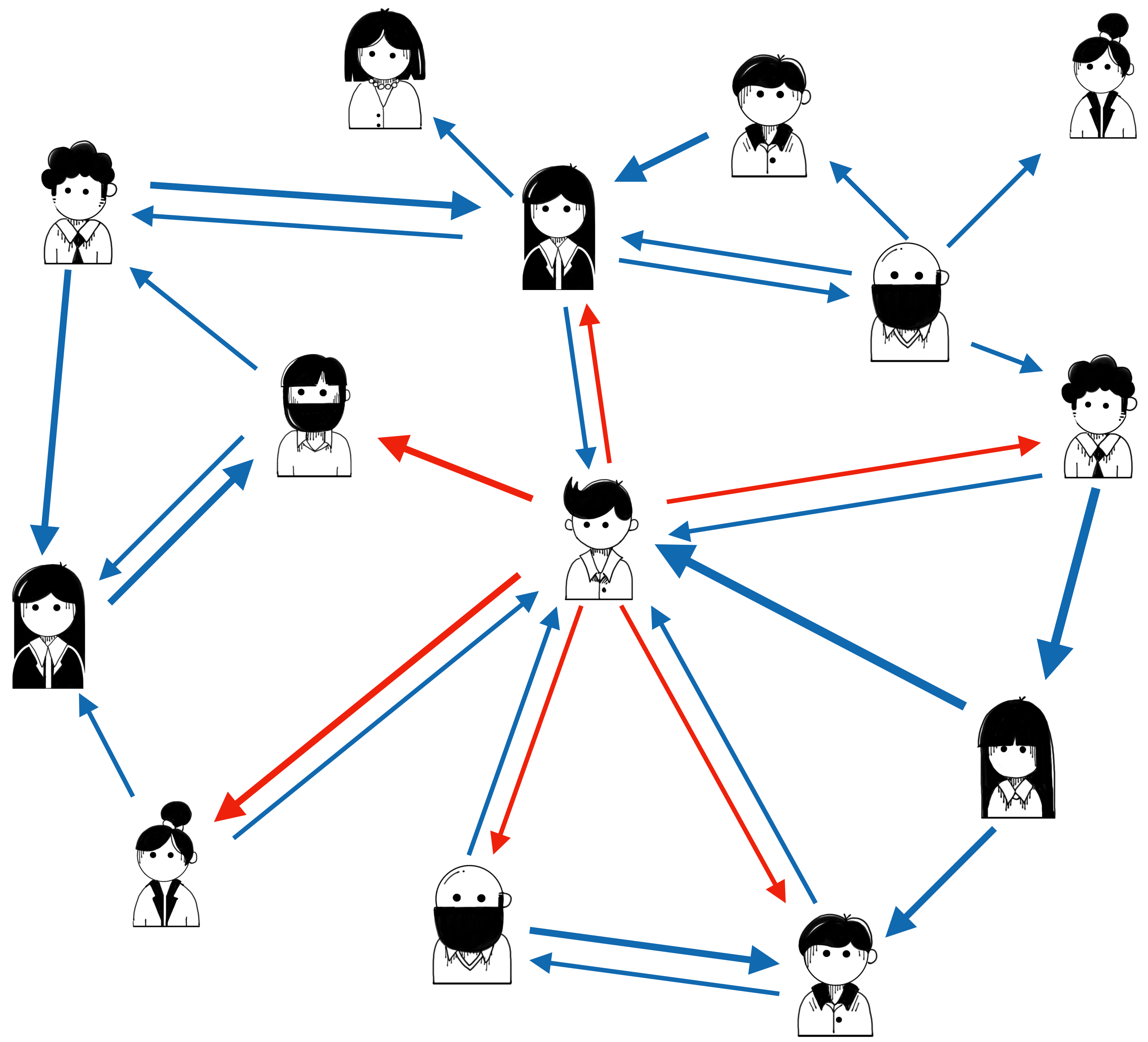
A typical action of agent i is:

$$a_i = \left[a_{i1}, \ldots, a_{i,i-1}, a_{i,i+1}, \ldots, a_{iN}\right] \in \mathcal{A} = [0,1]^{N-1},$$



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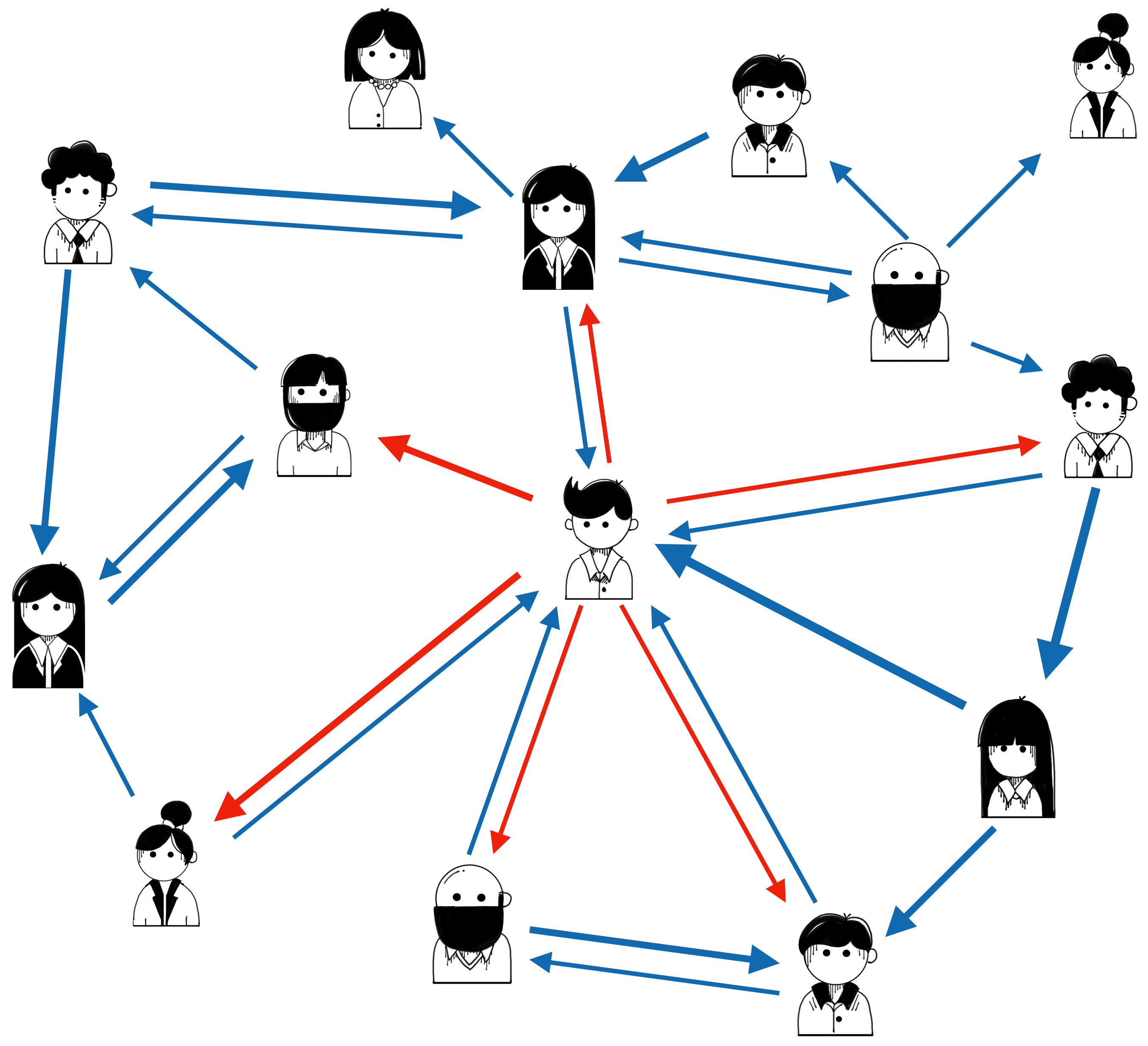
Every agent i is endowed with a payoff function V_i and is looking for

$$a_i^\star \in \arg\max_{a_i \in \mathscr{A}} V_i(a_i, \mathbf{a_{-i}})$$



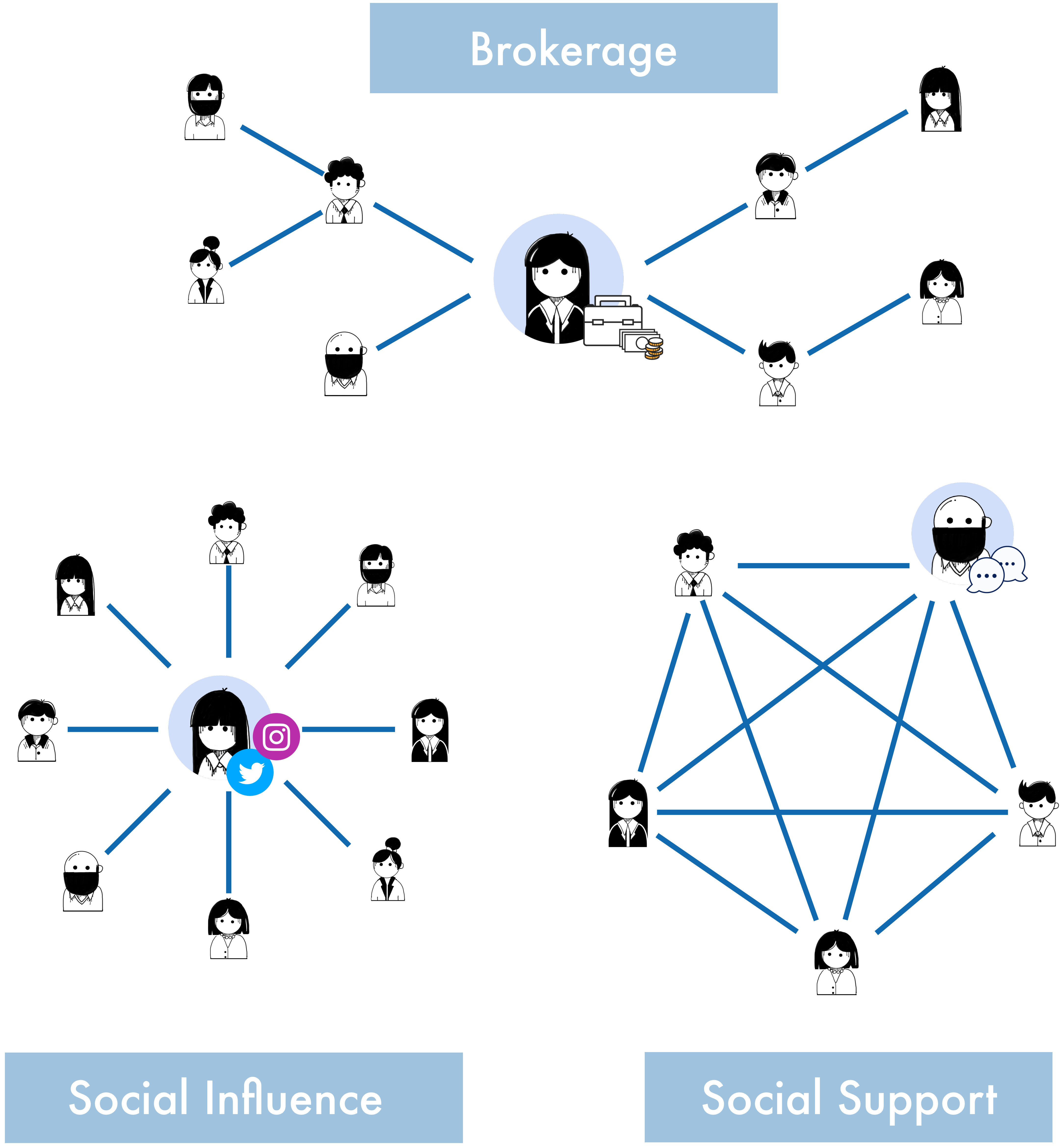
03

SOCIAL NETWORK FORMATION MODEL

Parametric payoff function:

$$V_i\left(a_i, \mathbf{a}_{-i}, \theta_i\right) = \theta_i^T \,\text{benefit}(a_i, \mathbf{a}_{-i}) - \text{cost}(a_i),$$

where $\theta_i \in \Theta$, are the **individual** parameters relative to the different contributions, e.g., social influence, social support, brokerage.



03

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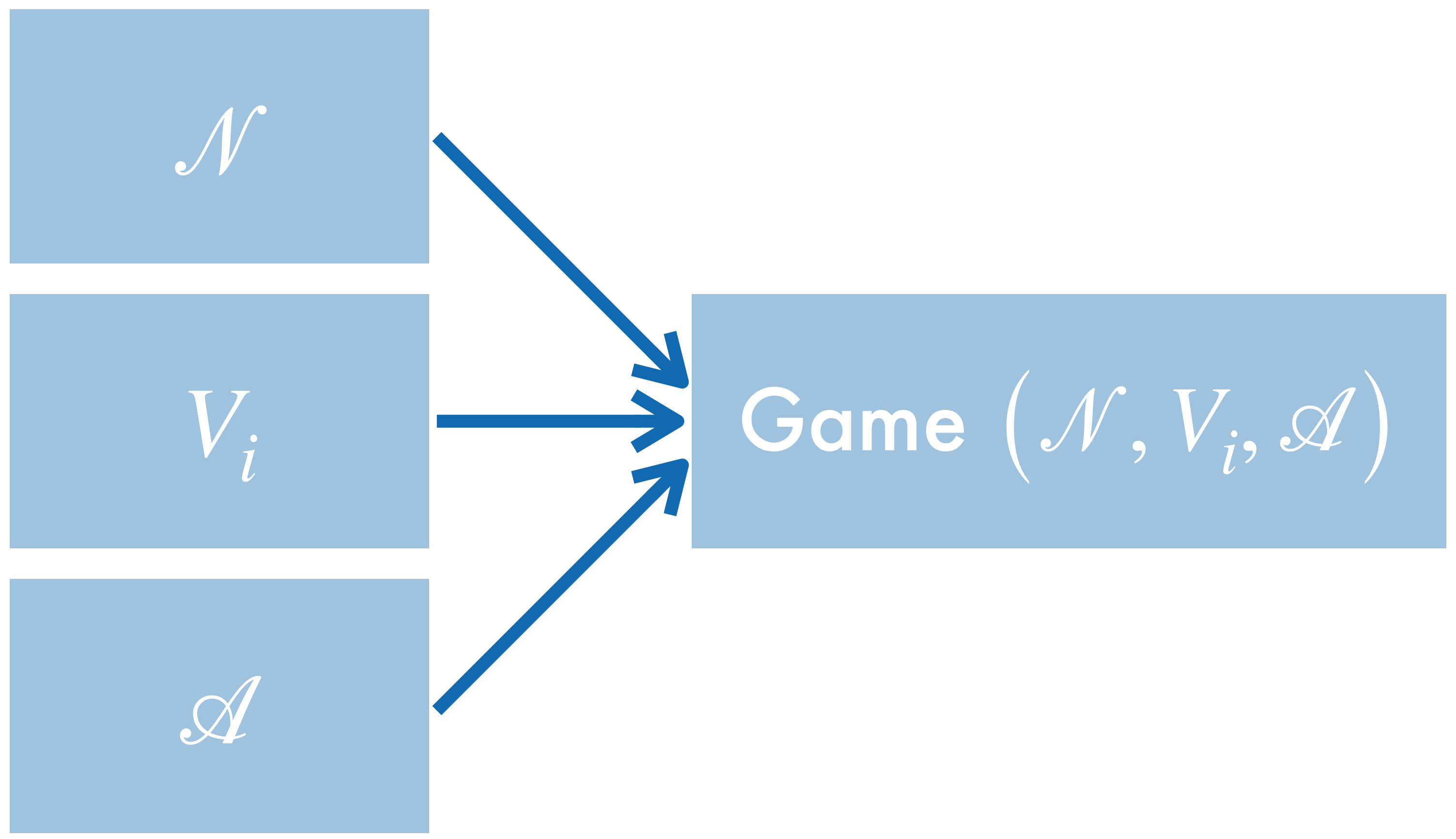
04

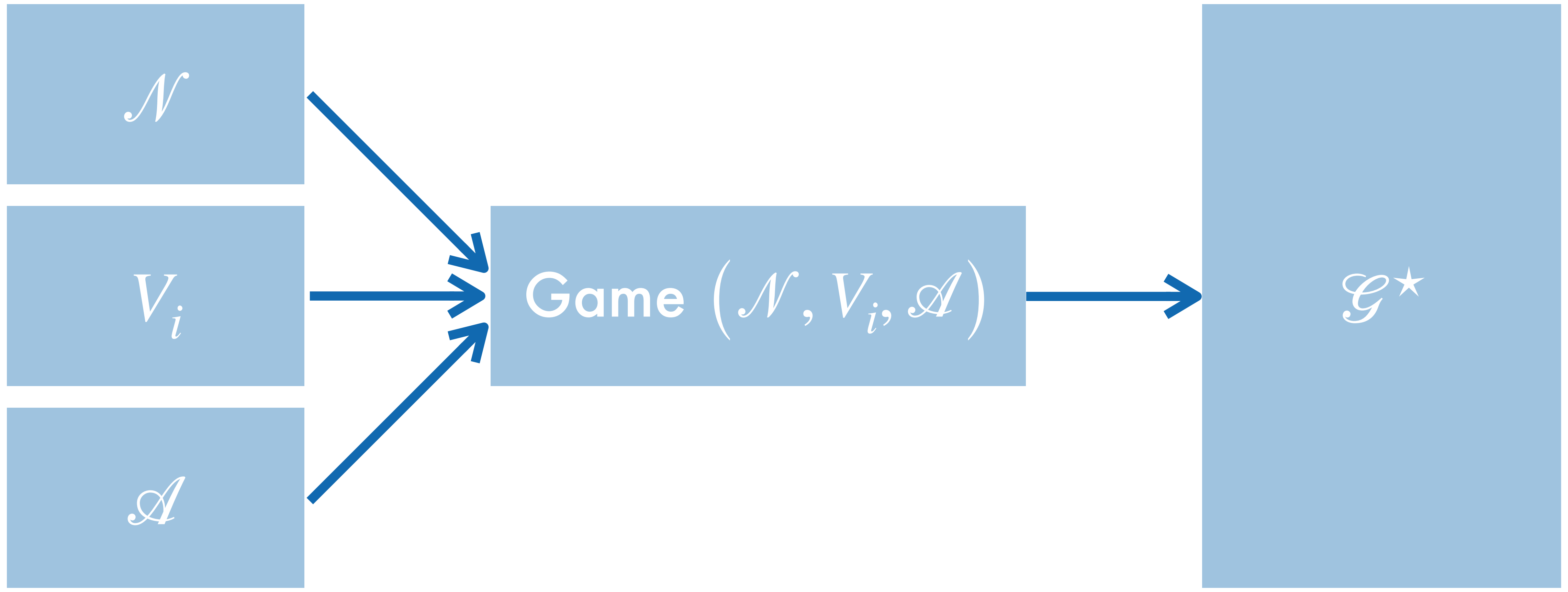


NASH EQUILIBRIUM

Definition.

The network $\mathscr{G}^\star$ is a Nash Equilibrium if for all agents i:

$$V_i\left(a_i, \mathbf{a_{-i}}^\star \mid \theta_i\right) \leq V_i\left(a_i^\star, \mathbf{a_{-i}}^\star \mid \theta_i\right), \forall a_i \in \mathscr{A}$$



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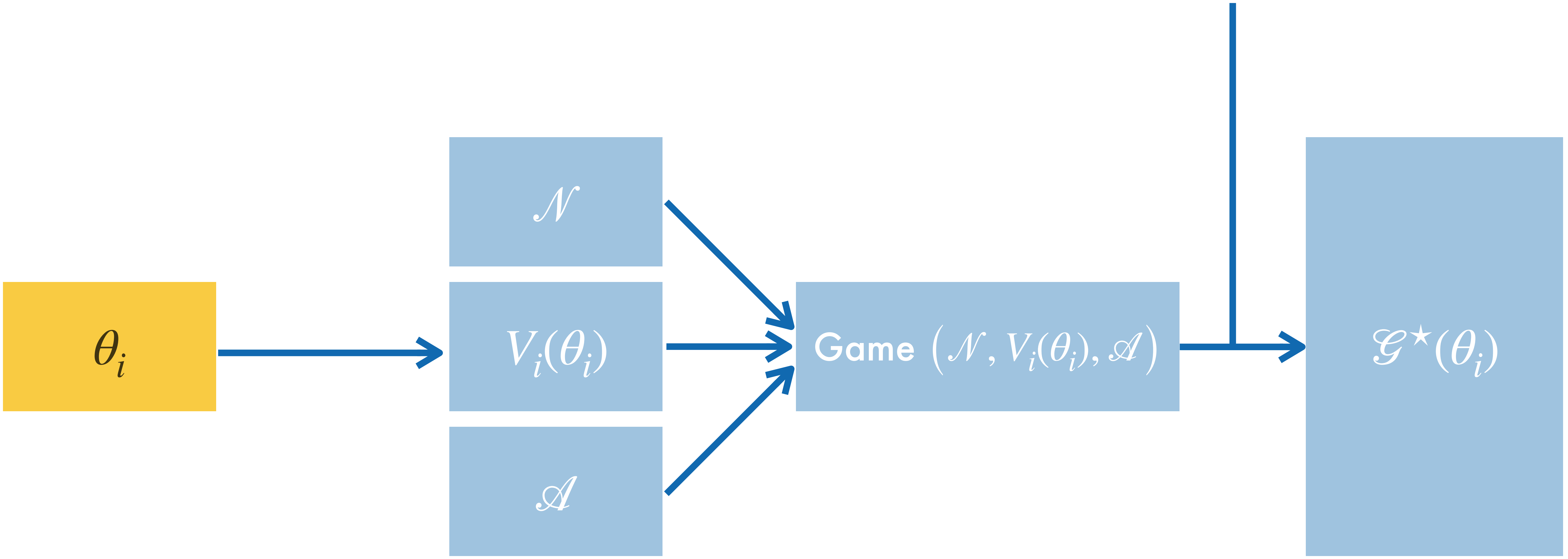
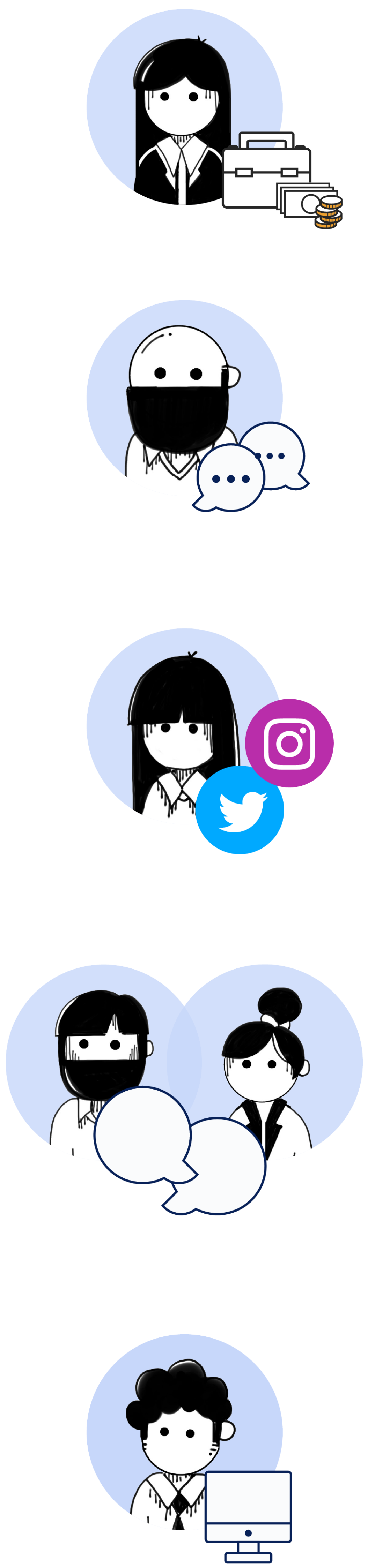


04

Question: Given θ_i, which $\mathscr{G}^\star$ is in equilibrium?

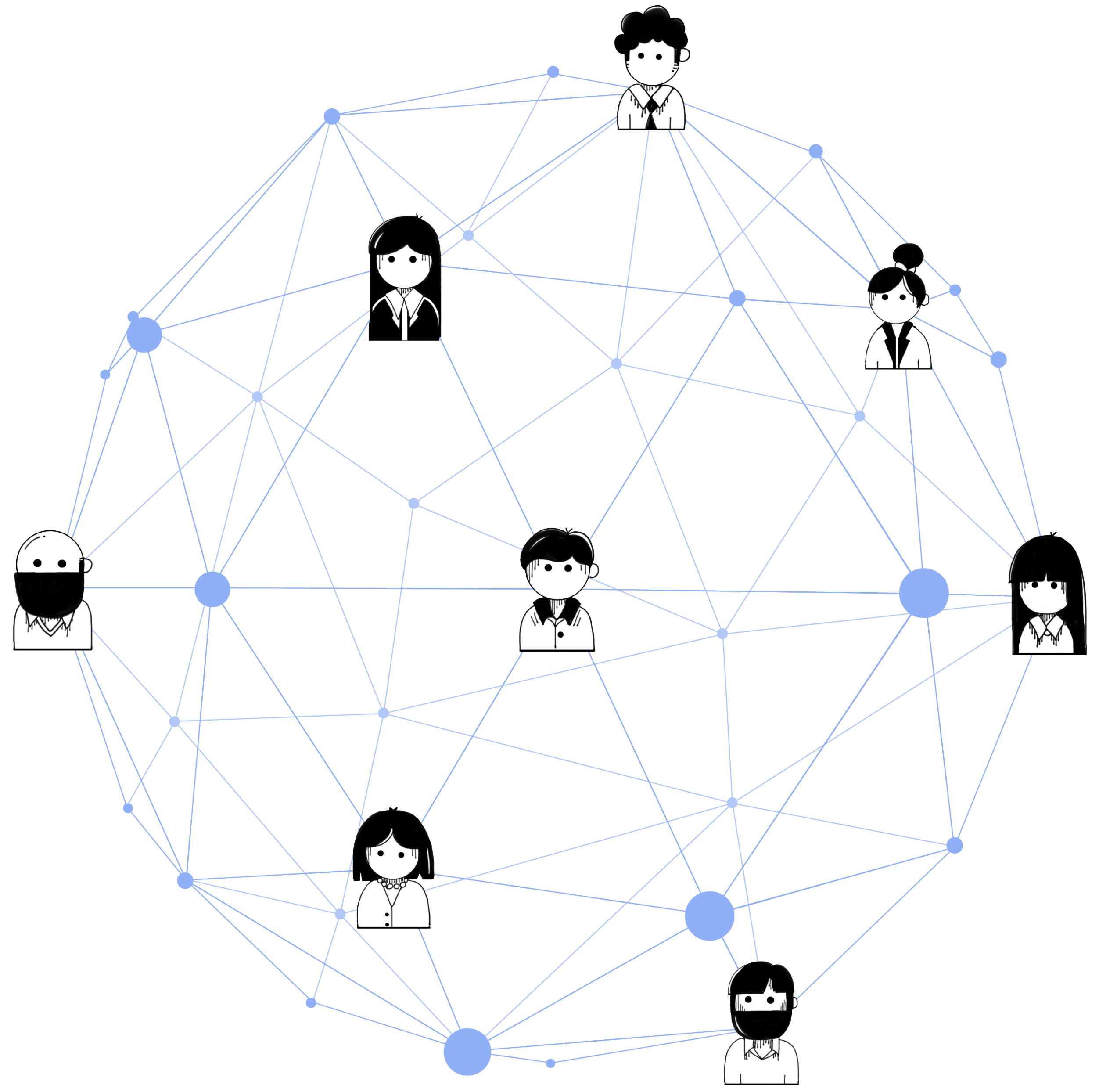
INDIVIDUAL BEHAVIOUR θ_i



DETERMINE

$$\forall i,\ a_i^\star \in \arg\max_{a_i \in \mathscr{A}} V_i\left(a_i, \mathbf{a}_{-i}^\star \mid \theta_i\right)$$

STRATEGIC NETWORK FORMATION MODEL



SOCIAL NETWORK STRUCTURE $\mathscr{G}^\star\left(\theta_i\right)$

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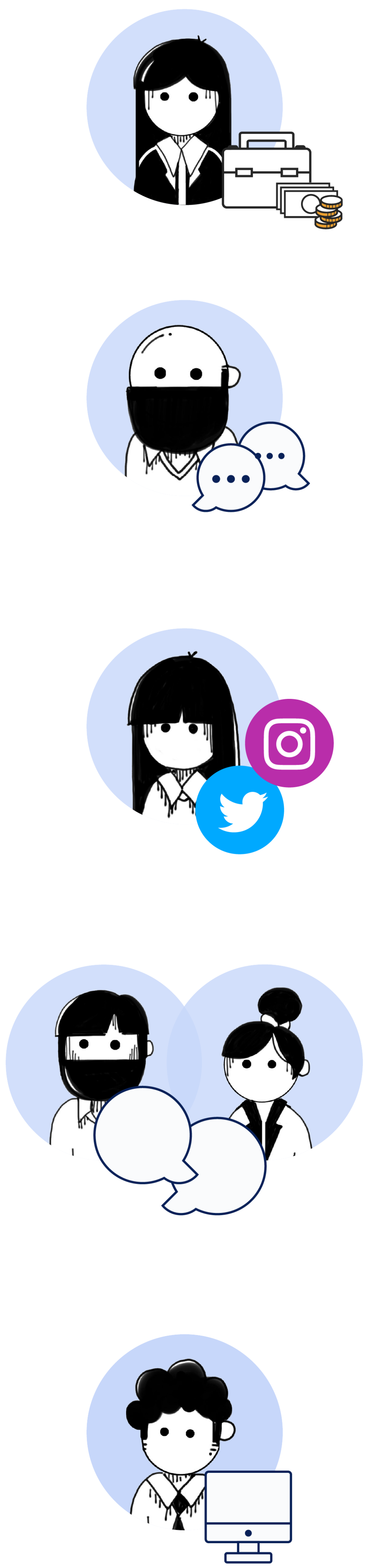
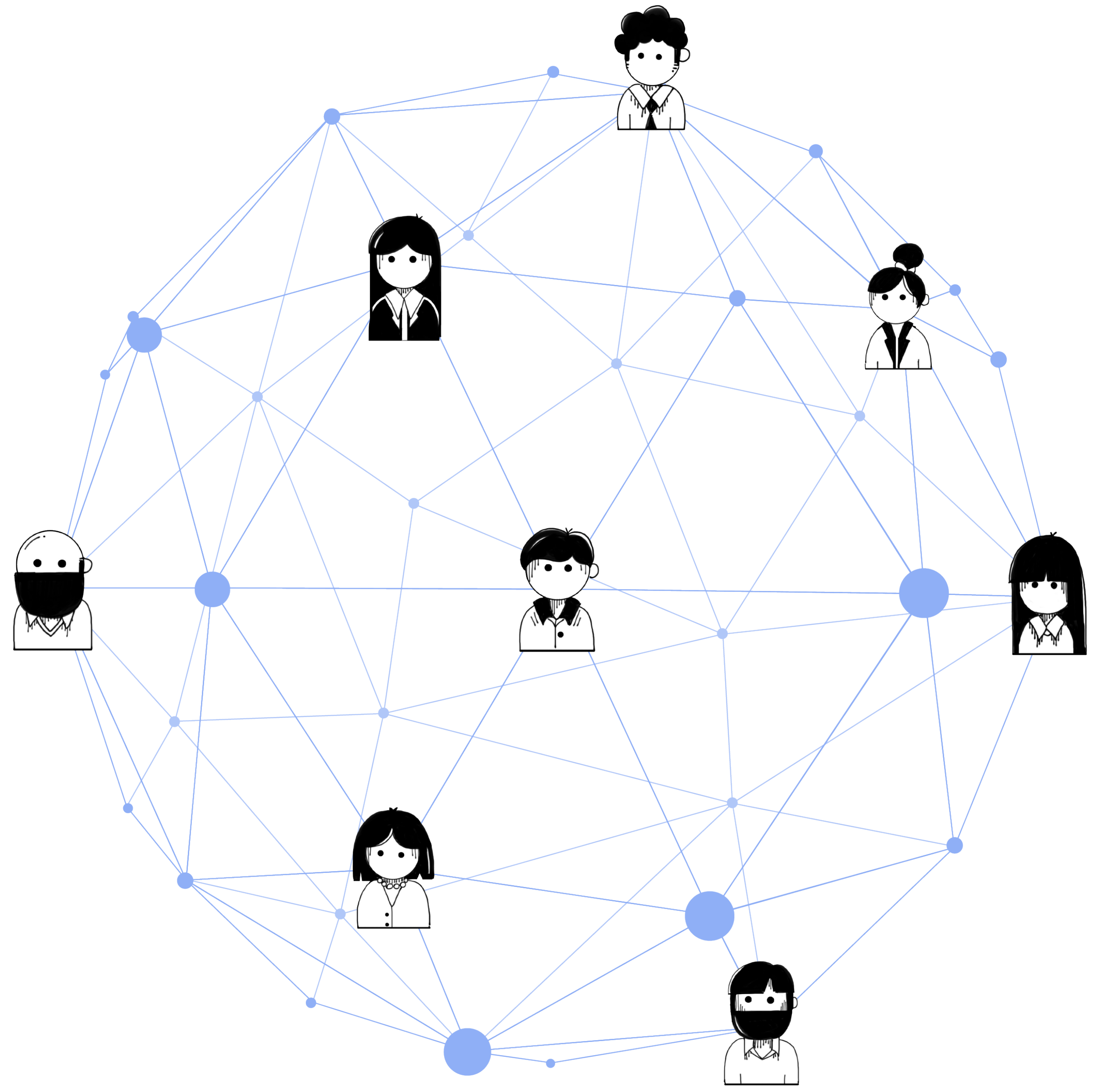
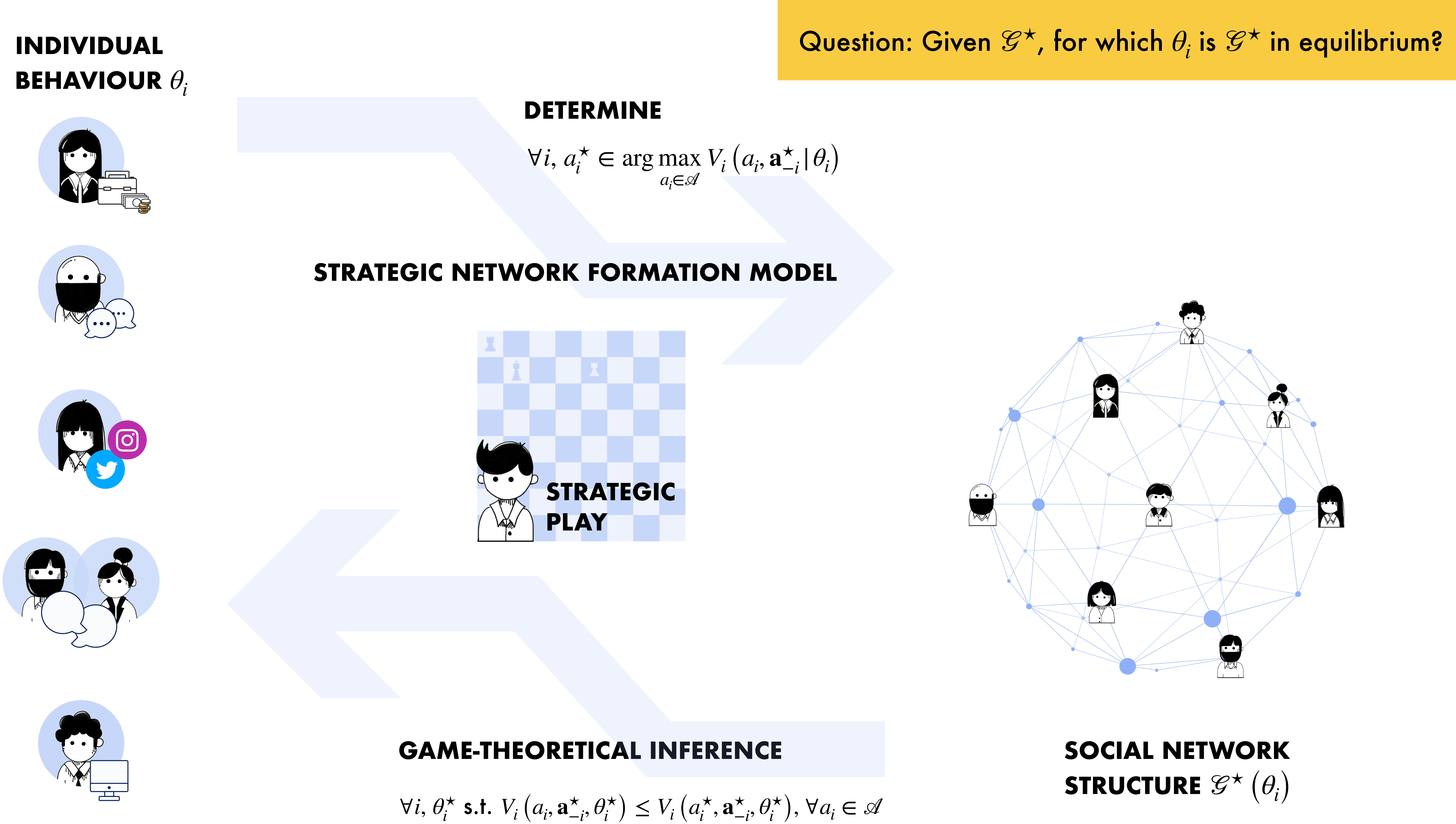


STOCHASTIC ACTOR-ORIENTED MODELS



SOCIAL NETWORK STRUCTURE $\mathscr{G}^\star(\theta_i)$



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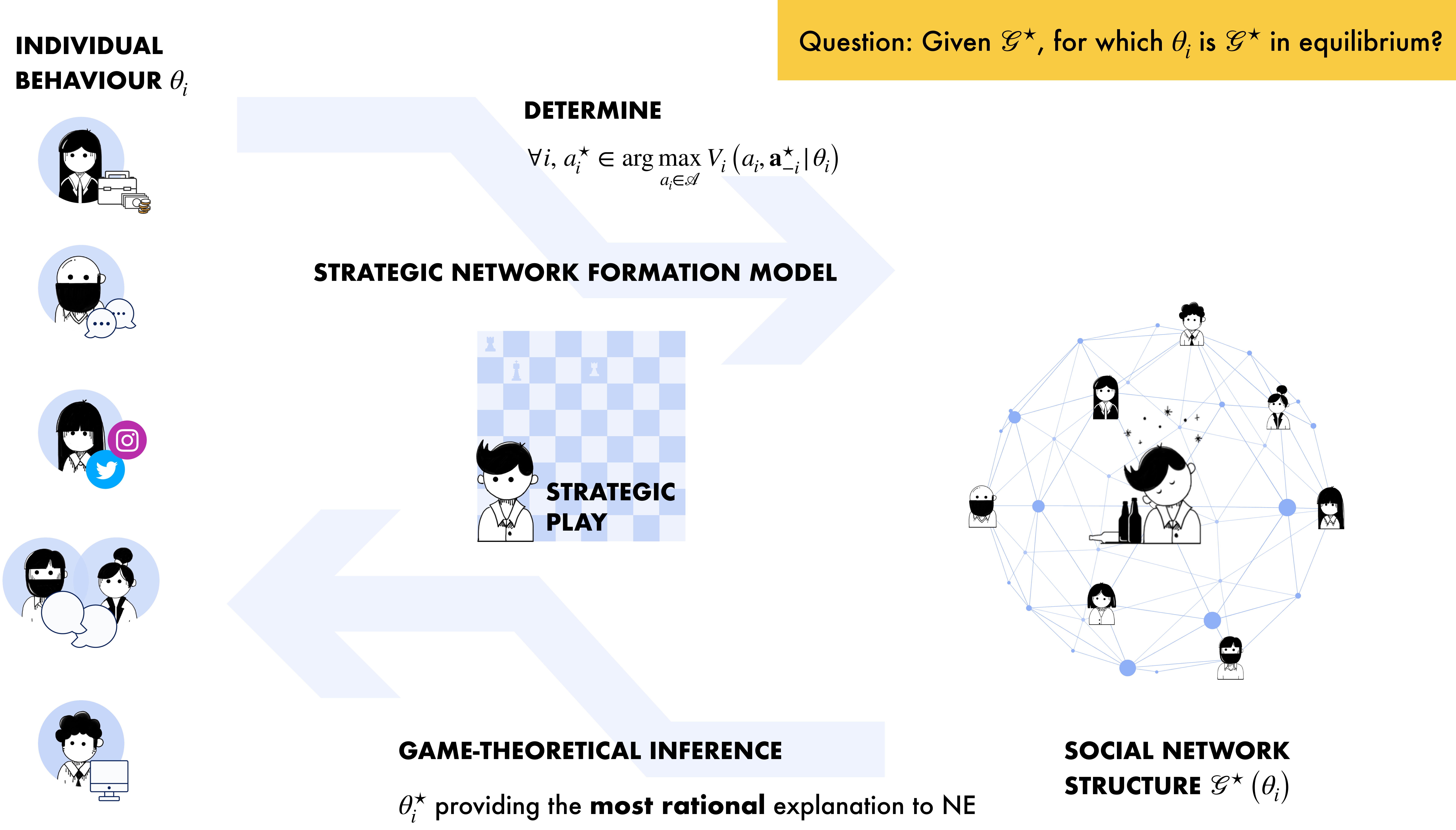
INDIVIDUAL BEHAVIOUR θ_i
DETERMINE
$\forall i,\ a_i^\star \in \arg\max_{a_i \in \mathscr{A}} V_i\left(a_i, \mathbf{a}_{-i}^\star \mid \theta_i\right)$
STRATEGIC NETWORK FORMATION MODEL
STRATEGIC PLAY
GAME-THEORETICAL INFERENCE
$\forall i,\ \theta_i^\star$ s.t. $V_i\left(a_i, \mathbf{a}_{-i}^\star, \theta_i^\star\right) \leq V_i\left(a_i^\star, \mathbf{a}_{-i}^\star, \theta_i^\star\right),\ \forall a_i \in \mathscr{A}$
SOCIAL NETWORK STRUCTURE $\mathscr{G}^\star\left(\theta_i\right)$



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STRATEGIC NETWORK FORMATION MODEL
STRATEGIC PLAY
GAME-THEORETICAL INFERENCE
$\theta_i^\star$ providing the **most rational** explanation to NE
SOCIAL NETWORK STRUCTURE $\mathscr{G}^\star\left(\theta_i\right)$

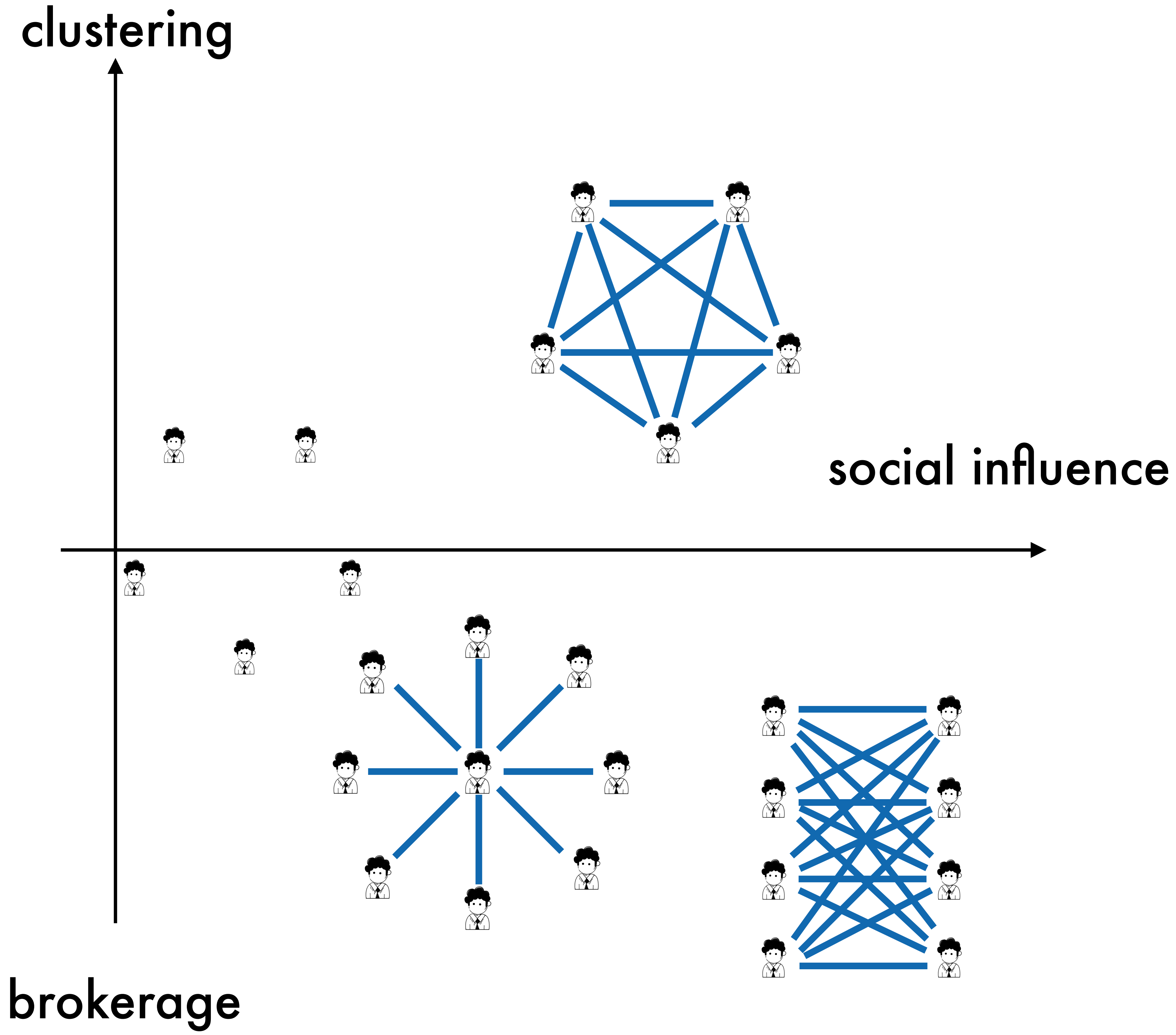
HOMOGENEOUS RATIONAL AGENTS

Assumption.
Individual preferences $\theta_i = \theta$, for all agents i, and fully rational behaviour.

For specific **network motifs**, analytical parametric *necessary* conditions can be derived through **Variational Inequality**. *Sufficiency* can also be established.

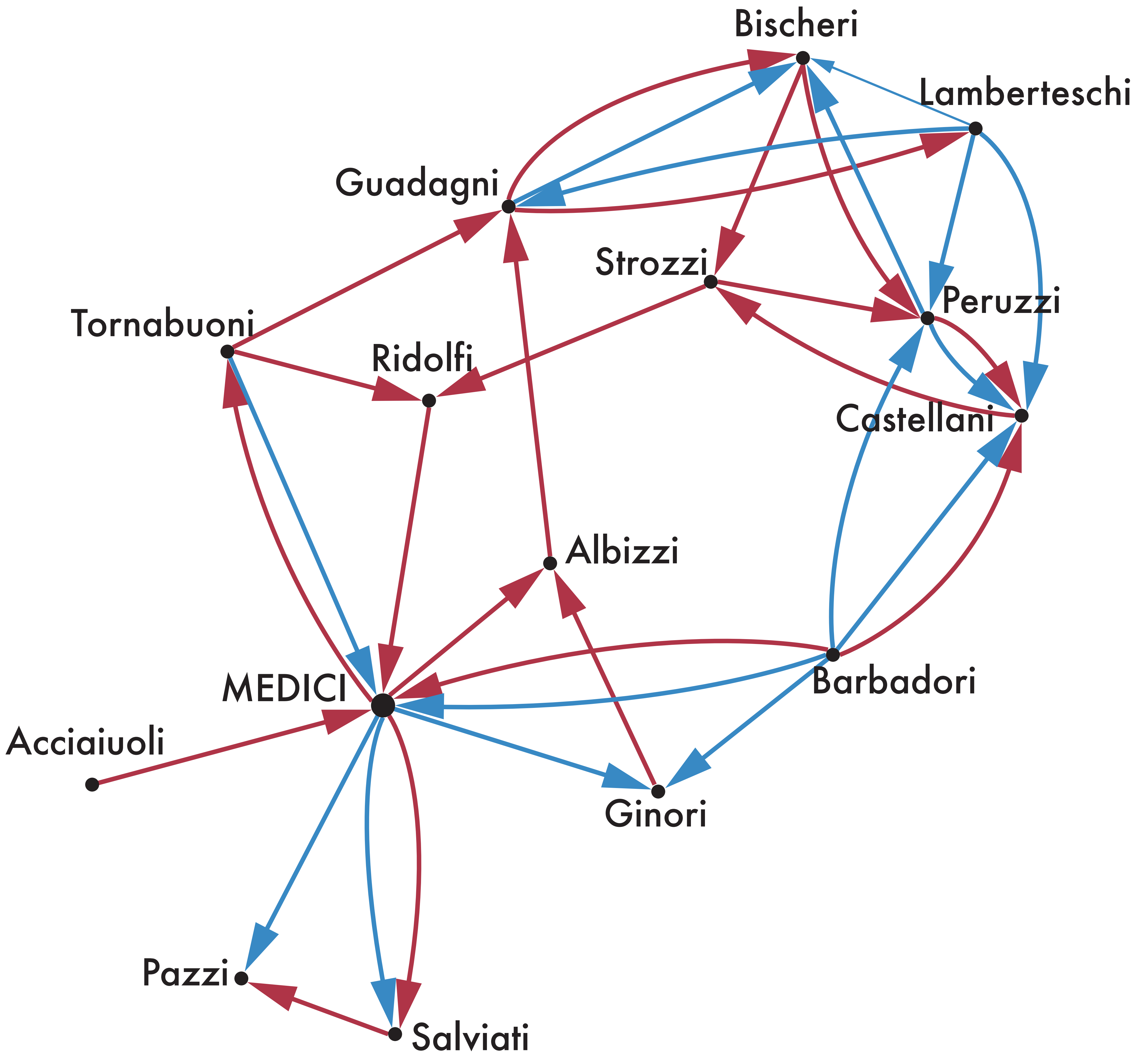
Confirm known results in Strategic Network Formation literature

IFF conditions, parameter space analysis, NE and Pairwise Nash Equilibrium results.



05

REAL WORLD NETWORKS?



06

INVERSE OPTIMIZATION PROBLEM

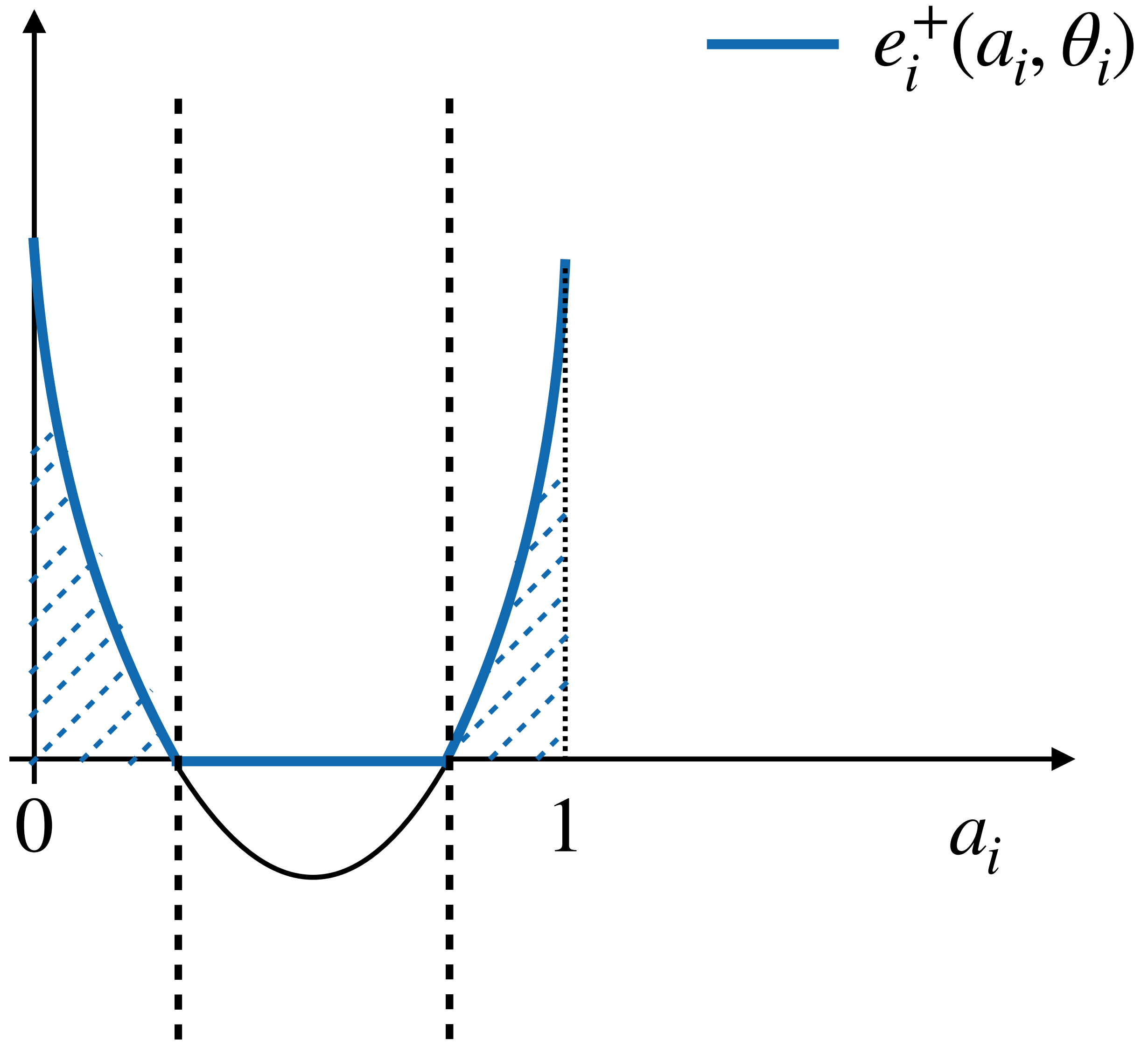
Error function.

$$e_i(a_i, \theta_i) := V_i\left(a_i, \theta_i \,|\, \mathbf{a}^{\star}_{-i}\right) - V_i\left(a_i^{\star}, \theta_i \,|\, \mathbf{a}^{\star}_{-i}\right)$$

$e_i^+(a_i, \theta_i) := \max\left\{0, e_i(a_i, \theta_i)\right\} > 0$ corresponds to a violation of the Nash equilibrium condition

Distance function.

$$d_i(\theta_i) := \left(\int_{\mathscr{A}} e_i^+(a_i, \theta_i)^2 da_i\right)^{1/2} = \|e_i^+(a_i, \theta_i)\|_{L_2(\mathscr{A})}$$



$e_i(a_i, \theta_i) < 0$

No violations: can be neglected

07

INVERSE OPTIMIZATION PROBLEM

Problem [Minimum NE-Distance Problem].

Given a network $\mathcal{G}^\star$ of N agents, for all agents i find the vectors of preferences $\theta_i^\star$ such that

$$\theta_i^\star \in \arg\min_{\theta_i \in \Theta} \; d_i^2(\theta_i)$$

Theorem [Convexity of the objective function].

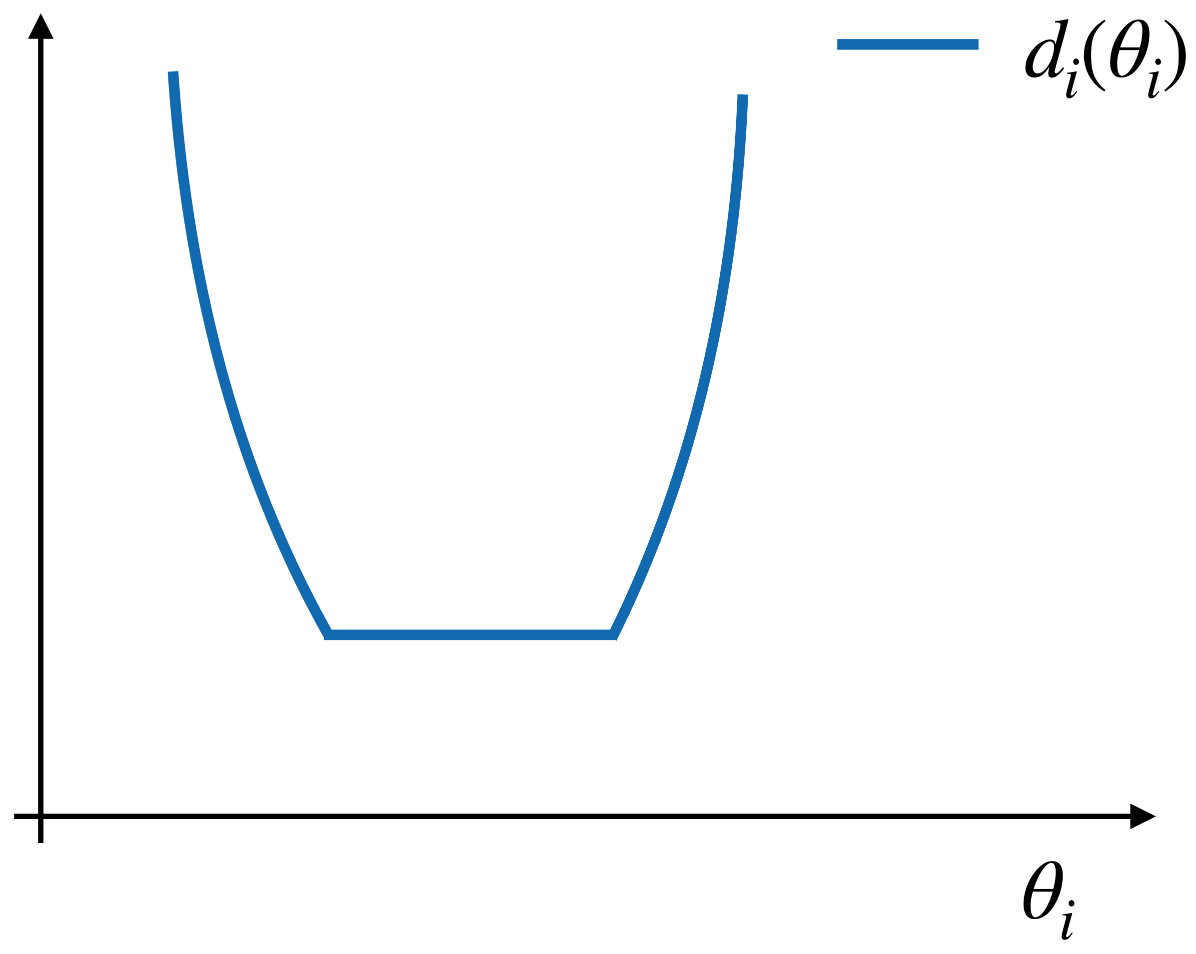
Let $e_i(a_i, \theta_i) : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}$ be a continuous function of $a_i \in \mathbb{R}^n$ and $\theta_i \in \mathbb{R}^p$, and linear in θ_i, and let $\mathcal{A}$ be a compact subset of $\mathbb{R}^n$. Consider the squared distance function:

$$d_i^2(\theta_i) := \int_{\mathcal{A}} \left(\max \left\{ 0,\, e_i(a_i, \theta_i) \right\} \right)^2 dx = \| e_i^+(a_i, \theta_i) \|^2_{L_2(\mathcal{A})}$$

Then $d_i^2(\theta_i)$ is continuously differentiable, and its gradient reads as

$$\nabla_\theta d_i^2(\theta) = \int_{\mathcal{A}} 2\, \nabla_{\theta_i} \big(e_i(a_i, \theta_i) \big) \max \left\{ 0,\, e_i(a_i, \theta_i) \right\} da_i \, .$$

Moreover, d_i^2 is convex.



INVERSE OPTIMIZATION PROBLEM - SOLUTION

First-order optimality condition

$$0 = \nabla_{\theta_i}\left(d_i^2(\theta_i)\right) = 2\int_{\mathscr{A}} \nabla_{\theta_i}\left(e_i(a_i,\theta_i)\right) \max\left\{0,\, e_i(a_i,\theta_i)\right\} da_i\,.$$

max operator within $(N-1)$ - dimensional integral

09

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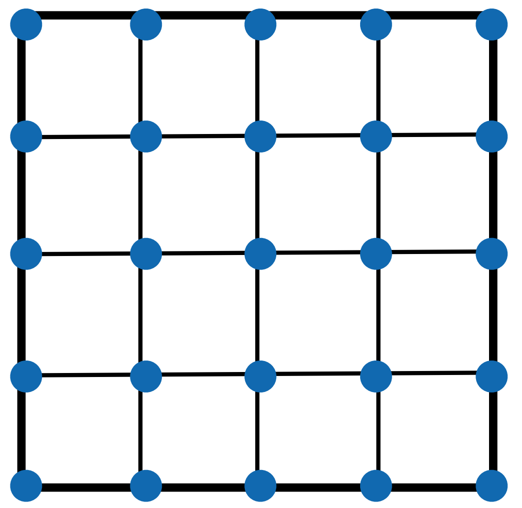
INVERSE OPTIMIZATION PROBLEM - SOLUTION

Search for an approximate solution. Consider a finite set of possible actions (samples)

$$\left\{a_i^j\right\}_{j=1}^{n_i} \subset \mathscr{A}$$



Let $e_i^j(\theta_i) = e_i(a_i^j, \theta_i)$ and $e_i^{j,+}(\theta_i) = e_i^+(a_i^j, \theta_i)$ be the corresponding error and positive error at the samples.

Approximate the distance function as

$$\tilde{d}_i(\theta_i) := \left(\sum_{j=1}^{n_i}\left(e_i^{j,+}(\theta_i)\right)^2\right)^{1/2} = \|\mathbf{e}_i^+\|_2$$

Problem [Discrete Minimum NE-Distance Problem].

Given a network $\mathscr{G}^\star$ of N agents, for all agents i find the vectors of preferences $\hat{\theta}_i$ such that

$$\hat{\theta}_i \in \arg\min_{\theta_i \in \Theta} \tilde{d}_i^2(\theta_i)$$

Same property of the original problem (Convexity)

09

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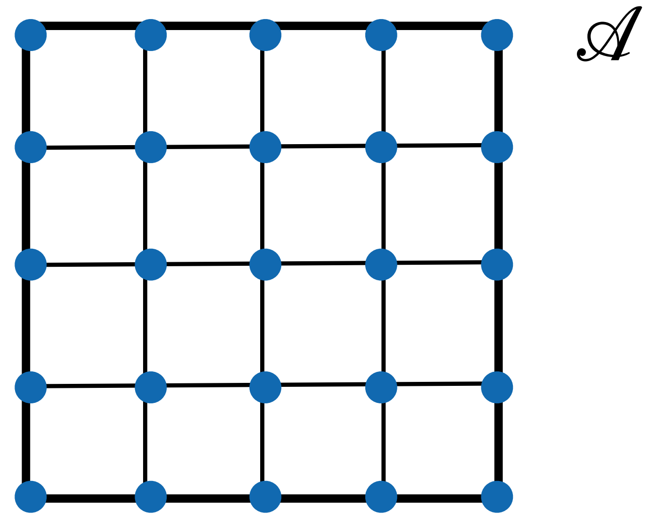

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Similar to Generalized Least Square Regression

RENAISSANCE FLORENCE NETWORK

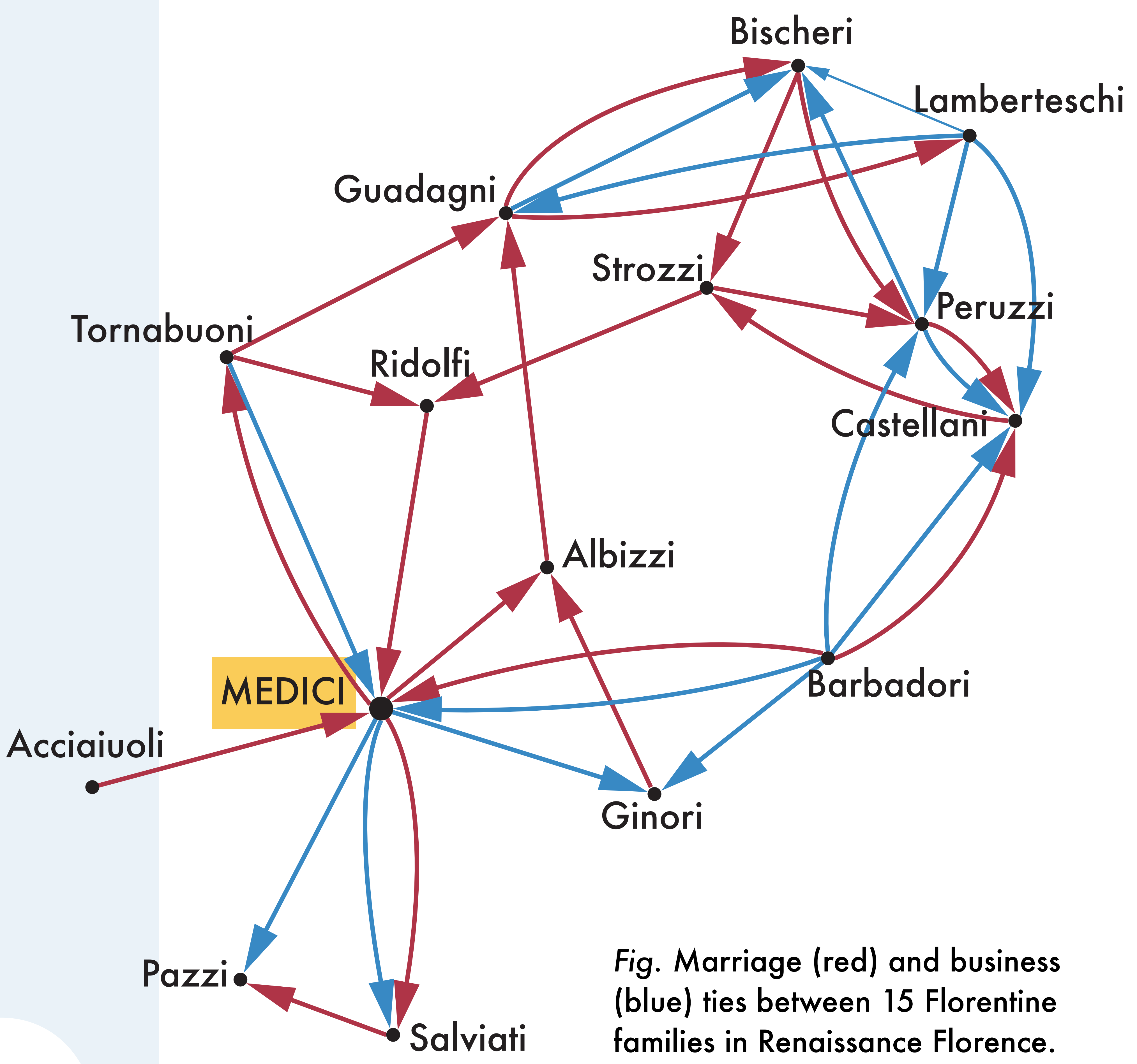
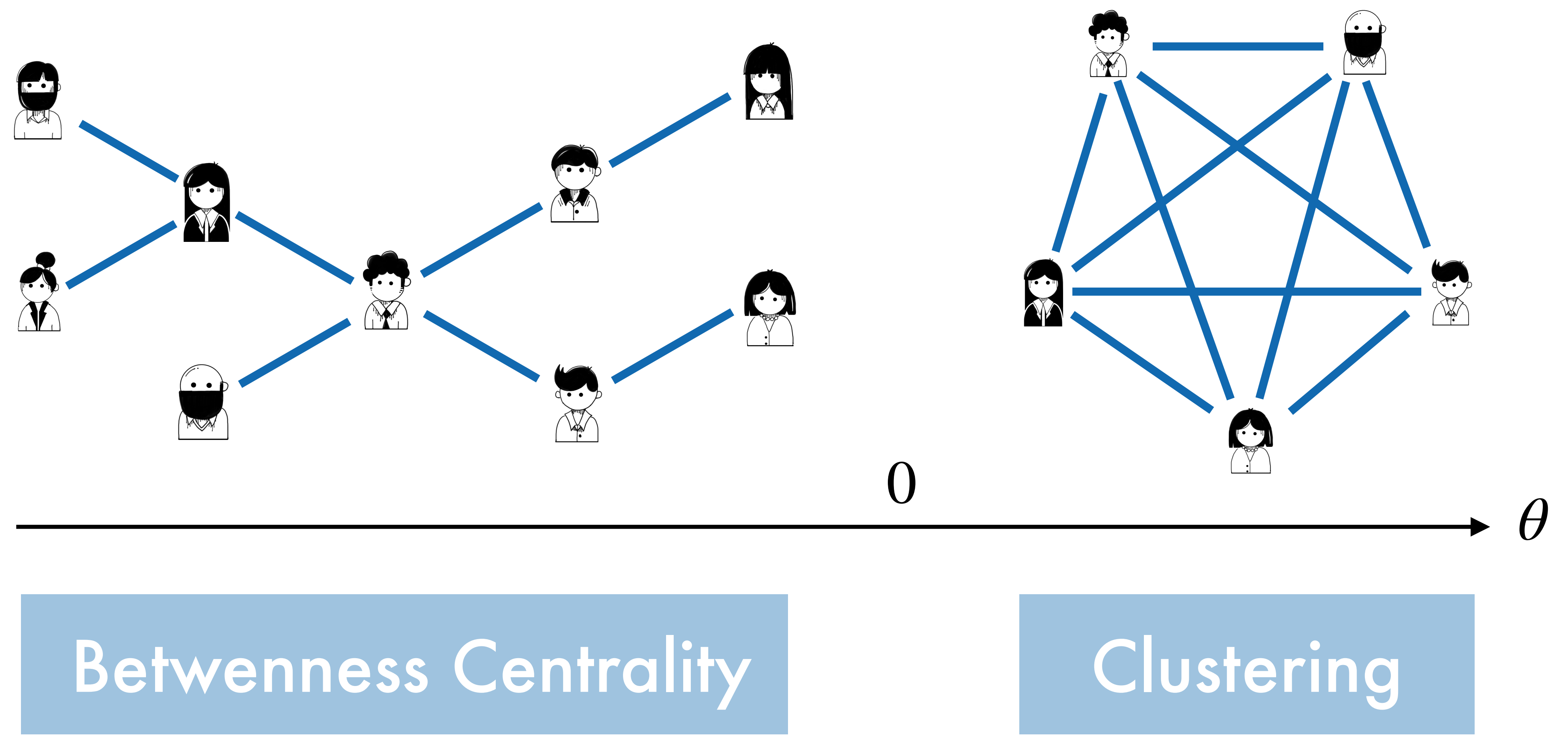
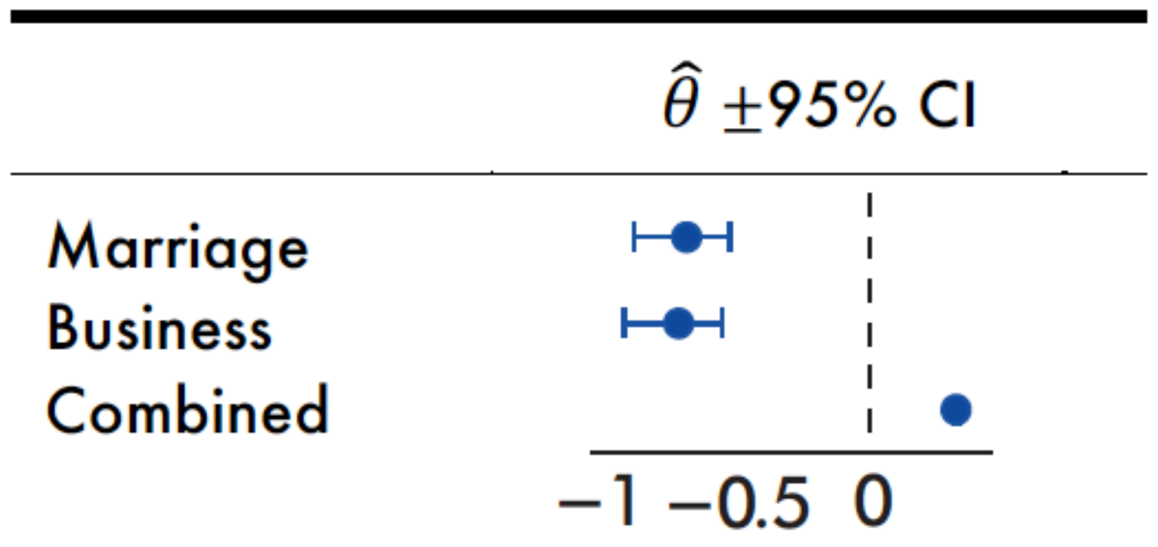


Fig. Marriage (red) and business (blue) ties between 15 Florentine families in Renaissance Florence.

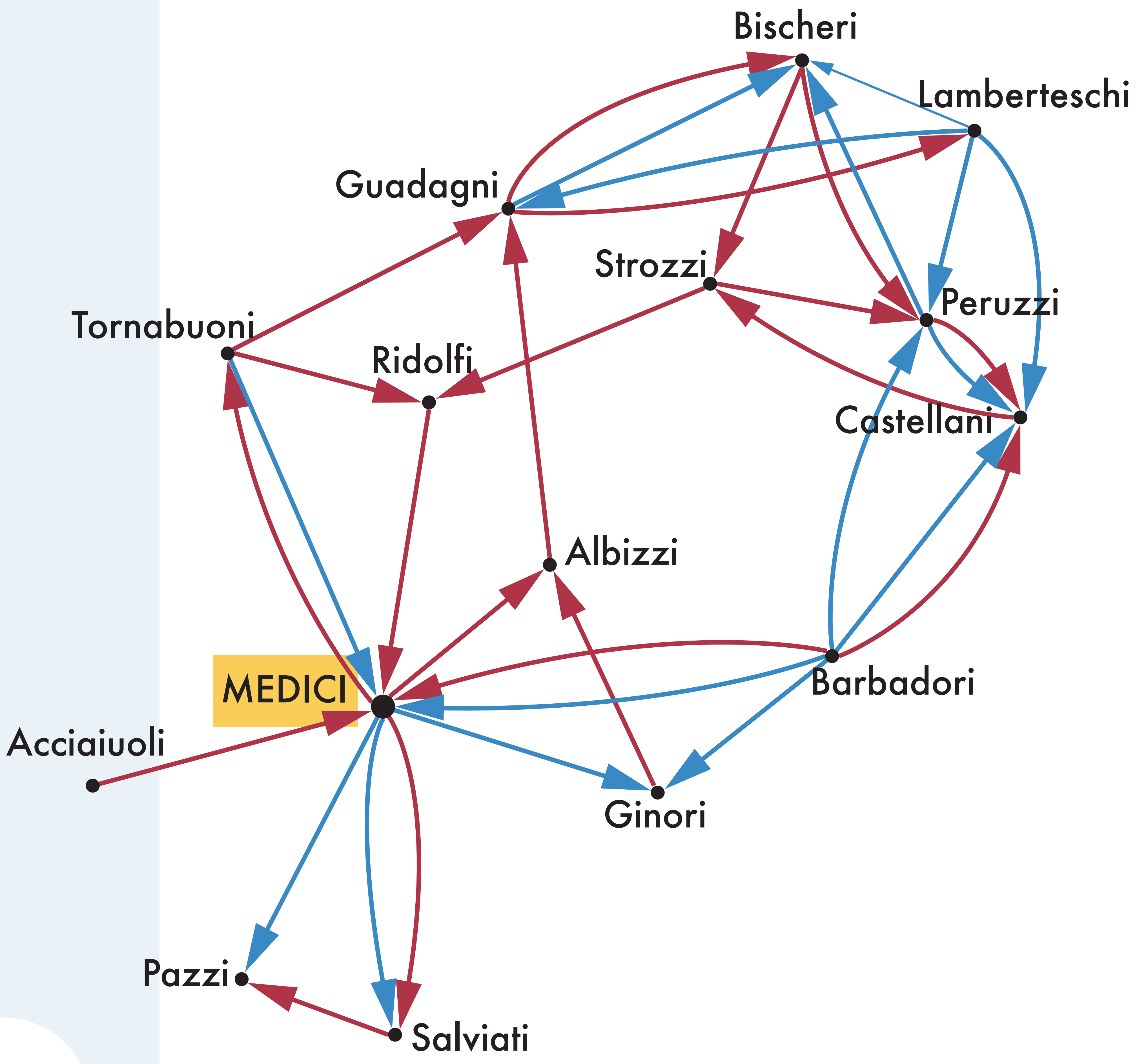


Behaviour estimation of the Medici family



Padgett, J. F., & Ansell, C. K. (1993). Robust Action and the Rise of the Medici, 1400-1434. *American journal of sociology*, *98*(6), 1259-1319.

PIECE OF ART



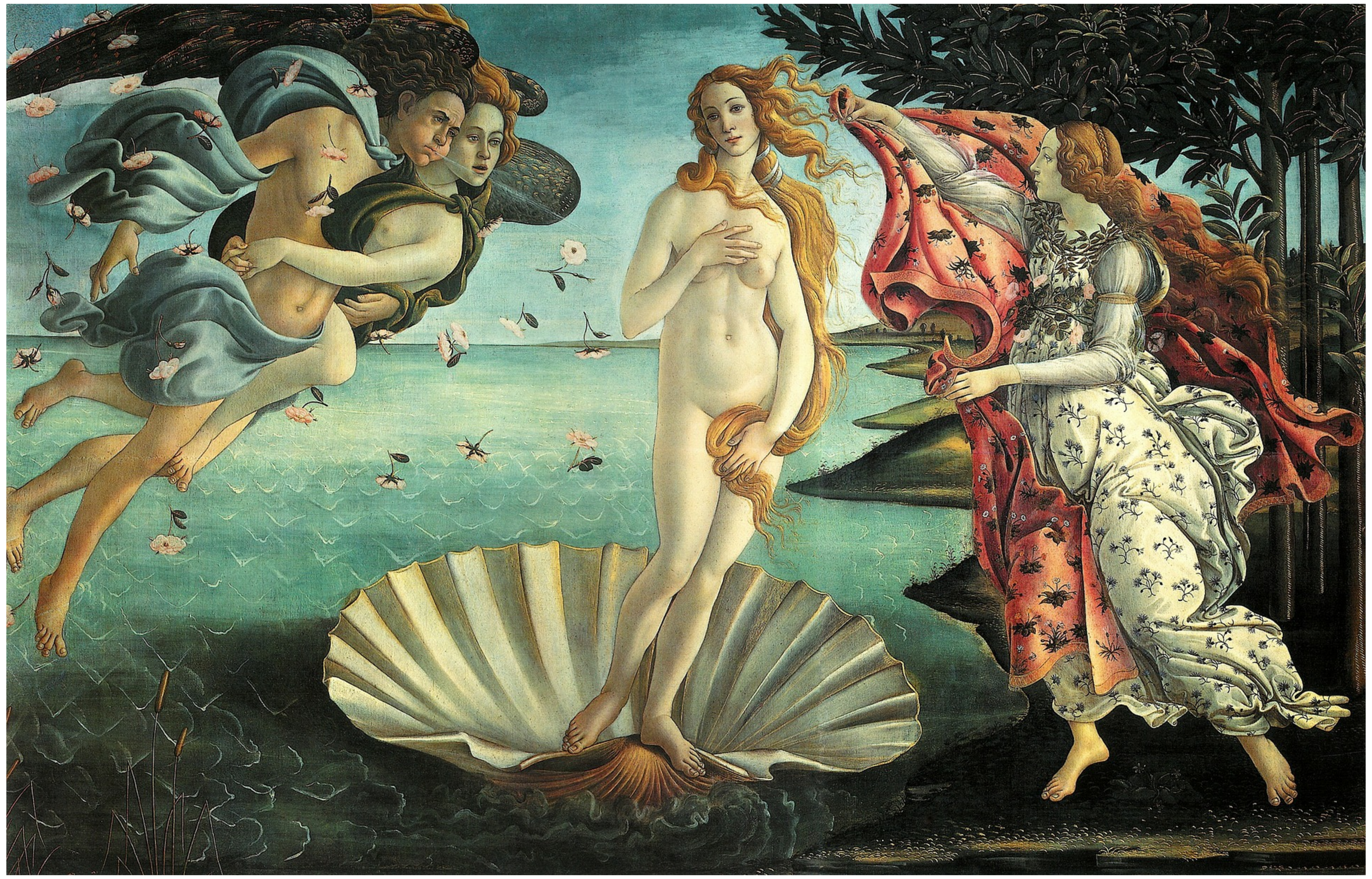
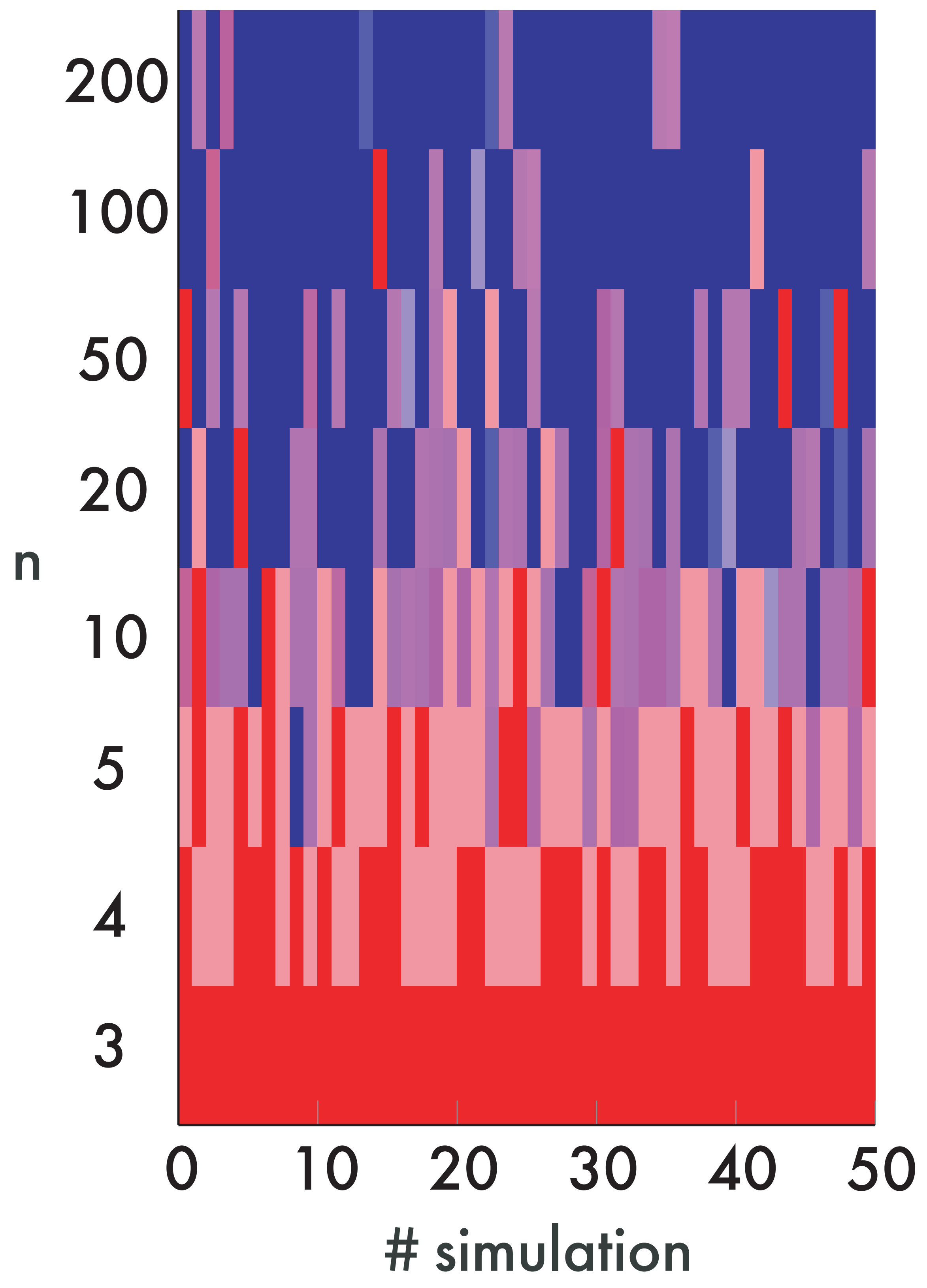
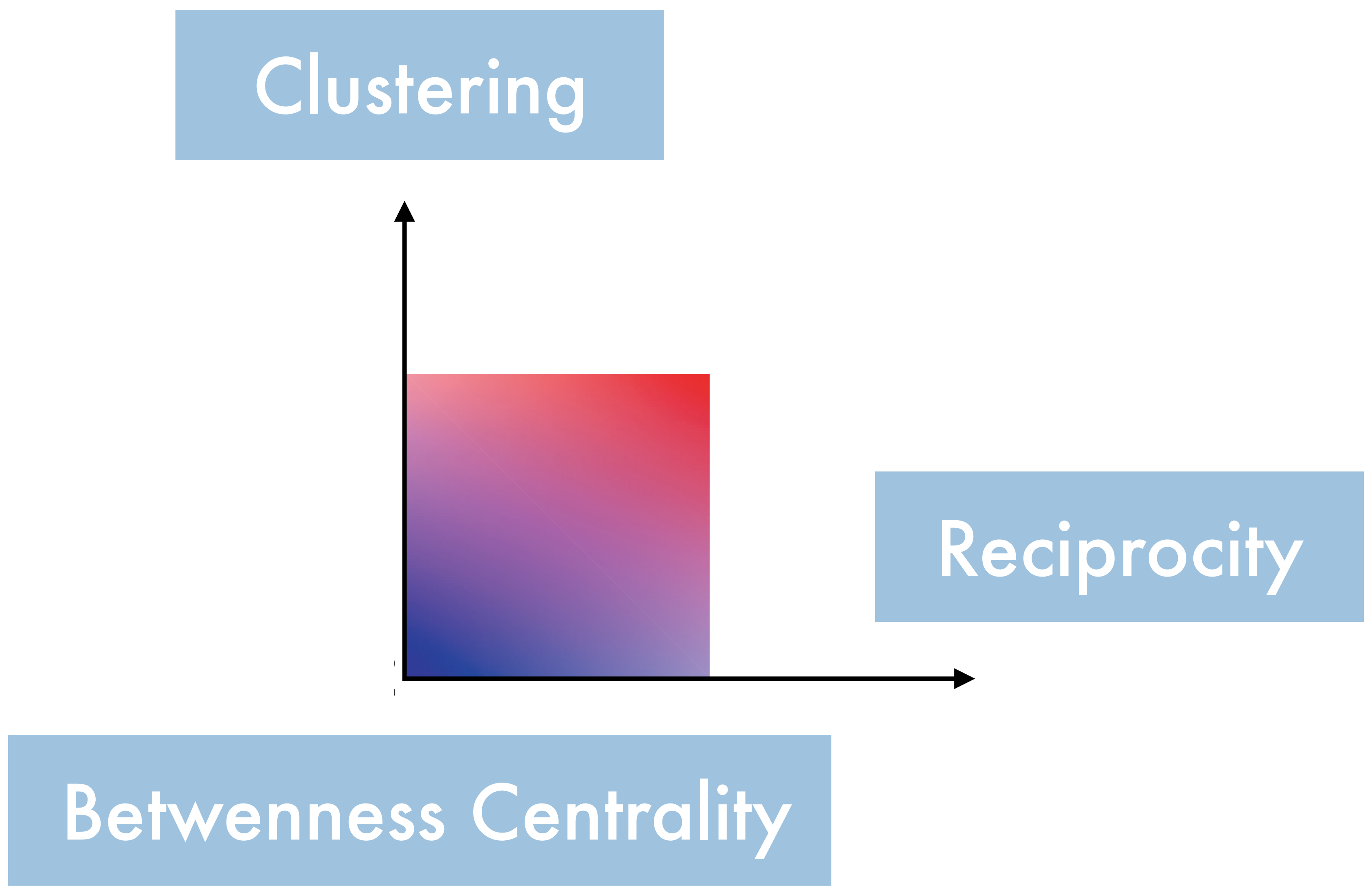


Fig. "La nascita di Venere", 1482-1485, Sandro Botticelli. Galleria degli Uffizi, Firenze.

PREFERENTIAL ATTACHMENT MODEL

Nodes are introduced sequentially.
Each newborn **receives 2 incoming links** from existing nodes (randomly selected, proportionally to the outdegree), and **creates 2 outgoing ties** to existing nodes (randomly selected, proportionally to the indegree).





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