Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in social networks

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Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

1. Introduction

2. Targeting in computer science and complex systems

3. Representing networks and centrality measures

4. Targeting by persuaders with extreme and centrist opinions

5. Analytical models of targeting in economics

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Outline

1. **Introduction**

2. Targeting in computer science and complex systems

3. Representing networks and centrality measures

4. Targeting by persuaders with extreme and centrist opinions

5. Analytical models of targeting in economics

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Introduction

- Identifying optimal targets is a crucial problem for achieving social impact (marketing, lobbying, voting and political campaigning, etc.).
- Some surveys: Jackson 2008, Acemoglu and Ozdaglar 2011, Bloch 2016, Bramoullé et al. 2016, etc.
- Targeting in networks studied in different fields: complex systems, computer science, mathematics, physics, economics, marketing and organizational science, political science, etc.
- Various methods and techniques: mathematical models, agent-based simulations, some attempts of the empirical (experimental) approach.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Classifications of targeting models

- Non-competitive versus competitive environments: social planner (e.g., monopolist) versus competing persuaders (e.g., oligopoly, perfect competition).
- Perfect information of the network structure versus limited information (e.g., consumers and firms only observe local neighborhods and the degree distribution of the network).
- Our focus: analytical models of targeting in economics, where targets are frequently characterized by new or existing centrality measures.
- Our analysis: Targeting individuals to diffuse information or opinions in a social network; pricing at different nodes of the social network when agents experience consumption externalities; determining and removing a key player to increase/reduce some activities.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Diffusion of information in social networks (1/3)

- Survey partially based on Bloch (2016)
- How can a firm use social interactions to diffuse information about a new product?
- The firm's problem: selecting a target in the (fixed) social network in order to maximize diffusion.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Diffusion of information in social networks (2/3)

- Diffusion of information (simple model):
  - Information as a binary signal $(0/1)$ = awareness of a new product, recommendation for the existing product.
  - An agent is in state $0/1$ if he is uninformed/informed or has not/has received a positive recommendation.
  - Agents' states evolve over time according to their interactions with others.
  - The social network g is fixed. At any time, a consumer may receive a message from one of his neighbours.
  - Once informed - he remains informed forever.
  - Diffusion of information is mechanical: a consumer does not control whether he sends information to his neighbors or not.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Diffusion of information in social networks (3/3)

Two main diffusion models:

- The linear threshold model:
  - An agent gets information about the product iff the number of informed neighbours is greater than a certain threshold.
  - If neighbors are heterogeneous, this threshold may be replaced by weighted averages of the status of the agents' neighbors (i.e., some agents are more influential than others, independently of their location in the social network).
- The independent cascade model:
  - Each neighbour of an agent sends information with an independent probability. Agent is informed iff at least one of his neighbours has sent the information.
- Given the (fixed) network structure, targeting = choosing the agent(s) to give first the product in order to diffuse information in the network as fast as possible.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Outline

1. Introduction

2. Targeting in computer science and complex systems

3. Representing networks and centrality measures

4. Targeting by persuaders with extreme and centrist opinions

5. Analytical models of targeting in economics

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in computer science and complex systems

- Algorithmic perspective for studying the target selection for the optimal adoption and diffusion of innovation; choosing influential sets of individuals as a problem in discrete optimization.

- Which set of individuals should we target if we aim to have a large cascade of adoptions of a new product or innovation?

- Influence maximization: Domingos and Richardson (2001), Richardson and Domingos (2002), Kempe, Kleinberg and Tardos (2003, 2005).

- Revenue maximization: Hartline, Mirrokni and Sundarajan (2008), Arthur, Motwani, Sharma and Xu (2009).

- Influence maximization with competition: Bharathi, Kempe and Salek (2007), Goyal and Kearns (2012), Dubey, Garg and de Meyer (2006).

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Influence maximization:
Domingos and Richardson (2001) (1/2)

- First work on targeting algorithms to maximize the probability of sales in a social network.
- Each consumer is of type 0 or 1 (buying the product or not).
- A consumers' probability of buying a product depends on marketing expenditures and the probability that the direct neighbours have bought the product.
- How can a firm optimally target consumers by directing marketing expenditures to specific agents?
- Three algorithms:
    - a single-pass algorithm (one iteration),
    - a greedy algorithm (which increases marketing expenditures when they increase payoffs),
    - hill-climbing algorithm (increases expenditures where matters most).

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Influence maximization:
Domingos and Richardson (2001) (2/2)

- They use data on an experimental program of movie recommendations, EachMovie, from 1996-1997.
- The authors compare three marketing strategies:
  - mass marketing where all agents receive uniform expenditures,
  - directed marketing where agents are targeted according to their individual characteristics but ignoring their influence on others,
  - targeted marketing taking into account agents' market values.
- They quantify the profit increase due to the use of targeted strategies with respect to directed and mass strategies.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Influence maximization:
Richardson and Domingos (2002)

- The same model, assuming that the influence is measured through a weight matrix $[w_{ij}]$, representing i's influence on j's choice, and that marketing decisions are continuous.

- The authors use data from the knowledge-sharing site Epinions, quantify agents's network values, and simulate the effect of different marketing policies.

- They test the robustness of the results w.r.t. removal of nodes from the network.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Influence maximization:
Kempe, Kleinberg and Tardos (2003) (1/2)

- The firm selects an initial set A of k nodes in the social network in order to maximize the total number of informed nodes.
- Information is diffused according to a linear threshold or independent cascade model.
- Results:
    - The optimal targeting problem is NP-hard (in general impossible to find a polynomial algorithm to compute the optimal target set A except in special cases, e.g., in Richardson and Domingos (2002)'s linear model where the optimal target is a solution of a system of linear equations).
    - Determining the approximation bound on the efficiency of the hill-climbing algorithm, which selects agents to place in the set A by looking sequentially at agents with the highest influence.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Influence maximization:
Kempe, Kleinberg and Tardos (2003) (2/2)

- For the approximation bound, the analysis of submodular functions in integer programming is used:
  - A function f is called submodular, if for any $S \subseteq T$ and any vertex $v \notin T$, $f(S \cup v) - f(S) \geq f(T \cup v) - f(T)$.
  - The following result is known: Let the function f be monotone, positive and submodular. Let S be the set of k elements obtained by adding one by one element which maximizes the increase in the function's value. Let S^* be the set of k elements which maximizes the functions' value. Then $f(S) \geq \left(1 - \frac{1}{e}\right) f(S^*)$, where e is the base of natural logarithms.

- The authors show that the influence function in the linear threshold and independent cascade models are submodular, and hence establish the approximation bound for the hill-climbing algorithm.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Influence maximization:
Kempe, Kleinberg and Tardos (2005)

- Similarly, the authors consider the problem of target selection in order to maximize the expected spread of an innovation or of some behaviour within a social network in the "word-of-mouth" referral framework.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Revenue maximization:
Hartline, Mirrokni and Sundarajan (2008) (1/3)

- A monopolistic seller of a new product chooses the sequence in which buyers are approached and the prices charged to the buyers.
- In the revenue maximization problem, buyers choose whether to purchase the good or not.
- The value of a consumer i is drawn at random from a distribution $F_i(V)$ which depends on the set V of consumers influencing i.
- Consumers are myopic and choose to purchase the good based on the current set of agents who have bought the good and not on the final set of consumers purchasing the product.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Revenue maximization:
Hartline, Mirrokni and Sundarajan (2008) (2/3)

- Additional layer of complexity w.r.t. the influence maximization problem: the seller has to compute optimal prices at each node taking into account the effect of prices on the diffusion of the good through the purchasing decisions of the buyers.

- The solution to the problem is NP hard, but for the complete network and all agents being symmetric, the optimal pricing strategy can be computed in polynomial time as the solution to a linear programming problem.

- The revenue generated by consumer i is assumed to be a monotone, submodular function of the set of agents V who influence agent i.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Revenue maximization:
Hartline, Mirrokni and Sundarajan (2008) (3/3)

- The authors derive lower bounds on the efficiency of a two-step EI algorithm (Exploit and Influence).
- The algorithm first searches for a set of initial agents A to whom the good is given for free. Then the seller visits agents in $N \setminus A$ in a random order and the algorithm selects for each buyer the optimal revenue maximizing price.
- In the linear threshold model, the approximation bound is equal to $\frac{2}{3}$.
- For general diffusion models, it achieves at least $\frac{1}{3}$ of the total value.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Revenue maximization:
Arthur, Motwani, Sharma and Xu (2009)

- Another EI algorithm to solve the revenue maximization problem.
- The algorithm first computes a minimum cost spanning tree of the graph with maximal number of leaves (NP hard problem) and the set A is formed by the internal nodes, who receive a fixed cashback for each consumer they refer, while leaves are charged a fixed price.
- The algorithm achieves a fraction of the total value which depends on the complexity parameters of the problem (this fraction is typically lower than the fraction computed by Hartline, Mirrokni and Sundarajan (2008)).

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Influence maximization with competition

- Instead of one firm, two firms are competing to seed the network.
- The influence maximization problem becomes much more complex.
- The firms play a noncooperative game, and hence besides inefficiencies due to the approximation algorithm, additional inefficiencies (w.r.t. the policy adopted by a single firm) are possible.
- A price of anarchy measures the worst ratio between the optimal value and the values obtained in the equilibrium of the game.
- Consumer's type: 0 (uninformed), A (buy product A), B (buy product B).
- We have two competing diffusion models: how a consumer exposed to both products A and B chooses the product to buy.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Influence maximization with competition: Bharathi, Kempe and Salek (2007)

- Extensions of the independent cascade model in a competitive environment.
- The consumer adopts the first product he becomes aware of.
- If he becomes aware of A and B simultaneously, an exogenous tie-breaking rule decides which of the two products is chosen.
- The hill climbing algorithm provides an approximation of the efficient policy, with the same approximation bound of $1 - \frac{1}{e}$ as in the monopoly influence maximization problem.
- The authors also compute the price of anarchy $= 2$ (the number of nodes informed at any equilibrium is $\geq 1/2$ of the total number of nodes informed at the optimum).

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Influence maximization with competition: Goyal and Kearns (2012)

- They study the optimal allocation of a fixed budget over nodes under a general threshold diffusion process characterized by two functions:
  - function f of the proportion of informed neighbours (specifies the probability that the agent is informed);
  - function g of the share of neighbours buying A and B (specifies the probability that the agent adopts one of the two products given that he is informed);
- Price of anarchy $= 4$ when f concave and g linear.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Influence maximization with competition: Dubey, Garg and de Meyer (2014)

- Extension of the linear model of Richardson and Domingos (2002) to multiple firms.
- Firms simultaneously choose marketing expenditures at each node.
- The inclination of a consumer to buy from a given firm is a linear function of the number of neighbors buying from the firm and the relative marketing expenditures of the firm.
- Since the model is linear, the best response functions can be computed in polynomial time by solving systems of linear equations.
- Unique Nash equilibrium: firms spend either zero resources on a consumer, or an amount which depends on the effective cost of marketing expenditures of the firms.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Simulations on targeting

- In general, optimal targeting has no analytical solution, and hence numerical simulations can be used to assess the importance of different parameters on the diffusion of new products.
- Goldenberg, Libai and Muller (2001):
    - First work using agent based modeling to study new product diffusion in complex social networks.
    - They use a cellular automata model, where each consumer forms a cell activated according to a fixed automaton.
- For last 10 years, many papers use agent-based models to analyze the role of different characteristics of the seeds on diffusion in random and actual networks.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Simulations on targeting: Watts (2002)

- He studies conditions under which global cascades occur in random networks where diffusion follows a threshold rule:
  - analytical derivations of the critical values of connectivity under which global cascades occur in random networks,
  - running simulations to clarify the relation between the average connectivity (measured by the fixed probability that a link is formed in a random Erdös-Rényi network) and the percentage of nodes informed in the long run.
- Findings:
  - The percentage of node informed is low when the graph has low connectivity (agents are likely not to receive information from neighbors), and high when agents have so many neighbors that the threshold condition becomes very difficult to satisfy.
  - Global cascades or full spread of new products across the social network arise for intermediate values of the connectivity parameter.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Simulations on targeting: Dodds and Watts (2007)

- They distinguish between two types of agents: influentials (belonging to the top 10 % of the degree distribution; more connected than others) and imitators.

- They study the likelihood that a global cascade arises, depending on the identity of the initial seed.

- The identity of the initial seed does not matter much in the likelihood of a global cascade.

- However, the size of the global cascade depends on the identity of the seed (influentials trigger larger cascades).

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Simulations on targeting (continued)

- Goldenberg, Han, Lehmann and Hong (2009):
  - Contrary to Dodds and Watts (2009), nodes with high connectivity play an essential role in information diffusion.
  - They use an independent cascade model.
  - The simulations are based on an actual network from a Korean social networking site.

- Stonedahl, Rand and Wilensky (2010):
  - They run an algorithm to select an optimal seeding strategy based on some characteristics like degree, clustering, etc.
  - They consider four stylized networks: random, lattice, small world and preferential attachment, and one real network (a sample of Twitter users).
  - A seeding strategy based on degree performs rather well in all four stylized networks, but not in the actual network of Twitter users.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Simulations on targeting (continued)

- Stephen, Dover and Goldenberg (2010):
    - They emphasize the role played by two characteristics of the seed: connectivity measured by the degree, and activity measured by the number of times at which the agent is active.
    - They show that both characteristics play a role in the diffusion process.
- Libai, Muller and Peres (2013):
    - They consider competing firms, and study seeding strategies based on random targeting, targeting influential nodes and targeting influential experts (more likely to be believed by consumers).
    - They run simulations on 12 real world networks.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Outline

1. Introduction

2. Targeting in computer science and complex systems

3. Representing networks and centrality measures

4. Targeting by persuaders with extreme and centrist opinions

5. Analytical models of targeting in economics

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Matrix representation of a network

- A network is represented by a graph (N, g), where
  - $N = \{1, 2, ..., n\}$ set of nodes (agents, players, vertices)
  - $g = [g_{ij}]$ real-valued $n \times n$ matrix (adjacency matrix)
- g_{ij} - relationship between i and j (possibly weighted and/or directed), also referred to as a link ij or an edge
- We assume that graphs are simple, i.e., $g_{ii} = 0$ for all $i \in N$ (no loops).
- In what follows we consider an unweighted and undirected network:

$$g_{ij} = \begin{cases} 1 & \text{if there is a link between } i \text{ and } j \\ 0 & \text{otherwise,} \end{cases}$$

and $g_{ij} = g_{ji}$ for all $i, j \in N$.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Walks and paths

- How can one node be reached from another one in g?
  - Walk = sequence of links $i_1 i_2, \cdots, i_{K-1} i_K$ such that $g_{i_k i_{k+1}} = 1$ for each $k \in \{1, \cdots, K-1\}$
    (a node or a link may appear more than once)
  - Trail = walk in which all links are distinct
  - Path = trail in which all nodes are distinct
- Geodesic between two nodes is a shortest path between them.
- $d(i, j; g)$ = geodesic distance between i and j in g
  If there is a path between i and j in g, then
  $d(i, j; g)$ = the number of links in a shortest path between i and j

$$d(i, j; g) = \min_{\text{paths } P \text{ from } i \text{ to } j} \sum_{(k, l) \in P} g_{kl}.$$

If there is no path between i and j in g, we set $d(i, j; g) = \infty$.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Number of walks of a given length

$g^k = k$th power of g

$g^0 := \mathbb{I}$ with $\mathbb{I} = n \times n$ identity matrix, where

g_{ij}^k = number of walks of length k that exist between i and j in g.
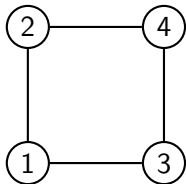
$$g = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \quad g^2 = \begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 2 & 0 & 0 & 2 \end{bmatrix} \quad g^3 = \begin{bmatrix} 0 & 4 & 4 & 0 \\ 4 & 0 & 0 & 4 \\ 4 & 0 & 0 & 4 \\ 0 & 4 & 4 & 0 \end{bmatrix}$$

E.g., walks of length 3 between 1 and 2: $(12, 24, 42)$, $(13, 34, 42)$, $(12, 21, 12)$, $(13, 31, 12)$.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Measuring centrality and prestige

- Given nodes that represent agents (players) and links that represent relationships between the agents (communication, influence, dominance ...), the following questions may appear:
  - How central is a node (player) in the network?
  - What is his position and prestige?
  - How influential is his opinion?
  - To which degree is the agent successful and powerful in collective decision making?
  - · · ·

- Centrality measures can be useful for the analysis of the information flows, bargaining power, infection transmission, influence, etc.

- Different centrality measures capture different aspects of centrality, and therefore can have highest values for different individuals.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Standard measures of centrality

- The concept of centrality captures a kind of prominence of a node in a network.
- Since the late 1940's a variety of different centrality measures that focus on specific characteristics inherent in prominence of an agent have been developed.
- Measures of centrality can be categorized into the following main groups (Jackson (2008)):
  - (1) Degree centrality - how connected a node is
  - (2) Closeness centrality - how easily a node can reach other nodes
  - (3) Betweenness centrality - how important a node is in terms of connecting other nodes
  - (4) Prestige- and eigenvector-related centrality - how important, central, or influential a node's neighbors are.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Degree centrality of a node

- The degree centrality (Shaw (1954), Nieminen (1974)):
  How connected is a node in terms of direct connections?
- The degree centrality $C_i^d(g)$ of node i in network g is given by

$$C_i^d(g) = \frac{d_i(g)}{n-1} = \frac{|N_i(g)|}{n-1} \in [0,1]$$

- Index of the node's communication activity: the more ability to communicate directly with others, the higher the centrality.
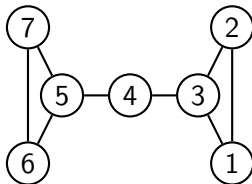
$C_i^d(g) = 0.5$ for $i \in \{3, 5\}$, $C_i^d(g) = 0.33$ for $i \notin \{3, 5\}$.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Closeness centrality of a node

- The closeness centrality (Beauchamp (1965), Sabidussi (1966)) is based on proximity: How easily can a node reach other nodes?
- The closeness centrality $C_i^c(g)$ of node i in network g is

$$C_i^c(g) = \frac{n-1}{\sum_{j \neq i} d(i,j;g)}$$

- Measure of the node's independence or efficiency: the possibility to communicate with others depends on a min. nb of intermediaries.
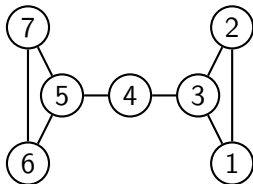
$C_4^c(g) = 0.60,\ C_3^c(g) = C_5^c(g) = 0.55,\ C_i^d(g) = 0.4$ otherwise.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Decay centrality of a node

- We introduce a decay parameter δ, with $0 < \delta < 1$, and consider the proximity between a given node and each other node weighted by the decay.

- The decay centrality of node i in network g is

$$C_i^{dc}(g, \delta) = \sum_{j \neq i} \delta^{d(i,j;g)}$$
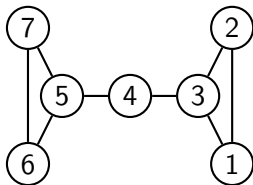
For $\delta = 0.5$, $C_i^{dc}(g, 0.5) = 2$ for $i \in \{3, 4, 5\}$, $C_i^{dc}(g, 0.5) = 1.5$ for $i \in \{1, 2, 6, 7\}$.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Betweenness centrality of a node (1/2)

- The betweenness centrality (Bavelas (1948), Freeman (1977, 1979)): How important is a node in terms of connecting other nodes?
- The betweenness centrality $C_i^b(g)$ of node i in network g is

$$C_i^b(g) = \frac{2}{(n-1)(n-2)} \sum_{k \neq j : i \notin \{k,j\}} \frac{P_i(kj)}{P(kj)}$$

  $P_i(kj) =$ number of geodesics between k and j containing $i \notin \{k,j\}$
  $P(kj) =$ total number of geodesics between k and j

- Index of the potential of a node for control of communication: the possibility to intermediate in the communications of others is of importance.
- If g is a star, then $C_i^b(g) = 1$ for i being the center and $C_i^b(g) = 0$ otherwise.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Betweenness centrality of a node (2/2)

$$C_i^b(g) = \frac{2}{(n-1)(n-2)} \sum_{k \neq j : i \notin \{k,j\}} \frac{P_i(kj)}{P(kj)}$$
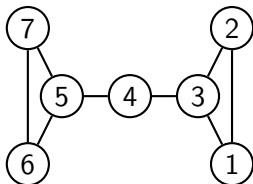
$C_4^b(g) = 0.60$

$C_3^b(g) = C_5^b(g) = 0.53$

$C_i^b(g) = 0$ for $i \in \{1, 2, 6, 7\}$

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Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Katz prestige

- Measures of centrality that are based on the idea that a node's importance is determined by the importance of its neighbours.
- The Katz prestige $C_i^{PK}(g)$ of node i in g is defined as

$$C_i^{PK}(g) = \sum_{j \neq i} g_{ij} \frac{C_j^{PK}(g)}{d_j(g)}$$

If j has more relationships, then i gets less prestige from being connected to j. This definition is self-referential.

- Calculating $C^{PK}(g)$ - finding the unit eigenvector of $\widetilde{g}$:

$$C^{PK}(g) = \widetilde{g} C^{PK}(g), \quad (\mathbb{I} - \widetilde{g}) C^{PK}(g) = \mathbf{0}$$

$\widetilde{g}$ - the normalized adjacency matrix g, $\widetilde{g}_{ij} = \frac{g_{ij}}{d_j(g)}$, $\widetilde{g}_{ij} = 0$ for $d_j(g) = 0$. $C^{PK}(g)$ - the $n \times 1$ vector of $C_i^{PK}(g)$, $i \in N$, $\mathbb{I}$ - the $n \times n$ identity matrix, $\mathbf{0}$ - the $n \times 1$ vector of 0's.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Eigenvector centrality

- If we do not normalize g, we get the eigenvector centrality $C^e(g)$ associated with g (Bonacich (1972)).

- The centrality of a node is proportional to the sum of the centrality of its neighbours.

$$\lambda C_i^e(g) = \sum_j g_{ij} C_j^e(g)$$

$$\lambda C^e(g) = g C^e(g)$$

and thus $C^e(g)$ is an eigenvector of g and λ is the corresponding largest eigenvalue of matrix g.

- The Katz prestige can be seen as a kind of eigenvector centrality with the network adjacency matrix being weighted.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Second prestige measure of Katz

- $C^{PK2}(g, a)$ = the second prestige measure of Katz (1953)
- Introducing an attenuation parameter a to adjust the measure for the lower 'effectiveness' of longer walks in a network.
- The prestige of a node is a weighted sum of the walks that emanate from it, and a walk of length k is of worth a^k, where $0 < a < 1$. The vector of prestige of nodes is

$$C^{PK2}(g, a) = ag\mathbf{1} + a^2 g^2 \mathbf{1} + \cdots + a^k g^k \mathbf{1} + \cdots$$

  where $\mathbf{1}$ is the $n \times 1$ vector of 1's.
- Each entry of the vector $g^k \mathbf{1}$ is the total number of walks of length k that emanate from each node; $g\mathbf{1}$ is the vector of degrees of nodes.
- For a sufficiently small, $C^{PK2}(g, a)$ is finite and

$$C^{PK2}(g, a) - ag C^{PK2}(g, a) = ag\mathbf{1}, \quad C^{PK2}(g, a) = (\mathbb{I} - ag)^{-1} ag\mathbf{1}.$$

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Bonacich centrality

- A two-parameter family of prestige measures which can be seen as a direct extension of $C^{PK2}(g, a)$.
- An agent can have some status which does not depend on its connections to others.
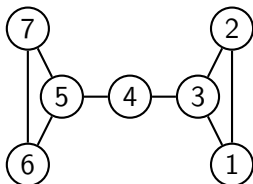- Bonacich centrality (Bonacich (1987)) is given by

$$C^B(g, a, b) = ag\mathbf{1} + abg^2\mathbf{1} + \cdots + ab^k g^{k+1}\mathbf{1} + \cdots$$

$$C^B(g, a, b) = (\mathbb{I} - bg)^{-1} ag\mathbf{1}$$

  where a and b are parameters, and b is sufficiently small.
  - b captures how the value of being connected decays with distance.
  - a captures the base value on each node.
  - For $b = 0$, $C^B(g, a, b)$ takes into account only walks of length 1 and reduces to $ad_i(g)$.
  - For $b > 0$, $C^B(g, a, b)$ takes into account more distant interactions.
  - $C^{PK2}(g, a)$ and $C^B(g, a, b)$ coincide when $a = b$.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Example (ctd)

Centrality measures ↓ Nodes →	1,2,6,7	3,5	4
Degree, Katz prestige	0.33	0.50	0.33
Closeness	0.40	0.55	0.60
Decay centrality, $\delta = 0.5$	1.5	2.0	2.0
Decay centrality, $\delta = 0.75$	3.1	3.7	3.8
Decay centrality, $\delta = 0.25$	0.59	0.84	0.75
Betweenness	0	0.53	0.60
Eigenvector centrality	0.47	0.63	0.54
Second Katz prestige, $a = 1/3$	3.1	4.3	3.5
Bonacich centrality, $a = 1$, $b = 1/3$	9.4	13	11
Bonacich centrality, $a = 1$, $b = 1/4$	4.9	6.8	5.4

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Outline

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

The point of departure and objectives

- Grabisch, Mandel, Rusinowska and Tanimura (2018):
  - extension of DeGroot (1974) by introducing two (stubborn) external players with extreme opinions who compete over the prominence in the network of non-strategic players
  - non-cooperative game played by the external players, the impact of the strategic aspects on the characterization of the key target
- Rusinowska and Taalaibekova (2019):
  - introducing the centrist persuader with a specific position that represents balance, neutrality, and equal combination of the extreme positions (e.g., three-candidate political or university elections)
  - studying the impact of the centrist persuader on the competition between the two extremist persuaders (opinion convergence and consensus reaching, characteristics of the targets)

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Motivation and applicability (1/2)

- The leading assumption – each persuader targets only one individual:
  - One target as a kind of outstanding and influencing master, a celebrity becoming an ambassador or a "face" of the brand
  - Example: Ambassador Marketing (a form of "word of mouth" marketing, where a person with specific influence or expertise participates in a brand's marketing strategy, by presenting the brand in a way that encourages the audience to purchase a product)

- Multi-target extension briefly discussed:
  - The persuaders can target more than one individual.
  - Convergence and consensus reaching when the persuaders target the same individuals.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Motivation and applicability (2/2)

- An extension to many more persuaders not so appealing in reality:
  - Using only iOS/Linux/Android software
    driving British/German/Japanese cars
    wearing American/French/Italian brands, ...
  - Example (football equipment manufacturers): Adidas, Nike and
    Puma. Each brand has a representative from the football world:
    Adidas has a contract with Leonel Messi, Nike – with Cristiano
    Ronaldo, and Puma – with Antoine Griezmann.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

The model with three persuaders (1/2)

- $N = \{1, \ldots, n\}$ society of individuals discussing a certain issue
- Each individual i has an initial opinion represented by $x_i(0) \in [0, 1]$ and interpreted as the intensity of i's opinion "yes" at time 0
- With no intervention, the individuals update their opinion as in DeGroot (1974), i.e., there is a $n \times n$ row-stochastic matrix of weights $W = [w_{ij}]$, where w_{ij} is the weight (trust) that i places on the opinion of j and:

$$\mathbf{x}(t) = W\mathbf{x}(t - 1) = W^t\mathbf{x}(0)$$

where $\mathbf{x}(t) = (x_1(t), \ldots, x_n(t))'$ is the opinion vector at time t
- W irreducible: $\forall \, i, j \in N \, \exists \, m(i, j)$ such that $w_{ij}^{(m(i,j))} > 0$

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

The model with three persuaders (2/2)

- The society is observed by three persuaders A, B and C with the fixed opinions 1, $\frac{1}{2}$ and 0, respectively.
- Each persuader chooses one individual in N (s_A, s_B and s_C) to form a link with in order to influence the opinion formation in the society.
- A, B and C are also characterized by possibly unequal (positive) persuasion impacts λ, γ and μ to adjust influence in the society.
- The same adjustment of influence holds for s_B and s_C being targeted by B and C, with impacts γ and μ, respectively.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Walks, cycles and weights of walks

- We associate to W a directed graph Γ on N such that there is an arc (i, j) from i to j (meaning that i listens to j) iff $w_{ij} > 0$.
- A walk from i to k is a sequence of nodes $(i_1 = i, i_2, \cdots, i_{j-1}, i_j = k)$ s.t. $w_{i_m i_{m+1}} > 0$ for each $m \in \{1, \cdots, j-1\}$.
- A cycle around i is a walk from i to i which does not pass through i between the starting and ending nodes.
- For any walk $p = (i_1, \ldots, i_j)$:
  $w(p) = $ "weight" of p measured according to W

$$w(p) := \prod_{m=1}^{j-1} w_{i_m, i_{m+1}}$$

- $C_i^j = $ set of cycles around i that pass through j, $B_i^j = $ set of walks that start from any node $\neq i$, end up in i and go through j

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

The influenceability and intermediacy

$$\text{For } i, j \in N, \ i \neq j, \quad c_i^j := \sum_{p \in \mathcal{C}_i^j} w(p), \quad b_i^j := \sum_{p \in \mathcal{B}_i^j} w(p)$$

- c_i^j = sum of weights of cycles around i that pass through j = probability for i to be reached by the influence of j before he receives the self-feedback (echo) of his own opinion

- $d_i c_i^j$ = *influenceability of individual i*, given that j is targeted by another persuader, with d_i being i's out-degree

- b_i^j = *intermediacy (influence, centrality) of j relatively to i* = sum of weights of walks to i that pass through j = sum of the probabilities for all agents other than i to be reached by the influence of j before this of i

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

The extended matrix of influence

One gets a new $(n+3) \times (n+3)$ influence matrix

$$
M_{\lambda,\gamma,\mu}(\mathbf{s}) = \begin{bmatrix} 1 & 0 & 0 & \mathbf{0} \\ 0 & 1 & 0 & \mathbf{0} \\ 0 & 0 & 1 & \mathbf{0} \\ \hline \Delta_{\lambda,\gamma,\mu}(\mathbf{s})E_{\lambda,\gamma,\mu}(\mathbf{s}) & & & \Delta_{\lambda,\gamma,\mu}(\mathbf{s})W \end{bmatrix}
$$

where

- the weight renormalization matrix $\Delta_{\lambda,\gamma,\mu}(\mathbf{s})$ is a diagonal matrix with elements $\frac{d_1}{d_1+\lambda\delta_{1,s_A}+\gamma\delta_{1,s_B}+\mu\delta_{1,s_C}}, \cdots, \frac{d_n}{d_n+\lambda\delta_{n,s_A}+\gamma\delta_{n,s_B}+\mu\delta_{n,s_C}}$

- the strategic influence matrix is $E_{\lambda,\gamma,\mu}(\mathbf{s}) = \left[\frac{\lambda}{d_{s_A}}e_{s_A} \quad \frac{\gamma}{d_{s_B}}e_{s_B} \quad \frac{\mu}{d_{s_C}}e_{s_C} \right]$

- e_i is the unit vector with coordinate 1 at i, δ is the Kronecker symbol: $\delta_{i,s_j} = 1$ if $i = s_j$ for $i \in N$, $s_j \in \{s_A, s_B, s_C\}$ and 0 otherwise.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

The updating rule under the persuaders' intervention

- The vector of opinions is extended to $\mathbf{x}(t) = [1 \ \frac{1}{2} \ 0 \ \mathbf{x}_N(t)]'$.
- The opinion updating rule is now determined by

$$\mathbf{x}(t+1) = M_{\lambda,\gamma,\mu}(\mathbf{s})\mathbf{x}(t) = (M_{\lambda,\gamma,\mu}(\mathbf{s}))^{t+1}\mathbf{x}(0)$$

which leads to the evolution law for the opinions of the individuals in N given by

$$\mathbf{x}_N(t+1) = \Delta_{\lambda,\gamma,\mu}(\mathbf{s})E_{\lambda,\gamma,\mu}(\mathbf{s}) \begin{bmatrix} 1 \\ \frac{1}{2} \\ 0 \end{bmatrix} + \Delta_{\lambda,\gamma,\mu}(\mathbf{s})W\mathbf{x}_N(t)$$

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Convergence of opinions

When the society gets a new persuader, the one with the centrist position, the opinion convergence is preserved in the society.

Proposition 1

For any initial vector of opinions $\mathbf{x}(0) := [1 \ \frac{1}{2} \ 0 \ \mathbf{x}_N(0)]'$, we have

$$\lim_{t \to +\infty} (M_{\lambda,\gamma,\mu}(\mathbf{s}))^t \begin{bmatrix} 1 & \frac{1}{2} & 0 & \mathbf{x}_N(0) \end{bmatrix}' = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \overline{\mathbf{x}}_N(\mathbf{s}) \end{bmatrix}'$$

where

$$\overline{\mathbf{x}}_N(\mathbf{s}) = [I - \Delta_{\lambda,\gamma,\mu}(\mathbf{s})W]^{-1} \Delta_{\lambda,\gamma,\mu}(\mathbf{s}) \left(\frac{\lambda}{d_{s_A}} e_{s_A} + \frac{\gamma}{2d_{s_B}} e_{s_B} \right)$$

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Consensus reaching (1/3)

If the three persuaders choose the same target, then the long run opinion in the society converges towards a consensus $\alpha \in [0, 1]$ which is determined by the three persuasion impacts.

Proposition 2

If $s_A = s_B = s_C$, then the individuals in N reach a consensus α given by

$$\alpha = \frac{2\lambda + \gamma}{2(\lambda + \gamma + \mu)}$$

In particular, if $\lambda = \mu$, then the consensus is $\alpha = \frac{1}{2}$.

Extension to a multi-target framework: This result holds independently of the number of the same targets, i.e., when the three persuaders can choose several targets for diffusion of information.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Consensus reaching (2/3)

$$\alpha = \frac{2\lambda + \gamma}{2(\lambda + \gamma + \mu)}$$

- When $\gamma \to 0$, the consensus is $\frac{\lambda}{\lambda + \mu}$ (Grabisch et al., 2018).
- When $\gamma \to +\infty$ and $\lambda, \mu \in \mathbb{R}_+$, the consensus tends to $\frac{1}{2}$.
- When $\lambda \to +\infty$ and $\gamma, \mu \in \mathbb{R}_+$, the consensus tends to 1.
- When $\mu \to +\infty$ and $\lambda, \gamma \in \mathbb{R}_+$, the consensus tends to 0.
- $\frac{2\lambda + \gamma}{2(\lambda + \gamma + \mu)} > \frac{\lambda}{\lambda + \mu}$ if and only if $\lambda < \mu$

  Under the same target, the presence of B always improves the situation of the weaker extreme persuader (moves the consensus closer to the opinion of the persuader with the smaller impact).

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Consensus reaching (3/3)

When persuaders A and C target the same individual, then the society ends up in a consensus, even if the centrist persuader targets another individual and independently of his own impact, but only if the extreme persuaders are equally strong. In this case, the consensus is equal to $\frac{1}{2}$.

Proposition 3

If $s_A = s_C$ and $\lambda = \mu$ then the individuals in N reach a consensus $\alpha = \frac{1}{2}$.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Game played by the persuaders (1/3)

- We consider a game $\mathcal{G}_{\lambda,\gamma,\mu}$ played between the three persuaders, with their set of strategies being N.
- Aim of each persuader: bringing the asymptotic average opinion as close as possible to the own opinion (1 for A, $\frac{1}{2}$ for B, 0 for C)
- The game-theoretic model is a system of minimization problems: given a strategy profile $\mathbf{s}=(s_A, s_B, s_C) \in N \times N \times N$

$$\pi^A_{\lambda,\gamma,\mu}(s_A, s_B, s_C) = \left(1 - \frac{1}{n}\mathbf{1}'\overline{\mathbf{x}}_N(\mathbf{s})\right)^2$$

$$\pi^B_{\lambda,\gamma,\mu}(s_A, s_B, s_C) = \left(\frac{1}{2} - \frac{1}{n}\mathbf{1}'\overline{\mathbf{x}}_N(\mathbf{s})\right)^2$$

$$\pi^C_{\lambda,\gamma,\mu}(s_A, s_B, s_C) = \left(\frac{1}{n}\mathbf{1}'\overline{\mathbf{x}}_N(\mathbf{s})\right)^2$$

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Game played by the persuaders (2/3)

- We denote the aggregate opinion formed in the society by

$$\tilde{x}_N(\mathbf{s}) := \mathbf{1}'\overline{\mathbf{x}}_N(\mathbf{s}) = \sum_{i \in N} \overline{x}_i(\mathbf{s})$$

where $\overline{\mathbf{x}}_N(\mathbf{s}) = [I - \Delta_{\lambda,\gamma,\mu}(\mathbf{s})W]^{-1}\Delta_{\lambda,\gamma,\mu}(\mathbf{s})\left(\frac{\lambda}{d_{s_A}}e_{s_A} + \frac{\gamma}{2d_{s_B}}e_{s_B}\right)$

Theorem 1

The payoffs of persuaders A, B and C, given the strategy profile (s_A, s_B, s_C) are as follows:

(i) If $s_A = s_B = s_C = i$, then

$$\tilde{x}_N(i,i,i) = \frac{n(2\lambda + \gamma)}{2(\lambda + \gamma + \mu)}$$

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Game played by the persuaders (3/3)

Theorem 1 (continued)

(ii) If $s_A = s_C = i$ and $s_B = k \neq i$, then:
$$\tilde{x}_N(i, k, i) = \frac{2\lambda(\gamma b_k^i + d_k c_k^i n) + \gamma((\lambda + \mu)b_i^k + d_i c_i^k n)}{2(\gamma d_i c_i^k + (\lambda + \mu)(d_k c_k^i + \gamma))}$$

(iii) If $s_A = k$ and $s_B = s_C = i \neq k$, then:
$$\tilde{x}_N(k, i, i) = \frac{2\lambda((\gamma + \mu)b_i^k + d_i c_i^k n)) + \gamma(\lambda b_k^i + d_k c_k^i n)}{2(\lambda d_i c_i^k + (\gamma + \mu)(d_k c_k^i + \lambda))}$$

(iv) If $s_A = s_B = i$ and $s_C = k \neq i$, then:
$$\tilde{x}_N(i, i, k) = \frac{(2\lambda + \gamma)(\mu b_k^i + d_k c_k^i n)}{2(\mu d_i c_i^k + (\lambda + \gamma)(d_k c_k^i + \mu))}$$

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Equal persuasion impacts (1/2)

- Let $\lambda = \gamma = \mu$. We use the simplified notations $\mathcal{G}_\lambda$, π_λ^A, π_λ^B, and π_λ^C.

Theorem 2

A profile of strategies (i, i, i) is an equilibrium of the game $\mathcal{G}_\lambda$ if and only if for all $k \in N \setminus \{i\}$

$$b_k^i - 2b_i^k \geq \frac{n}{\lambda} \left(d_i c_i^k - d_k c_k^i \right)$$

- In the extended three-persuader model, the condition to reach the equilibrium (i, i, i) requires more from the intermediacy of i over k than under the two extreme persuaders: i must be even more influential (central) among others to compensate the impact of the two other persuaders.

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Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Equal persuasion impacts (2/2)

- As the number of individuals in the society increases, the relative importance of intermediacy compared to influenceability goes down.
- Conversely, the relative importance of intermediacy goes up with the level of λ, the impact of the persuaders.

Proposition

(i) (i, i, i) is an equilibrium of $\mathcal{G}_\lambda$ as $\lambda \to 0$ if and only if for all $k \in N$
$$d_k c_k^i \geq d_i c_i^k$$

(ii) (i, i, i) is an equilibrium of $\mathcal{G}_\lambda$ as $\lambda \to +\infty$ if and only if for all $k \in N$

$$b_k^i \geq 2b_i^k$$

Unequal persuasion impacts: If $\gamma, \mu > 0$ are fixed and the impact λ of A is sufficiently large, then $\mathcal{G}_{\lambda, \gamma, \mu}$ has only equilibria in mixed strategies.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Examples (1/2)

- Perfectly symmetric society: $d_i = d_k$, $c_i^k = c_k^i$, $b_i^k = b_k^i$.
  While (i, i, i) was always an equilibrium in the model with two persuaders, the equilibrium condition does not hold here, but the game has non-symmetric NE. E.g., for $n = 3$ there exist 12 NE: 6 profiles (i, j, i) with $i \neq j$ leading to the consensus $\frac{1}{2}$, 6 profiles (i, j, k) with $i \neq j \neq k$ and $i \neq k$ for which $\tilde{x}_N(i, j, k) = \frac{3}{2}$ but the individual opinions are different from each other.

- Star society: $d_i = n - 1$, $d_k = 1$ for all $k \neq i$.
  $c_k^i = 1$, $c_i^k = \frac{1}{n-1}$, $b_k^i = n - 1$, $b_i^k = 1$.
  The equilibrium condition for (i, i, i) is always satisfied (unless $n < 3$). E.g., NE for $n = 3$ are $(2, 2, 2)$ and 16 non-symmetric NE: 6 profiles (i, j, i) with $i \neq j$, 6 profiles (i, j, k) with $i \neq j \neq k$ and $i \neq k$, and 4 other NE in which two "neighbouring" persuaders target the center, i.e., $(1, 2, 2)$, $(2, 2, 1)$, $(2, 2, 3)$ and $(3, 2, 2)$.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Examples (2/2)

- Directed circle: $d_i = 1$ for every $i \in N$, for any $k \neq i$, $c_i^k = c_k^i = 1$, $b_k^i = l(k, i)$, $b_i^k = l(i, k)$, where $l(k, i)$, $l(i, k)$ are the lengths of the (unique) shortest walk from k to i, and from i to k, respectively. If $\lambda = \gamma = \mu$ then no pure strategy symmetric NE exists, as in the case with two persuaders. Similarly, non-symmetric equilibria do not exist.

- Line network: Two types of nodes: $d_1 = d_n = 1$ and $d_j = 2$ for each $j \neq 1, n$. No symmetric equilibrium (i, i, i) exists, but the game admits non-symmetric NE, e.g., for $n = 4$: $(2, 3, 2)$, $(3, 2, 3)$, $(2, 4, 2)$ and $(3, 1, 3)$, all leading to the consensus $\frac{1}{2}$. Under equilibrium the extreme persuaders target one of the "middle" individuals while the centrist persuader chooses either another "middle" individual or the "end" individual which is not the neighbour of the extreme persuaders' target.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Conclusion (1/2)

- Consensus can emerge if the three persuaders target the same individual. Additionally under the equal impacts, the centrist one has no effect on the social opinion, but the outcome is ideal for him.

- The existence of a pure strategy NE depends on the network structure (e.g., no NE in circular networks).

- A symmetric equilibrium in pure strategies emerges when the persuaders exert an equal impact. It is characterized by two features of the targets: influenceability and intermediacy (centrality).

- With three persuaders, the relative influence of a potential target must be at least twice higher than the one of any other individual in the network (the persuaders are demanding higher centrality from the target to compensate the impact of the additional persuader).

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Conclusion (2/2)

- Influenceability gains importance versus intermediacy as the size of the network grows or the impact of the persuaders decreases.

- When the persuasion impacts are unequal, the high-impact persuader aims at ensuring preeminence on the network by increasing his centrality and diminishing the influenceability of his opponents' target.

- The low-impact persuaders seek to keep a minimal level of influence by hiding their target from the opponent's impact.

- A growing number of the persuaders does not affect too much the game when the persuaders are weak.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Outline

1. Introduction

2. Targeting in computer science and complex systems

3. Representing networks and centrality measures

4. Targeting by persuaders with extreme and centrist opinions

5. Analytical models of targeting in economics

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Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting and pricing in economics and operations research

- Many studies of targeting in the economics literature characterize targets by existing and new centrality measures.
- Ballester et al. 2006; Banerjee et al. 2013, 2017; Candogan et al. 2012, Tsakas 2014, 2017, Bimpikis et al. 2016, Galeotti et al. 2017, Demange 2017, Grabisch et al. 2018, Rusinowska and Taalaibekova 2019, ...
- Consumption externalities:
  - Monopoly pricing with consumption externalities: Candogan et al. 2012, Bloch and Querou 2013, Fainmesser and Galeotti 2013, ...
  - Competitive pricing in social networks: Banerji and Dutta 2009, Galeotti 2010, Jullien 2011, ...
- We will briefly discuss some of these works.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Consumption externalities

- Network externalities = agents' consumption of a good is affected by the number of agents consuming the same good (Katz and Shapiro 1985, Farrell and Saloner 1985).

- Network externalities arise in many environments, e.g., in telecommunications (agents benefit from others using the same communication device), in the software industry (development of application is driven by the number of users), etc.

- Early works model the consumption externality as global phenomenon rather than network based feature (the valuation of consumers is defined as a function of the total number of users).

- More recent models study consumption externalities based on a given social network.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Monopoly pricing with consumption externalities: Candogan et al. 2012, Bloch and Querou 2013 (1/5)

Candogan, Bimpikis and Ozdaglar (*Operations Research* 2012)
Bloch and Querou (*Games and Economic Behavior* 2013)

- The society consists of a set of n agents embedded in a social network represented by the adjacency matrix $G = [g_{ij}]$, where g_{ij} measures the influence of agent j on i's consumption, $g_{ij} \geq 0$, $g_{ii} = 0$ for all i.

- A monopolist introduces a divisible good in the market and chooses a vector **p** of prices, $(p_1, \ldots, p_n)$, from the set of pricing strategies **P**, where p_i is the price offered by the monopolist to agent i for one unit of the good.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Monopoly pricing with consumption externalities: Candogan et al. 2012, Bloch and Querou 2013 (2/5)

- The utility of agent i is given by the following quadratic expression:

$$u_i(q_1, \ldots, q_n, p_i) = a_i q_i - \frac{1}{2} b_i q_i^2 + q_i \sum_j g_{ij} q_j - p_i q_i$$

where $q_i \geq 0$ is the amount of the good that agent i consumes.

- u_i increases with the consumption q_j of direct neighbors j.
- The positive externality implies that, given any vector of prices **p**, the consumption levels are computed as the equilibrium of a non-cooperative games played among consumers.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Monopoly pricing with consumption externalities: Candogan et al. 2012, Bloch and Querou 2013 (3/5)

- Two-stage pricing-consumption game:
  - Stage 1 (Pricing): The monopolist chooses the pricing strategy $(p_1, \ldots, p_n)$ in order maximize profits:

  $$\Pi = \sum_i (p_i - c) q_i$$

  where c denotes the marginal cost of producing a unit of the good, and q_i denotes the amount of the good that i purchases in the second stage of the game.
  - Stage 2 (Consumption): Agent i chooses to purchase q_i units of the good as to maximize his utility given the prices chosen by the monopolist and $\mathbf{q}_{-i}$.
- We look for the subgame-perfect equilibria of the two-stage pricing-consumption game.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Monopoly pricing with consumption externalities: Candogan et al. 2012, Bloch and Querou 2013 (4/5)

- Two assumptions:
  - (A1) $2b_i > \sum_j g_{ij}$ for all i (the optimal consumption level of each agent is bounded)
  - (A2) $a_i > c$ for all i (when the monopolist sets prices optimally, all consumers purchase a positive amount of the good)

- Under (A1) and (A2), the optimal price vector is given by:

$$\mathbf{p} = \mathbf{a} - (\mathbf{\Lambda} - \mathbf{G}) \left(\mathbf{\Lambda} - \frac{\mathbf{G} + \mathbf{G}^T}{2} \right)^{-1} \frac{\mathbf{a} - c\mathbf{1}}{2}$$

where Λ is a diagonal matrix with terms b_i on the diagonal, and $\mathbf{a}$ is the vector of a_i's.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Monopoly pricing with consumption externalities: Candogan et al. 2012, Bloch and Querou 2013 (5/5)

- Corollary: Under (A1) and (A2), when the matrix G is symmetric (influence is undirected), the monopoly sets a uniform price:

$$\mathbf{p} = \frac{\mathbf{a} + c}{2}$$

  i.e., the optimal prices do not depend on the network structure.
- The authors obtain an alternative characterization of the optimal prices (using the Bonacich centrality).
- The optimal price consists of three components:
  - (i) a nominal term that is independent of the network structure,
  - (ii) a discount term proportional to the influence that the agent exerts over the network (quantified by the agent's Bonacich centrality),
  - (iii) a markup term proportional to the influence that the network exerts on the agent.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Monopoly pricing with consumption externalities: Fainmesser and Galeotti (2013)

- They analyze discriminatory pricing in the quadratic utility model when the monopoly only knows the degree distribution and degrees.
- Consumers have different in- and out-degrees, where:
  - the in-degree of i = the number of agents influenced by i
  - the out-degree of i = the number of agents that influence i.
- They compute the optimal price of a monopoly which discriminates according to out-degrees or in-degrees.
- Prices are monotonic:
  - a consumer with higher out-degree pays more
  - a consumer with higher in-degree pays less.
- Equilibrium consumption increases:
  - in the out-degrees when the monopolist discriminates on out-degrees
  - both in the in- and out-degrees when the monopolist discriminates on in-degrees.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Competitive pricing in social networks: Jullien (2011)

Jullien (*American Economic Journal: Microeconomics 2011*)

- Consumers are divided into groups, with a fixed matrix of external effects across groups.
- Two firms compete by setting prices for each group of consumers.
- The firms choose their prices sequentially, with firm A moving as the leader and firm B as the follower.
- The author shows that the follower can target some consumer group to which it offers a low price, while raising the price of another consumer group which benefits highly from cross-group externalities with the first group.
- By choosing optimally its target group, the follower can conquer the market even when it is less efficient than the leader.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Competitive pricing in social networks: Galeotti (2010)

Galeotti (*International Economic Review 2010*)

- A model of duopoly pricing with social interactions.

- A different role of the social network: instead of capturing consumption externalities, the network describes how consumers learn information about prices collected by other consumers.

- Prices and profits are not necessarily monotonic in the level of connectivity of the network.

- Consumers with more neighbors get more info but their incentive to search goes down. The two effects work in opposite direction:
  - High values of the search cost: the 1st effect dominates, the market becomes more competitive, reducing equilibrium prices and profits.
  - Low values of the search cost: the 2nd effect dominates, the market becomes less competitive, raising equilibrium prices and profits.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics:
Ballester, Calvo-Armengol and Zenou (2006) (1/5)

Ballester, Calvo-Armengol and Zenou (*Econometrica 2006*)

- They study a model in which individuals located in a network choose actions (criminal activities) which affect the payoffs of other individuals within the network.

- Which individuals should be eliminated from the network if the objective is to minimize crime?

- A noncooperative (network) game with local payoff complementarities.

- Each player $i = 1, \ldots n$ selects an effort $x_i \geq 0$ and gets the utility:
$u_i(x_1, \cdots, x_n) = \alpha_i x_i + \frac{1}{2} \sigma_{ii} x_i^2 + \sum_{j \neq i} \sigma_{ij} x_i x_j$
which is strictly concave in own effort ($\frac{\partial^2 u_i}{\partial x_i^2} = \sigma_{ii} < 0$).

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics:
Ballester, Calvo-Armengol and Zenou (2006) (2/5)

$$u_i(x_1, \cdots, x_n) = \alpha_i x_i + \frac{1}{2}\sigma_{ii}x_i^2 + \sum_{j \neq i} \sigma_{ij}x_i x_j$$

- We set $\alpha_i = \alpha$ and $\sigma_{ii} = \sigma < 0$ for all i.
- Bilateral influences are captured by the cross-derivatives:
  $\frac{\partial^2 u_i}{\partial x_i \partial x_j} = \sigma_{ij}$ for $i \neq j$.
- If $\sigma_{ij} > 0$, then efforts of i and j are strategic complements from i's perspectives (an increase in j's effort triggers a upwards shift in i's response).
- If $\sigma_{ij} < 0$ then efforts of i and j are strategic substitutes from i's perspectives.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics:
Ballester, Calvo-Armengol and Zenou (2006) (3/5)

- Let $\sum = [\sigma_{ij}]$ be the square matrix of cross-effects, also used for denoting the simultaneous move n-player game with the strategy space $\mathbb{R}_+$. It is additively decomposed into:

$$\sum = -\beta \mathbf{I} - \gamma \mathbf{U} + \lambda \mathbf{G}$$

where $\mathbf{I}$ is the n-square identity matrix, $\mathbf{U}$ is the n-square matrix of ones, $\mathbf{G} = [g_{ij}]$ is a zero-diagonal nonnegative adjacency matrix of the network g of relative payoff complementarities across pairs, $\beta > 0$, $\gamma \geq 0$, $\lambda > 0$.

- Bilateral influences result from the combination of an idiosyncratic effect, a global interaction, and a local interaction.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics:
Ballester, Calvo-Armengol and Zenou (2006) (4/5)

- Consider a network g with adjacency n-square matrix $\mathbf{G} = [g_{ij}]$ and a scalar $a \geq 0$ such that $[m_{ij}(g, a)] = [\mathbf{I} - a\mathbf{G}]^{-1}$ is well defined and non-negative, where $m_{ij}(g, a) = \sum_{k=0}^{\infty} a^k g_{ij}^{[k]}$ and

$$b_i(g, a) = \sum_{j=1}^{n} m_{ij}(g, a)$$

measures total number of walks starting in i.

- $\mathbf{b}(g, a)$ is obtained from the Bonacich centrality by an affine transformation:

$$\mathbf{b}(g, a) = \mathbf{1} + C^{PK2}(g, a)$$

where $C^{PK2}(g, a)$ is the second prestige of Katz.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics:
Ballester, Calvo-Armengol and Zenou (2006) (5/5)

- For the matrix $[\beta\mathbf{I} - \lambda\mathbf{G}]^{-1}$ well defined and nonnegative, the game $\sum$ has a unique Nash equilibrium (NE).
- The NE action of a player is proportional to his Bonacich centrality.
- In order to decrease aggregate effort, the key player (to target) is identified by an intercentrality measure which takes into account i's centrality and his contribution to the centrality of the others:

$$c_i(g, a) = \frac{b_i(g, a)^2}{m_{ii}(g, a)}$$

- While the Bonacich centrality of i counts the number of walks in g that stem from i, the intercentrality counts the total number of such walks that hit i: it is the sum of i's Bonacich centrality and i's contribution to every other player's Bonacich centrality.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics:
Galeotti and Goyal (2009) (1/3)

Galeotti and Goyal (*The RAND Journal of Economics* 2009)

- They model networks in terms of degree distributions and study influence strategies in the presence of local interaction.
- They consider two groups of players, where the first group M (the firm) chooses a strategy with the aim of influencing members of the second group N to choose certain actions.
- Optimal influence strategies will target individuals with low or high connectivity, depending on the nature of the interaction.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics:
Galeotti and Goyal (2009) (2/3)

- The firm only knows the degree distribution of consumers in the network and consumers' degrees.

- A targeted marketing strategy assigns a different advertising expenditure on each consumer on the basis of his degree.

- Each consumer: out-/indegree (number of influencing/influenced agents).

- The profit of reaching a consumer with (out)degree k and spending marketing expenditures x is represented by $\phi_k(x)$.

- $\phi_k(x)$ exhibits increasing (decreasing) marginal returns in degree if, for any $x > x'$, $\phi_{k+1}(x) - \phi_{k+1}(x') > (<)\phi_k(x) - \phi_k(x')$.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics:
Galeotti and Goyal (2009) (3/3)

- Two models:
  - one step diffusion version of the independent cascade model with $\phi_k(x) = 1 - (1 - x)^{k+1}$
  - one step version of the threshold model with $\phi_k(x) = (1 - x)xk^{1-\beta}$ with $\beta > 1$.

- Monotonicity of the optimal targeting policy: if $\phi_k(x)$ exhibits increasing/decreasing marginal returns to degree (threshold model/ cascade model), nodes with higher/lower degree receive more advertising.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics: Campbell (2013)

- Campbell (*American Economic Review* 2013)
- The techniques for large random graphs to study monopoly pricing and targeting.
- A monopoly sets a price, consumers decide according to a random valuation whether to buy the product (information flows to others) or not (information stops, the network breaks into smaller components).
- There exists a critical price P^{crit} such that a giant component emerges only when the price $P < P^{crit}$.
- The firm's targeting strategy given a fixed price P:
  if $P > P^{crit}$, the firm should target (= offer the product to) the consumer with the highest degree;
  if $P < P^{crit}$, it should target the consumer with the smallest degree.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics: Banerjee, Chandrasekhar, Duflo and Jackson (2013, 2019) (1/2)

Banerjee, Chandrasekhar, Duflo and Jackson (*Science* 2013, forthcoming in the *Review of Economic Studies* 2019)

- Identifying the most influential agents in a gossip process.
- Players generate some information about particular people, which is then stochastically passed from neighbour to neighbour.
- Diffusion centrality of a node i in a network with an adjacency matrix g, a probability p of passing the information and T iterations is defined as the ith entry of the vector

$$DC(g, p, T) = \big[\sum_{t=1}^{T} (pg)^t \big] \mathbf{1}$$

  which measures how extensively the information spreads from i.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics: Banerjee, Chandrasekhar, Duflo and Jackson (2013, 2019) (2/2)

- Consider T iterations of information passing from a single initially informed node i where at each iteration every informed node tells each neighbor with probability p. The diffusion centrality of node i then corresponds to the expected total number of times that all nodes taken together get the information.
- If $T = 1$, diffusion centrality is proportional to degree centrality.
- As $T \to +\infty$, it becomes proportional to either Katz-Bonacich centrality or eigenvector centrality, depending on whether p is smaller or greater than the inverse of the first eigenvalue of the adjacency matrix g.
- In the intermediate region of T, diffusion centrality differs from existing measures.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics: Tsakas (2017a) (1/2)

- There are two firms A and B that produce products A and B, respectively.
- The firms introduce their products simultaneously to a new market using fixed advertising budgets.
- The quality of the two products is ex ante uncertain.
- The firm A allocates its budget uniformly to the whole population, thus making every agent aware of its product.
- The firm B (the firm) allocates its budget, i.e., targets a subset $T \subset N$ of the population to advertise the product, where the cardinality t of T is given exogenously.
- The author provides a condition that characterizes the optimal targeting strategy for any network structure.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics: Tsakas (2017a) (2/2)

- The geodesic distance in g between a group of agents T and an agent $j \notin T$:
$$d(T, j; g) = \min_{i \in T} d(i, j; g)$$

- Let $T^c = N \setminus T$. Consider a decay parameter $\delta \in (0, 1)$. The group decay centrality of set T in g is defined as $\sum_{j \in T^c} \delta^{d(T,j;g)}$.

- The optimal strategy is to target a set that maximizes some expression involving the group decay centrality and p, where p determines the probability of product B being perceived to be of higher quality than product A.

- E.g., when the firm can target a single agent, the optimal targeting strategy is to target an agent with the maximum decay centrality.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics: Tsakas (2017b)

Tsakas (*Journal of Economic Behavior and Organization* 2017)

- Targeting agents in diffusion based on social imitation.
- Agents choose between two alternative actions and revise their choices by imitating the most successful past action of their neighbours.
- The optimal targeting strategy of a planner depends on the likelihood of the action being more successful than the alternative action and on the planner's patience.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics: Demange (2017)

Demange (*Games and Economic Behavior* 2017)

- The optimal targeting strategies of a planner (a governmental agency, a firm) aiming to increase the aggregate action of a population.

- The agents interact through a social network and react to their exposure to neighbours' actions.

- The reaction function (e.g., best response in a strategic game, mechanical influence in a contagion disease, mimetic behaviour) is increasing in exposure, resulting in complementarity in actions.

- When the reaction function is linear, the optimal planner's strategies are characterized by well-known centralities indices.

- When it is concave or convex, the optimal strategies depend on the impacts and the pattern of agents' attentions.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics:
Galeotti, Golub and Goyal (2017)

- Individuals interact strategically with their network neighbours.
- A planner's goal: maximizing welfare or minimizing volatility.
- The planner can shape incentives to achieve his goal.
- A method of decomposing any potential intervention into principal components determined by the network.
- A particular ordering of principal components describes the planner's priorities across a range of network intervention problems.
- If actions are strategic complements, the optimal intervention changes all agents' incentives in the same direction in proportion to their eigenvector centralities.
- If actions are strategic substitutes, the optimal intervention moves neighbours' incentives in opposite directions.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics:
Goyal, Heidari and Kearns (2019)

Goyal, Heidari and Kearns (*Games and Economic Behavior* 2019)

- See also Goyal and Kearns (2012).
- Competing firms allocate budgets to implant their respective products in some set of initial users.
- The final market shares are determined by a competitive contagion dynamics.
- The authors focus on the efficiency of the equilibrium strategies.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics:
Bimpikis, Ozdaglar and Yildiz (2016) (1/2)

Bimpikis, Ozdaglar and Yildiz (*Operations Research* 2016)

- A game-theoretic model where two strategic competing firms seek to optimally allocate their marketing budgets to maximize the product awareness resulting from social influence.

- Agents receive a message that can either be one of the two brands or the status quo.

- The probabilities of the different messages are determined by the agents' awareness levels for each of the two brands.

- The authors provide a characterization of the optimal targeted advertising strategies and highlight their dependence on the underlying social network structure.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

Targeting in economics:
Bimpikis, Ozdaglar and Yildiz (2016) (2/2)

- They provide conditions under which it is optimal for the firms to asymmetrically target a subset of the individuals.
- At equilibrium firms invest inefficiently high in targeted advertising and the extent of the inefficiency is increasing in the centralities of the agents they target.
- Agent i's absorption centrality = expected number of visits to node i before absorption at either node $n + 1$, $n + 2$, or $n + 3$ (corresponding to the two firms and the status quo) for a random walk started at a node other than i.
- The limiting average awareness levels are a weighted sum of the firms' marketing efforts where the weights are given by the absorption centralities of the agents.

Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

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Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

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Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

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Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

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Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

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Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

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Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

References (7/9)

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Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

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Introduction
Targeting in computer science and complex systems
Representing networks and centrality measures
Targeting by persuaders with extreme and centrist opinions
Analytical models of targeting in economics

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