

Information Design in Non-atomic Games: Computation, Repeated Setting and Experiment

Workshop on Algorithmic Game Theory, Mechanism Design, and Learning
Politecnico di Torino
8 Nov 2022



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Xtelligent

Joint work with: Yixian Zhu
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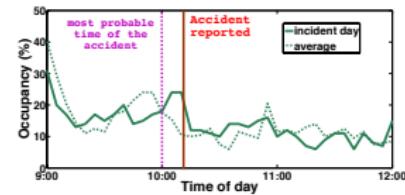
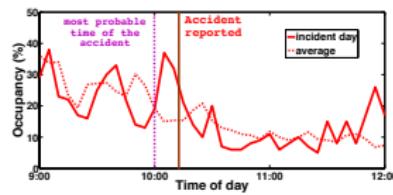
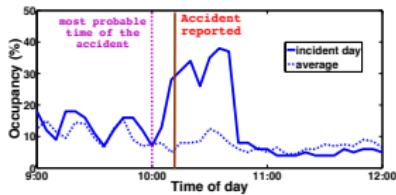
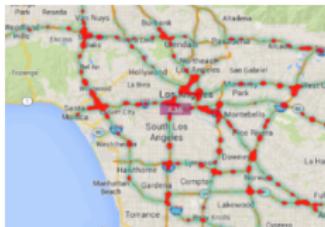
Routing Decisions in Urban Traffic Networks

- resource sharing among **non-atomic** agents
- uncertain and unpredictable environment



Routing Decisions in Urban Traffic Networks

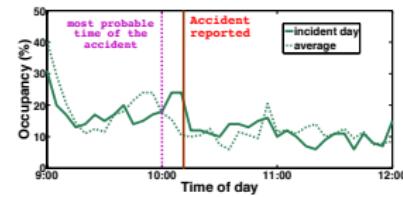
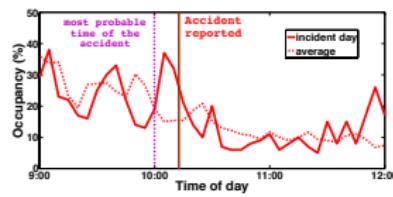
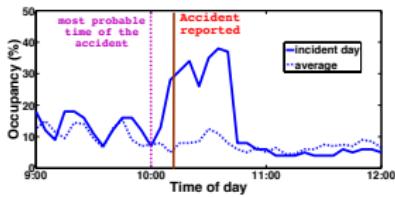
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Routing Decisions in Urban Traffic Networks



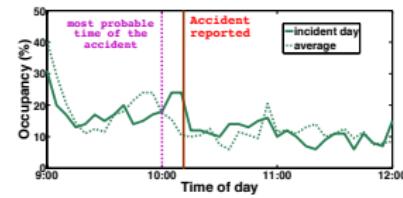
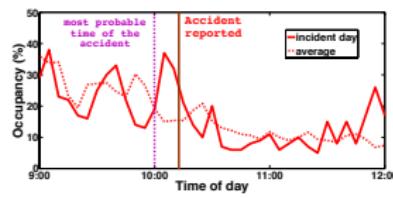
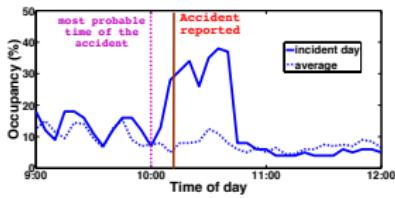
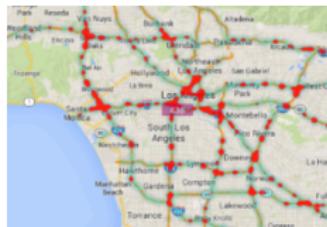
- resource sharing among **non-atomic** agents
- uncertain and unpredictable environment
- repeated interactions



Routing Decisions in Urban Traffic Networks



- resource sharing among **non-atomic** agents
- uncertain and unpredictable environment
- repeated interactions



Information Design Setting

- Full info not optimal [AMMO:18]

[AMMO:18] Acemoglu, Makhdoumi, et al, "Informational Braess paradox: The effect of information on traffic congestion", *Operations Research*, 2018

Information Design Setting

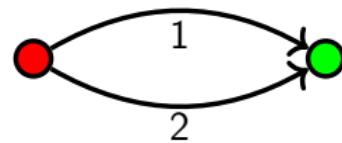
- Full info not optimal [AMMO:18]

$$\min_{\text{signal}} E[\text{cost}(\text{signal})]$$



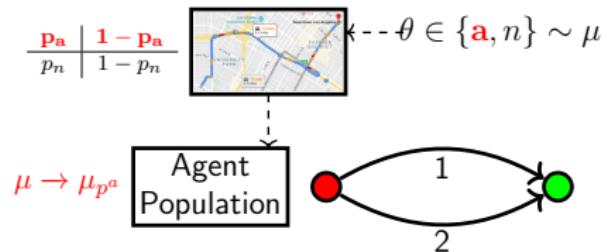
$$\theta \in \{a, n\} \sim \mu$$

Agent Population



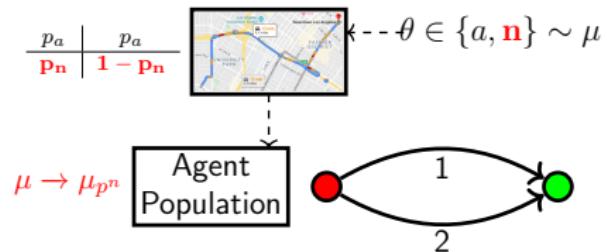
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Information Design Setting

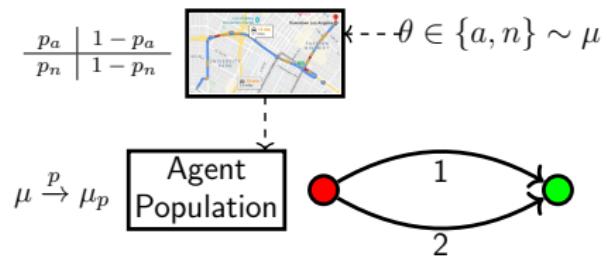
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Information Design Setting

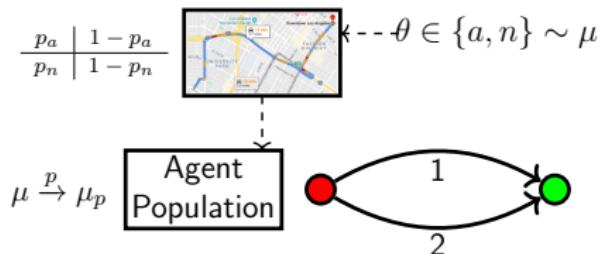
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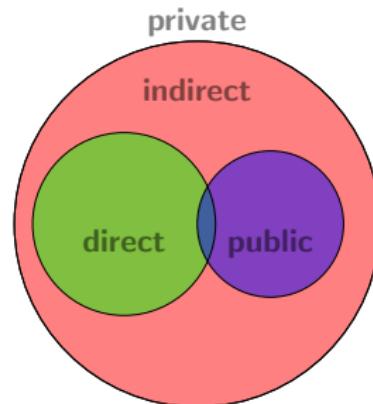
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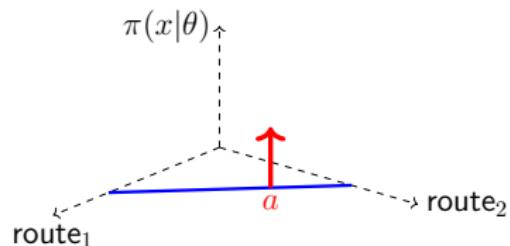


Signal Types

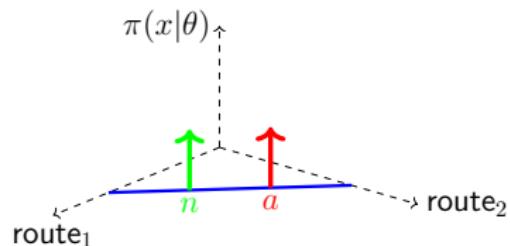
- private vs. public
- direct vs. indirect



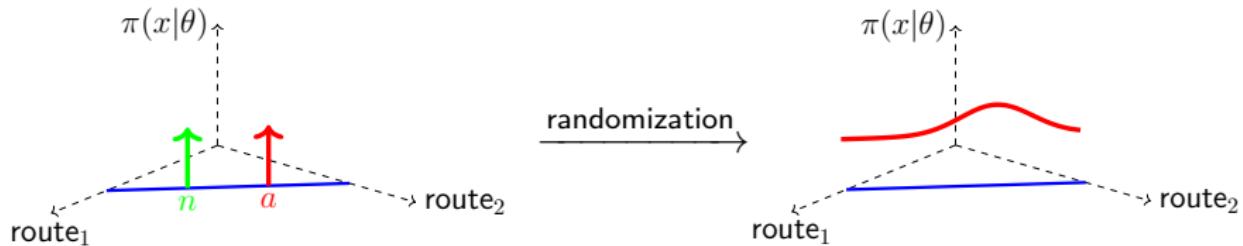
Randomization



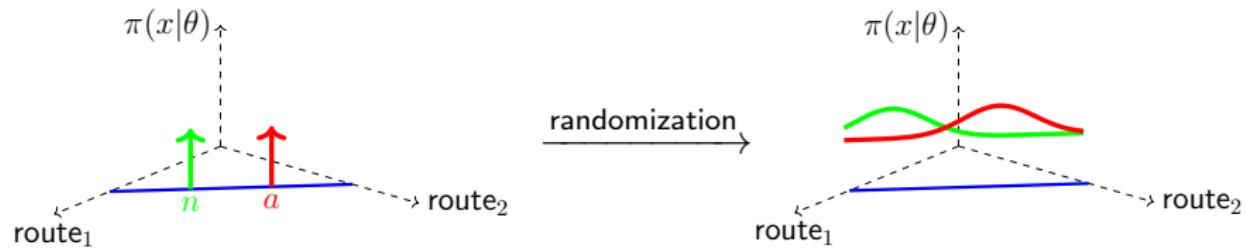
Randomization



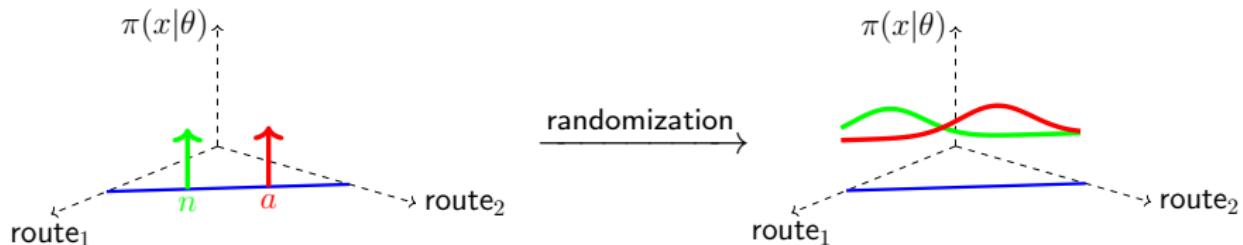
Randomization



Randomization



Randomization



$$\mu \sim \theta \xrightarrow{\pi} x \rightarrow \text{posterior}_i(x, \theta) \propto x_i \pi(x|\theta) \mu(\theta)$$

- μ and π are public knowledge

Information Design

$$\min_{\pi} \sum_{\theta,i} \int x_i \ell_i^\theta(x_i) \pi(x|\theta) \mu(\theta) dx \quad \text{s.t. obedience constraint}$$

- ℓ : link latency
- Existing works limited to stylized settings: [DKR:17], [TT:19]

[DKR:17]: S. Das, E. Kamenica, and R. Mirka, "Reducing congestion through information design", *Allerton 2017*

[TT:19]: H. Tavafoghi, D. Teneketzis, "Strategic information provision in routing games", 2019

Information Design

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Obedience Constraint

Recommendation i is *obeyed* if:

$$\sum_{\theta} \int (\ell_i^\theta(x_i) - \ell_j^\theta(x_j)) \underbrace{\text{posterior}_i(x, \theta)}_{\propto x_i \pi(x|\theta) \mu(\theta)} dx \leq 0 \quad \forall j$$

Information Design

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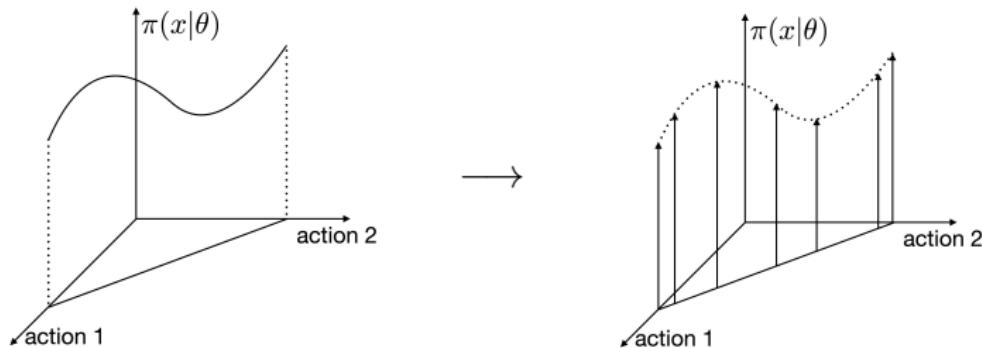
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infinite dimensional (non-convex) optimization!

Support Discretization



Support Discretization



$$\sum_{\theta} \int (\ell_i^{\theta}(x_i) - \ell_j^{\theta}(x_j)) x_i \pi(x|\theta) \mu(\theta) dx \rightarrow \sum_{k, \theta} (\ell_i^{\theta}(x_i^{(k)}) - \ell_j^{\theta}(x_j^{(k)})) x_i^{(k)} \pi(x^{(k)}|\theta) \mu(\theta)$$

Support Discretization



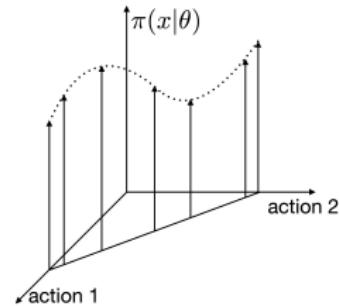
$$\sum_{\theta} \int (\ell_i^{\theta}(\mathbf{x}_i) - \ell_j^{\theta}(\mathbf{x}_j)) \mathbf{x}_i \pi(\mathbf{x}|\theta) \mu(\theta) d\mathbf{x} \rightarrow \sum_{k, \theta} (\ell_i^{\theta}(\mathbf{x}_i^{(k)}) - \ell_j^{\theta}(\mathbf{x}_j^{(k)})) \mathbf{x}_i^{(k)} \pi(\mathbf{x}^{(k)}|\theta) \mu(\theta)$$

finite dimensional but non-convex

Equivalence under Finite Discretization

Optimize over Grid Points

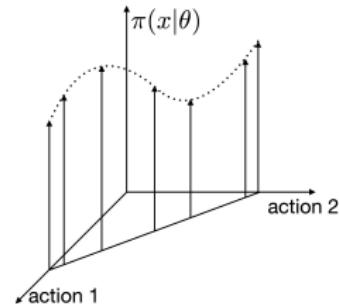
$$\min_{\pi, \{x^{(k)}\}} \sum_{\theta, i, k} x_i^{(k)} \ell_i^\theta(x_i^{(k)}) \pi(x^{(k)} | \theta) \mu(\theta)$$



Equivalence under Finite Discretization

Optimize over Grid Points

$$\min_{\pi, \{x^{(k)}\}} \sum_{\theta, i, k} x_i^{(k)} \ell_i^\theta(x_i^{(k)}) \pi(x^{(k)} | \theta) \mu(\theta)$$



+

- polynomial ℓ

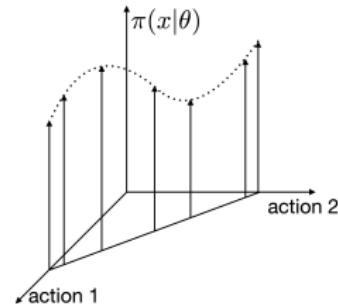
Example

BPR function is 4th order

Equivalence under Finite Discretization

Optimize over Grid Points

$$\min_{\pi, \{x^{(k)}\}} \sum_{\theta, i, k} x_i^{(k)} \ell_i^\theta(x_i^{(k)}) \pi(x^{(k)} | \theta) \mu(\theta)$$



+

- polynomial ℓ

Example

BPR function is 4th order

↓

equivalent poly optimization if $\# \text{ grid pts} \geq (\frac{\text{poly order} + \text{num links}}{\text{poly order} + 1})$

Proof Sketch: Cubature Formula

Tchakaloff Thm

- $\exists \{x^{(k)}\}$ s.t.

$$\int \text{poly}(x) \pi(x) dx = \sum_k \text{poly}(x^{(k)}) \pi(x^{(k)}) \quad \forall \text{ poly}$$

Proof Sketch: Cubature Formula

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- $\exists \{x^{(k)}\}$ s.t.

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Problem of Moments

$$\begin{aligned} & \min_{\pi} \sum_{\theta,i} \int \underbrace{x_i \ell_i^\theta(x_i)}_{\text{moments}} \pi(x|\theta) \mu(\theta) dx \\ \text{s.t. } & \sum_{\theta} \int \left(\ell_i^\theta(x_i) - \underbrace{\ell_j^\theta(x_j)}_{\text{moments}} \right) x_i \pi(x|\theta) \mu(\theta) dx \leq 0 \quad \forall j \end{aligned}$$

Information Design as Polynomial Optimization

optimize over sufficiently many grid pts + polynomial ℓ



polynomial optimization (non-convex)

Information Design as Polynomial Optimization

optimize over sufficiently many grid pts + polynomial ℓ
↓
polynomial optimization (non-convex)

- arbitrarily tight lower bound by hierarchy of semi-definite relaxations [L01]
- low hierarchies sufficient in practice
 - e.g., first hierarchy is tight for 2 links and affine ℓ

[L01]: J. B. Lasserre, “Global optimization with polynomials and the problem of moments”, *SIAM Journal on Optimization*, 2001.

Extension to Heterogeneous Agents

- only a fraction participate in information design

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$$\min_{\pi, \mathbf{y}} \sum_{\theta, i} \int (x_i + \mathbf{y}_i) \ell_i^\theta(x_i + \mathbf{y}_i) \pi(x|\theta) \mu(\theta) dx$$

$$\text{s.t. } \sum_{\theta} \int (\ell_i^\theta(x_i + \mathbf{y}_i) - \ell_j^\theta(x_j + \mathbf{y}_j)) x_i \pi(x|\theta) \mu(\theta) dx \leq 0 \quad \forall i, j \quad (\text{obedience})$$

Extension to Heterogeneous Agents

- only a fraction participate in information design

$$\min_{\pi, \mathbf{y}} \sum_{\theta, i} \int (x_i + \mathbf{y}_i) \ell_i^\theta(x_i + \mathbf{y}_i) \pi(x|\theta) \mu(\theta) dx$$

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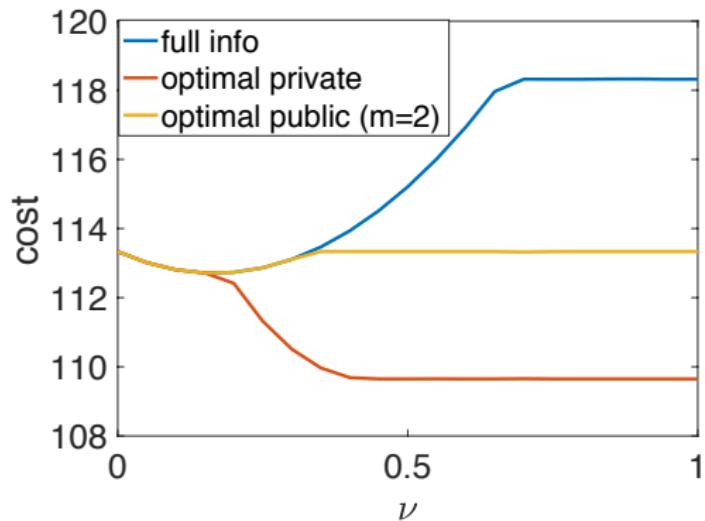
$$\sum_{\theta} \int (\ell_i^\theta(x_i + y_i) - \ell_j^\theta(x_j + y_j)) y_i \pi(x|\theta) \mu(\theta) dx \leq 0 \quad \forall i, j \quad (\text{Nash})$$

Public vs Private Signals

- Private: optimal social cost **non-increasing** with participation rate

Public vs Private Signals

- Private: optimal social cost **non-increasing** with participation rate
- Public: optimal social cost **may increase** with participation rate



Towards Repeated Setting

Obedience involves agents computing $\sum_{\theta} \int \ell_i(x_i) \underbrace{x_i \pi(x|\theta) \mu(\theta)}_{\text{posterior}} dx$

Towards Repeated Setting

Obedience involves agents computing $\sum_{\theta} \int \ell_i(x_i) \underbrace{x_i \pi(x|\theta) \mu(\theta)}_{\text{posterior}} dx$

obedience constraint

- Bayesian calculation ✗
- long evaluation phase ✗
- requires knowledge of π, μ ✗

Towards Repeated Setting

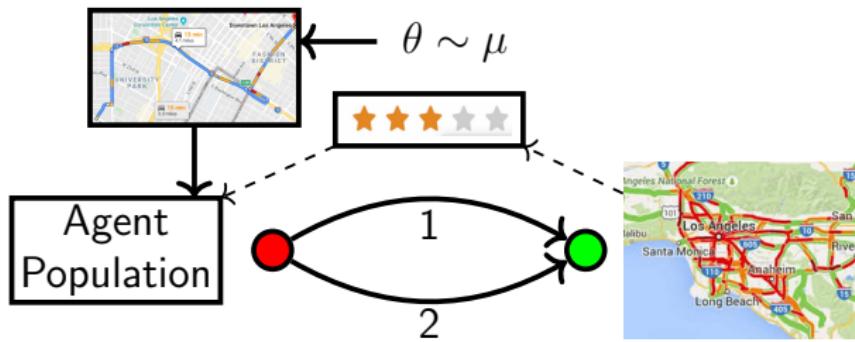
Obedience involves agents computing $\sum_{\theta} \int \ell_i(x_i) \underbrace{x_i \pi(x|\theta) \mu(\theta)}_{\text{posterior}} dx$

obedience constraint

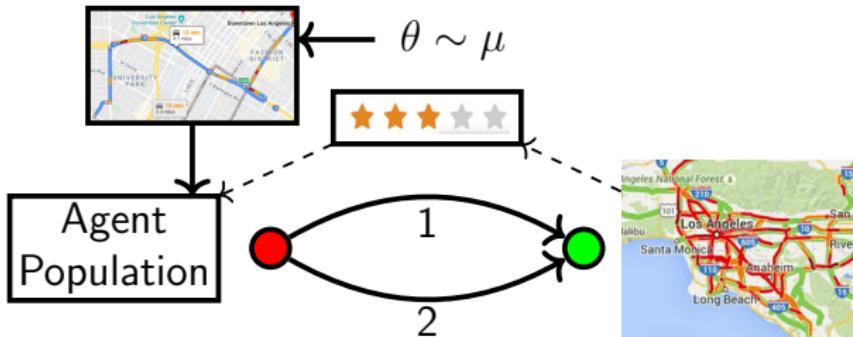
“empirical” obedience?

- Bayesian calculation ✗
 - long evaluation phase ✗
 - requires knowledge of π, μ ✗
-
- myopic decision ✓
 - dynamic obedience ✓
 - no knowledge of π, μ ✓

A Dynamic Model of Obedience



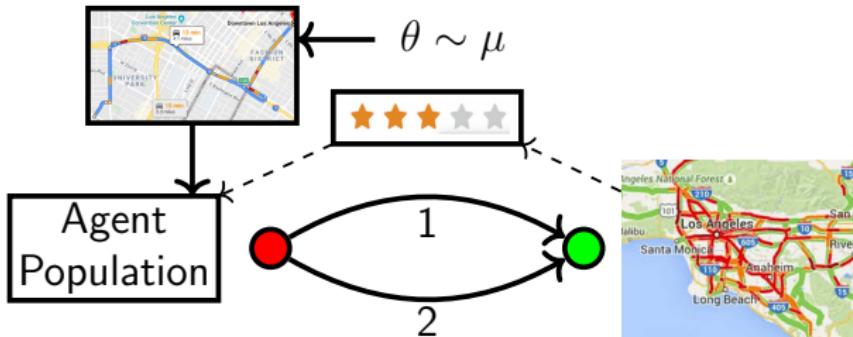
A Dynamic Model of Obedience



Participating Agents

- action space $\equiv \{\text{obey, do not obey}\}$
- $\Pr(\text{obey}) \sim \underbrace{\sum_{\text{agents, stages}}}_{\text{socio-temporal}} \frac{\ell_{\text{reco}} - \ell_{\text{alternate}}}{\text{regret}}$

A Dynamic Model of Obedience

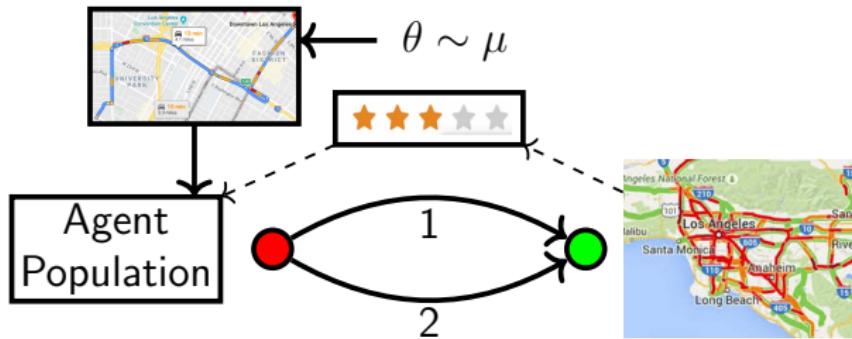


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	alternate			
reco	0	*	...	*
	*	0	...	*
	:	:	:	*
	*	*	...	0

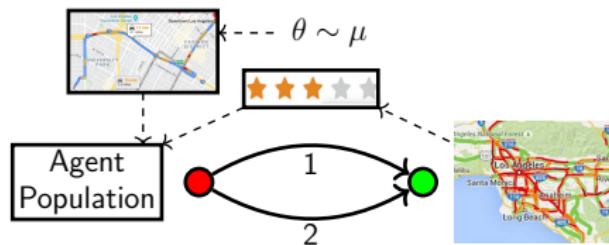
A Dynamic Model of Obedience



Non-participating Agents

- strategy space $\equiv \{1, 2\}$
- best response to $\Pr(\text{obey})$

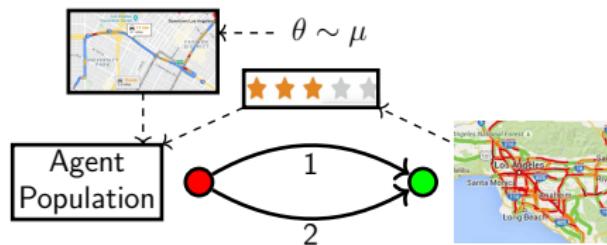
A Dynamic Model of Obedience



Participating Agents: ν fraction

- $\underbrace{\Pr(\text{obey})}_{q} \propto \star \star \star \star \star \xleftarrow{\text{projection}} (\text{socio-temp}) \text{ avg regret}$

A Dynamic Model of Obedience

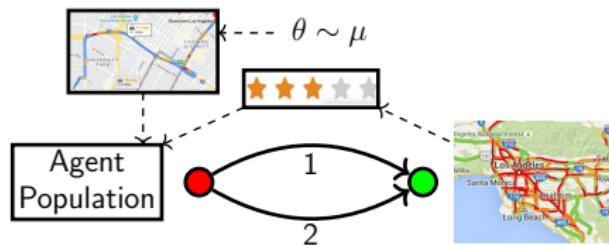


Participating Agents: ν fraction

- $\Pr(\text{obey}) \propto$ projection (socio-temp) avg regret

$$\underbrace{x(q)}_q = \nu q \underbrace{\pi(.|\theta)}_{\text{actual link flows}} + \nu (1 - q) \underbrace{P^T}_{\text{non-obedient routing}} \pi(.|\theta)$$
reco link flows
non-obedient routing

A Dynamic Model of Obedience



Participating Agents: ν fraction

$$\bullet \underbrace{\Pr(\text{obey})}_{q} \propto \underbrace{\star \star \star \star \star}_{\text{actual link flows}} \xleftarrow{\text{projection}} \underbrace{\pi(.|\theta)}_{\text{reco link flows}} + \nu(1-q) \underbrace{P^T}_{\text{non-obedient routing}} \pi(.|\theta)$$

Non-participating Agents: $1 - \nu$ fraction

- $y(q)$: Bayes Nash flow for given q

Model Summary

Key Assumptions

- $\Pr(\text{obey}) \propto \star\star\star\star\star$
- avg regret $\xrightarrow{\text{projection}} \star\star\star\star\star$
- \exists non-obedient routing matrix P

Model Summary

Key Assumptions

- $\Pr(\text{obey}) \propto \star\star\star\star\star$
- avg regret $\xrightarrow{\text{projection}} \star\star\star\star\star$
- \exists non-obedient routing matrix P

Contrast with Literature: [HM:00,FV:97,...]

- finite agents
- action space $\equiv \{\text{route 1, route 2, ...}\}$
- agent's action based on *personal* regret associated with action pairs

[HM:00]: S. Hart and A. Mas-Colell, "A simple adaptive procedure leading to correlated equilibrium", 2000.

[FV:97]: D. P. Foster and R. V. Vohra, "Calibrated learning and correlated equilibrium", 1997.

Experiment Procedure

Average Rating: 5.0 ★

Scenario 1

The map displays a green area representing a park or protected land. Three blue lines represent different routes: Route 1 (top), Route 2 (middle), and Route 3 (bottom). Labels on the map include Bryn Mawr, Haverford College, and Fairmount Park.

Select a Route:
 Route 1 Route 2 Route 3
Select

Route 1

Minutes	Percent
5	10
11	25
14	5
16	40
20	15

Route 2

Minutes	Percent
12	25
16	5
17	15
20	40
25	10

Route 3

Minutes	Percent
5	5
16	45
20	25
24	15

Experiment Procedure

Average Rating: 5.0 ★

Scenario 1

1: Traffic Network & Routes

The map displays a traffic network in Philadelphia, highlighting three specific routes. Route 1 is a blue line starting from the top left and ending at Fairmount Park. Route 2 is an orange line starting from the top left and ending at Fairmount Park. Route 3 is a red line starting from the bottom right and ending at Fairmount Park. The map also shows various neighborhoods and landmarks like Haverford College, Bryn Mawr, and the Philadelphia Museum of Art.

Route 1

Route 2

Route 3

Select a Route:

Route 1 Route 2 Route 3

Select

Percent

Minutes

Route 1

Minutes	Percent
5	10
11	25
14	5
16	35
20	5
20	20

Route 2

Minutes	Percent
12	25
16	5
17	10
17	20
20	35
25	5

Route 3

Minutes	Percent
5	10
16	45
20	25
24	15

Experiment Procedure

Average Rating: 5.0 ★

Scenario 1

1: Traffic Network & Routes

The map displays a traffic network in the Philadelphia area, including major roads like I-95, I-76, and I-476. Three specific routes are highlighted: Route 1 (blue line) starts near Bryn Mawr and ends at Fairmount Park; Route 2 (orange line) starts near Haverford College and ends at Fairmount Park; Route 3 (yellow line) starts near Bryn Mawr and ends at the Philadelphia Museum of Art. Various neighborhoods and landmarks are labeled across the map.

Select a Route:
 Route 1 Route 2 Route 3
Select

2: Travel Time Forecasts

Route 1

Minutes	Percent
5	8
11	25
14	5
16	40
20	15

Route 2

Minutes	Percent
12	25
16	5
17	20
20	40
25	10

Route 3

Minutes	Percent
5	5
16	45
20	25
24	20

Experiment Procedure

3: Average Rating

Average Rating: 5.0 ★

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1: Traffic Network & Routes

Select a Route:

Route 1 Route 2 Route 3

Select

2: Travel Time Forecasts

Route 1

Minutes	Percent
5	10
11	25
14	10
16	40
20	15

Route 2

Minutes	Percent
12	25
16	5
17	15
20	40
25	5

Route 3

Minutes	Percent
5	10
16	45
20	25
24	15

Experiment Procedure

3: Average Rating

Average Rating: 5.0 ★

1: Traffic Network & Routes

The map displays a traffic network for the Philadelphia area, specifically focusing on the northern part. Three routes are highlighted: Route 1 (blue line) starts near Bryn Mawr and ends at Fairmount Park; Route 2 (orange line) starts near Haverford College and ends at Fairmount Park; Route 3 (green line) starts near Bryn Mawr and ends at Fairmount Park. The map also shows various neighborhoods like Bryn Mawr, Haverford, and West Philadelphia, along with major roads and landmarks.

4: Menu

Select a Route:

Route 1 Route 2 Route 3

Select

2: Travel Time Forecasts

Route 1

Minutes	Percent
5	10
11	25
14	10
16	40
20	10

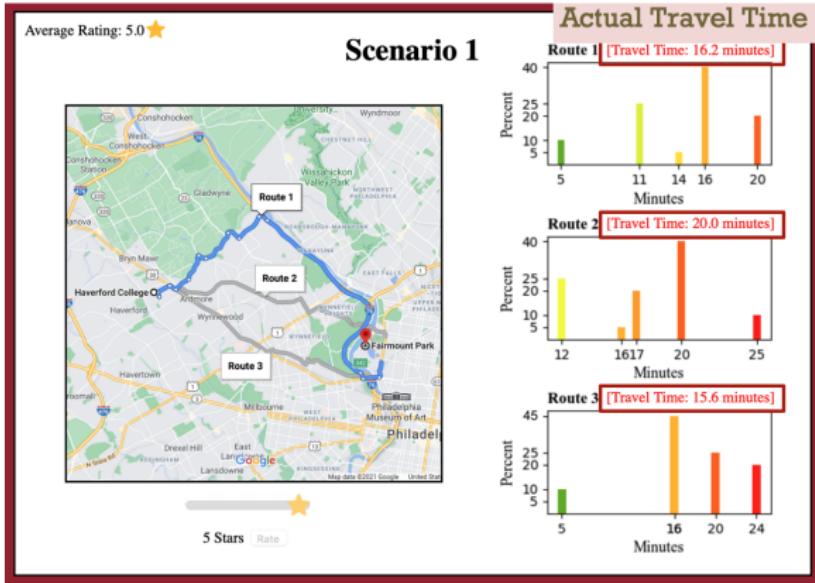
Route 2

Minutes	Percent
12	25
16	10
17	20
20	40
25	10

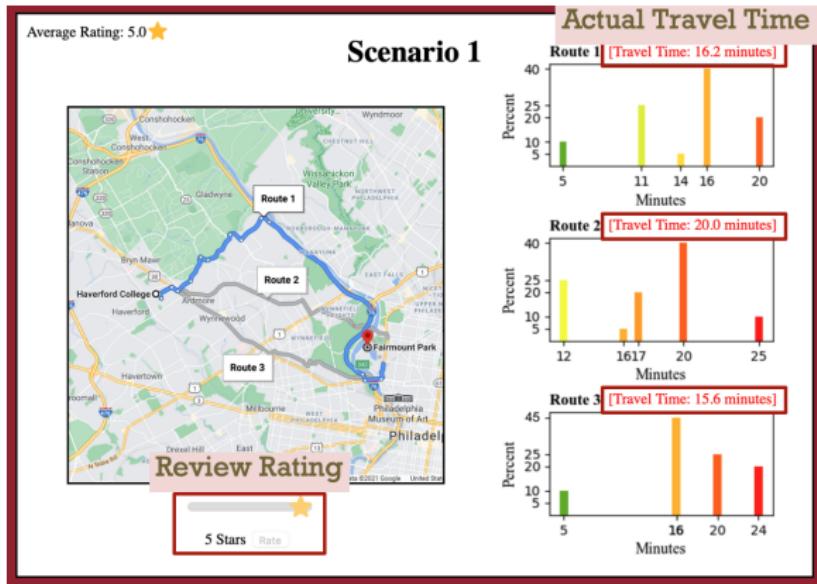
Route 3

Minutes	Percent
5	10
16	45
20	25
24	15

Experiment Procedure



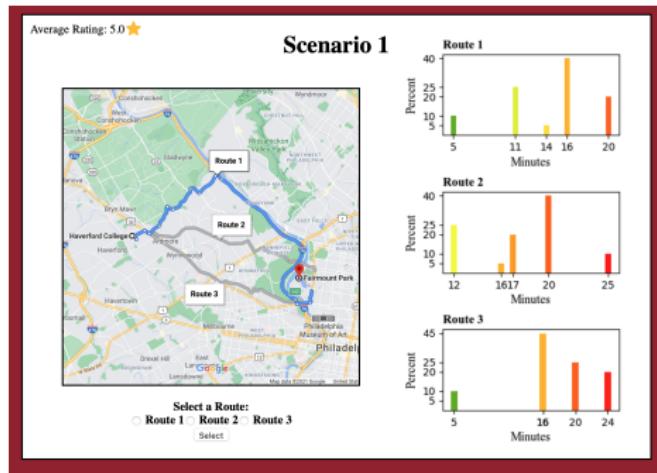
Experiment Procedure



- $\nu = 1$
- 100 scenarios/participant \times 33 participants

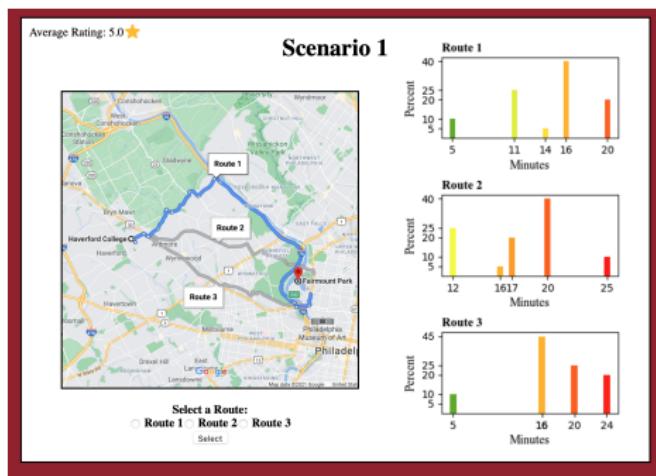
Connecting Experiment to Theory

- $\underbrace{\star\star\star\star\star}_{\text{rating}}(t+1) = \frac{t}{t+1} \text{rating}(t) + \frac{1}{t+1} \cdot \text{avg historical reviews}$
- $x = \text{rating} \cdot \pi(\cdot | \theta) + (1 - \text{rating}) \hat{P}^T \pi(\cdot | \theta)$



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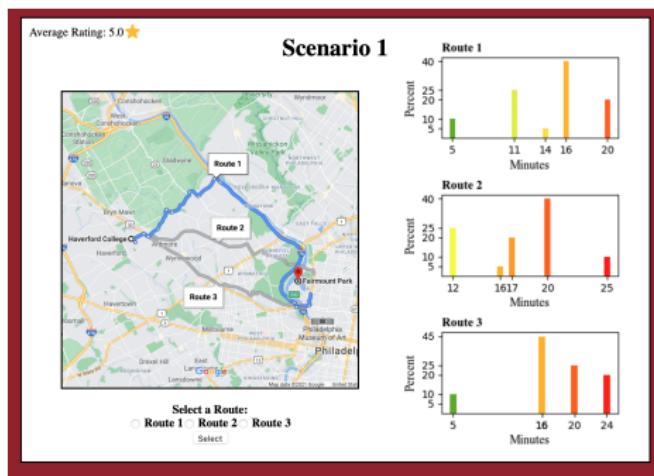


Hypotheses

- $\Pr(\text{obey}) \propto \text{rating} ?$
- convergence of \hat{P} ?

Connecting Experiment to Theory

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Hypotheses

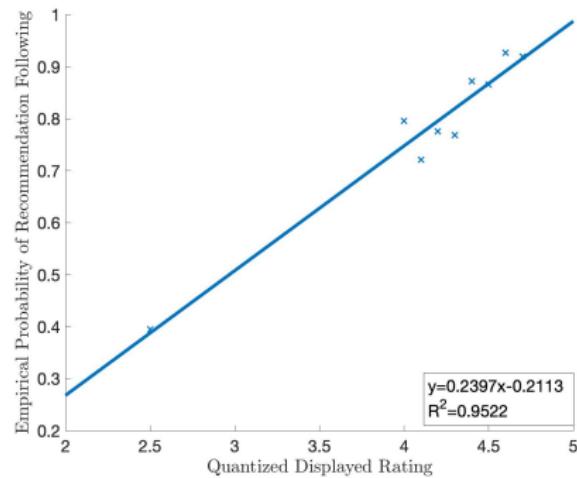
- $\Pr(\text{obey}) \propto \text{rating} ?$
- convergence of \hat{P} ?
- $\text{rating} \propto -\text{avg regret}?$
- $\text{rating}(t) \rightarrow \star\star\star\star\star ?$

Obedience vs Displayed Rating

$\Pr(\text{obey}) \propto \text{rating} ?$

Obedience vs Displayed Rating

$\text{Pr}(\text{obey}) \propto \text{rating ?}$



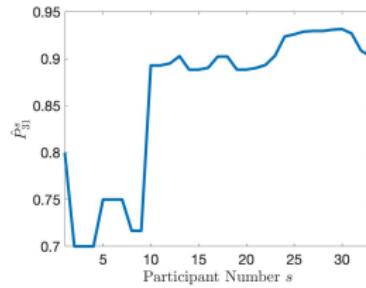
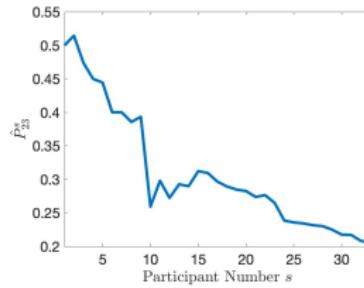
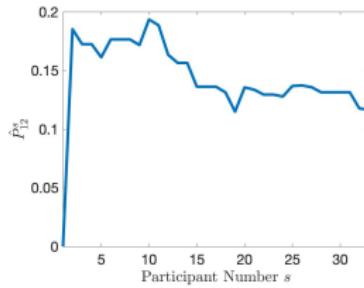
strong positive correlation ($R^2 = 0.9522$)

Long Run Behavior of \hat{P}

\hat{P} converges with increasing s ?

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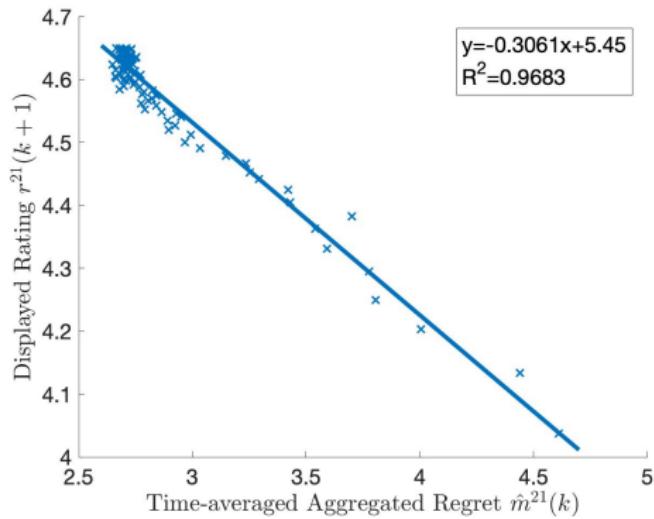


Displayed Rating vs Average Regret

rating $\propto -\text{avg regret}$?

Displayed Rating vs Average Regret

rating $\propto -\text{avg regret ?}$



Strong negative correlation ($R^2 = 0.9683$) for an unbiased individual

Convergence

For all obedient π , $q(t) \rightarrow 1$ a.s.



Convergence

For all obedient π , $q(t) \rightarrow 1$ a.s. 

Proof Sketch

$$\bullet \underline{\nu = 1} : \text{ avg regret} \rightarrow \underbrace{\sum_{\theta} \mu(\theta) \pi^T(x|\theta) (I - P)}_{\leq 0: \text{ obedience constraint}} \overbrace{- \underbrace{\dots}_{\geq 0: \text{ quadratic form}}}^{\text{all obey}}$$

Convergence

For all obedient π , $q(t) \rightarrow 1$ a.s. 

Proof Sketch

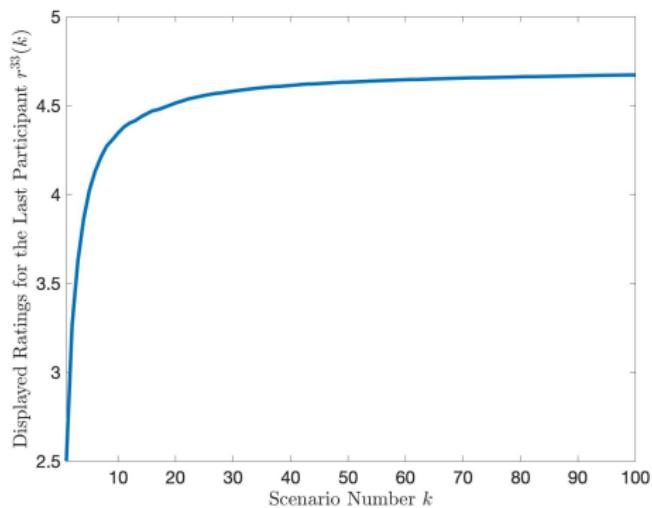
- $\nu = 1$: avg regret $\rightarrow \sum_{\theta} \mu(\theta) \pi^T(x|\theta) (I - P) \overbrace{\ell^*}^{\text{all obey}} - \underbrace{\dots}_{\geq 0: \text{quadratic form}}$
 $\leq 0: \text{obedience constraint}$
- $\nu < 1$:
 - \exists avg regret subsequence $\rightarrow \leq 0$
 - deviation of parent sequence is sufficiently small asymptotically

Long Run Behavior of the Displayed Rating

rating(t) → ★★★★?

Long Run Behavior of the Displayed Rating

rating(t) → ★★★★★ ?

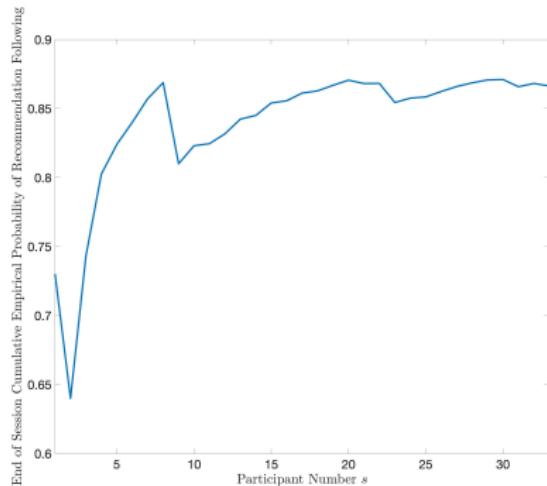


Long Run Behavior of $\Pr(\text{obey})$

$\boxed{\Pr(\text{obey}) \rightarrow 1 ?}$

Long Run Behavior of $\Pr(\text{obey})$

$\Pr(\text{obey}) \rightarrow 1 ?$



- $\Pr(\text{obey}) \rightarrow 0.87$
- consistent with $\Pr(\text{obey}) \rightarrow 0.91$ from the regression model

Summary

Key Takeaways

- Information design for non-atomic games as *finite* optimization
- A learning model for correlated equilibrium in non-atomic games
- Empirical evidence and analytical convergence

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- Information design for non-atomic games as *finite* optimization
- A learning model for correlated equilibrium in non-atomic games
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Ongoing Work

- Approximation algorithms
- Learning model for non participating agents
- Other decision-making settings: e.g., scheduling

Relevant Publications

- Y. Zhu, K. Savla, “*Information Design in Non-atomic Routing Games with Partial Participation: Computation & Properties*”, IEEE Transactions on Control of Network Systems, 2022.
- Y. Zhu, K. Savla, “*Convergence in a Repeated Non-atomic Routing Game with Partial Signaling*”, arxiv 2022.
- Y. Zhu, K. Savla, “*An Experimental Study on Learning Correlated Equilibrium in Routing Games*”, arxiv 2022.

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<https://viterbi-web.usc.edu/~ksavla/publications.html>

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